Probabilistic models for axially loaded bored shafts

Modèles probabilistes pour des fuits forés avec charge axiale

F.OBONI, Dr, President, Oboni & Associates, Inc., Morrens, Switzerland

SYNOPSIS:This paper considers the probabilistic evaluation of the performances of a single slender elastic element such as a bored pile installed in a multi-layered soil system without noticeable displacement of the soils. The element is loaded with axial forces applied at its head, near the ground surface. The principal parameters of the study are considered as stochastic variables. The evaluation of the vertical and horizontal stresses and the shear resistance along the shaft are based on classical soil mechanics. The proposed procedure is used to evaluate the performance of an idealized deep foundation.

1. INTRODUCTION

In 1983 an investigation concerning the various aspects of the analysis of deep foundations was carried out. The results (Focht & O'Neil,1985) dramatically point out a generalized lack of confidence towards classical methods of analysis and the desire of practitioners to use basic soil mechanics. The number of existing empirical methods shows very well the difficulties that still exist in the modelisation of the load transfer from the shaft to the soil (Chaplin,1977).

The aforementioned investigation yields the formulation of a number of general interest questions such as the following:

1)Is it possible to formulate simple models allowing to forecast behaviour of a deep foundation in a more realistic way than with the help of classical methods?

2)How can the factor of safety be replaced by a more rational set of safety qualifiers?

3)Is it possible to take into account the scatter of the pertinent parameters?

4)How is the reliability of a bored shaft influenced by the increasing level of information acquired on a particular site?

5)How can we solve in a simple and pragmatic way the problem of the interface between the shaft and the soil?

This paper deals with probabilistic models (Oboni,1988) that attempt to reply to the aforementioned questions. The models apply to single slender elastic elements embedded in a multi-layered soil system loaded by an axial force at their head. Only the elements installed without noticeable displacement of the soil are taken into account. The stress-strain relationship for each of the soil layers is the perfectly rigid-plastic Mohr-Coulomb failure criterion, or a model taking into account the progressive mobilization of the shear resistance as well as the post failure behaviour, titled the elastoplastic model.

2. PHENOMENOLOGICAL ASPECTS

Skempton (1959), Whitaker & Cooke (1966), Reese & al. (1969), O'Neil & Reese (1972), Biarez (1977) and Broms (1982) have effected, among others, numerous observations on the behaviour of shafts during load tests, which can be roughly summarized as follows:

- The axial stresses on a loaded pile decrease in function of the depth.
- The lateral friction is mobilized in function of the displacement under loading.
- The shaft tip is stressed when the applied load is high enough to mobilize an important fraction of the shaft resistance.
- The shaft resistance is generally mobilized for displacements of the head not exceeding 2% of the diameter.
- The tip resistance is generally mobilized for displacements approximatively equal to 10% of the diameter.
- The shearing occurring along the shaft surface can be considered as a drained shear (Chandler & Martins,1982).
- Only a very thin cylinder of soil around the shaft is actually deformed when the load is applied (Chandler & Martins, 1982, Potts & Martins,1982).

3. CLASSICAL METHODS

Classical methods of analysis of the bearing capacity implicitly assume the independence of the resistance of the shaft and the resistance of the tip. Since these two resistances can be fully mobilized for different displacements, it would be necessary to compute them with different values of the parameters, especially when dealing with dense or overconsolidated soils. The use of partial factors of safety can be helpful in dealing with this, but certainly does not help in understanding the actual phenomenon.

Elastic settlement analysis methods became quite popular (Poulos & Davis,1968, Poulos & Mattes, 1969, Mattes & Poulos,1969) because they allow the analysis at a preliminary design stage and can be used by means of design charts. Methods based on the empirical load transfer analysis were less used because of the necessity of transfer functions that require costly investigations. Finally, the methods based on the theoretical load transfer analysis demand the determination of costly parameters (Kraft, Ray & Kagawa,1981).

4. CHOICE OF THE PERTINENT STOCHASTIC VARIABLES

In order to choose the pertinent stochastic variables, sensitivity analyses on the variances of the functions of these variables have been performed. Finally, six parameters have been chosen because their variability had the biggest impact on the variance of the functions in which they enter.

a) Applied load, P_o.
b) Depth of the water table, h_w.
c) Overconsolidation ratio, OCR (each layer).
d) Elastic modulus, E, of the soil below tip.
e) Shear effective parameter, c' (each layer).
f) Shear effective parameter, ϕ' (each layer).

The unit weight γ, the shaft diameter d, and the elastic modulus of the shaft E_p, have been considered as deterministic variables, which represents a considerable economy in computational time.

5. DEVELOPMENT OF A RIGID-PLASTIC MODEL

A first model has been proposed based on the rigid-plastic behaviour of the soils in the multi-layered system and on an elastic reaction of the soils below the shaft tip. Although basic classical theories are used, the present study differs from known state of the art methods because a new transfer method based on the local mobilization of the shear resistance along the shaft has been developed for the anticipated ultimate load and for the evaluation of the load-deformation behaviour. This model is based on simple equations of limit equilibrium, solved in probabilistic terms by using the Rosenblueth's (1975) two point estimate method (PEM) for independent skewed variables.

5.1 Analysis of the shaft

The shaft is discretized into n elements, numbered from 1 at the soil surface to n at the tip. This discretization is carried out in such a manner that the interstrata correspond with an inter-element joint and the elements are approximatively of the same size. If the soil can be considered homogeneous over a large depth, then spatial variability techniques leading to the determination of the autocorrelation length can be used to define the discretization mesh.

The analysis is carried out from element 1 down to n in the following manner.

The stresses acting normally to the shaft σ_{11} and the lateral resistance τ'_{11} are evaluated for the first element (equations 1 to 6).

The Meyerhof (1975) standards are used, namely:

σ_v' = effective vertical stress due to depth
ϕ', c' = effective shear parameters
$K = \alpha \cdot K_{o,OCR}$

For the reduction factor α, Meyerhof suggests 0.75, whereas we use 1.00, but one has to consider in the determination of $K_{o,OCR}$, the formulation of Mayne & al. (1982) is applied, which leads to significantly lower values of $K_{o,OCR}$ than those used by Meyerhof.

$$\sigma_{vi}=\sum_{k=1}^{i} (\gamma_k \cdot z_k) \tag{1}$$

$$u_i=(\sum_{k=1}^{i}(z_k)-h_w)\cdot \gamma_w \tag{2}$$

$$\sigma'_{vi}=\sigma_{vi}-u_i \tag{3}$$

Following the Mayne & al. (1982) simplified relationship for $K_{o,OCR}$ one has:

$$K_{o,OCR} =(1-\sin\phi')\cdot OCR^{\sin\phi'} \tag{4}$$

$$\sigma'_{11}=K\cdot \sigma'_v \tag{5}$$

$$\tau'_{11}=\sigma'_{11}\cdot \tan\phi_1'+c_1' \tag{6}$$

$$C_1=\tau'_{11}\cdot p_1\cdot l_1 \tag{7}$$

where:

τ'_{11} = shear stress along the first element
p_1 = perimeter of the element section
l_1 = length of the considered element

The evaluation of the capacity (equation 7) using the rigid-plastic method is reliable given certain restrictions such as a) soils without a peak value in their stress-strain relationship (i.e. to use a peak-valued soil one would literally have to "guess" its resistance not knowing actual deformations) and, b) soils with maximum resistances obtained over very small displacements. For most slender shafts the displacement/deformation is large enough in the upper portion to reach the maximum resistance while, in the lower portion of the shaft, only a fraction of this resistance is mobilized.

The demand D_1, equal for the first element to the applied load P_o.

The safety margin of the first element is

$$SM_1=C_1-D_1 \tag{8}$$

The SM is a force with a positive sign if the capacity overcomes the demand, negative if the reverse is true. One can define the SM as a performance function of the structure and the equation SM=0 as the performance equation of the considered system.

Equation 9 defines the reliability index β, given the first two moments of SM.

$$\beta=\overline{SM}/\sigma_{SM} \tag{9}$$

The transfer force to the element i+1 is defined by equation 10.

$$P_1=D_1-C_1=-SM_1 \tag{10}$$

The analysis is performed for the second element of the shaft and the operation continued for all the elements down to n with the equations 11 to 15.

$$D_i=P_{(i-1)} \tag{11}$$
$$\tau'_{i1}=\sigma'_{i1}\cdot\tan\phi_i'+c_i' \tag{12}$$
$$C_i=\tau\cdot i_1\cdot p_i\cdot l_i \tag{13}$$
$$SM_i=C_i-D_i \tag{14}$$
$$P_i=-SM_i \tag{15}$$

The intensity of the transfer force P_i decreases for each element towards the tip as shown in Figure 1.

It is interesting to note that in the preceding formulas there is no restriction on the sign of the transfer force thus leading to results that could be physically inconsistent with the meaning of the transfer force. As the evaluation of the moments of the functions of stochastic variables is performed by using a second moment method (Rosenblueth,1981), it is necessary to make assumptions on the distribution of the force P_i.

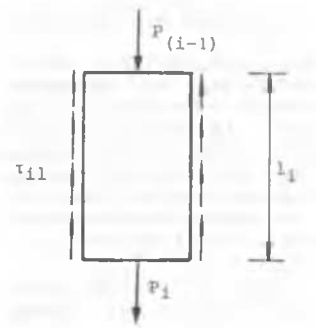

Figure 1. Decrease of the transmitted force due to the load transfer to the soil.

As P_i is the "mirror image" of SM_i it would be a priori correct to use for P_i the same distribution as for SM_i. Unfortunately, since the normal distribution does not have finite bounds, its use can lead to erroneous results if the variable should be physically limited to zero and its expected value is low.

In order to obtain physically reasonable results it is therefore necessary to use computational tricks such as a "cut-off function" that transforms the normal distribution (with a non-negligible negative branch) into a distribution with a lower bound equal to zero (Figure 2).

5.2 Analysis of the tip
At the tip, the remaining transferred force P_n has to be compared with the available resistance P_{bu} which can be evaluated by using the classical formulas.

$$P_{bu}=A_b\cdot(\sigma'_{vb}\cdot N_q)-W' \tag{16}$$

where:

A_b= tip surface
σ'_{vb}= effective stress at the tip depth
N_q= shape factor
W'= bouyant weight of the shaft

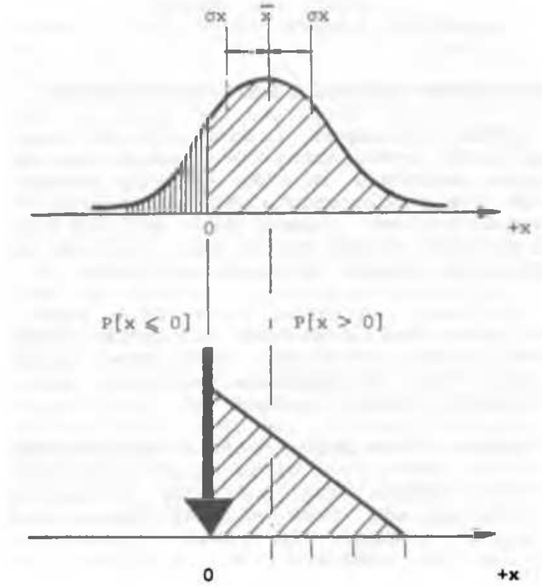

Figure 2. Cut-off function for compressive forces distributions.

If one assumes that $W'\approx A_b\cdot\sigma'_{vb}$ the following is obtained:

$$P_{bu}=W'\cdot(N_q-1) \tag{17}$$

The available tip resistance is evaluated by assuming that the shape factor N_q is

$$N_q=10^{(0.08\phi'-0.9)} \tag{18}$$

5.3 Analysis of deformations
The deformation analysis is based on the evaluation of the elastic shortening dl_i of the elements with cross section A_i, stressed by the normal load N_i.

$$dl_i=\frac{N_i\cdot l_i}{A_i\cdot E_p} \tag{19}$$

The settlement of the soil mass below the tip is carried out (Coyle & Reese,1966 and Kraft & al.,1981) with the help of Boussinesq's theory.

The settlement Φ_b of a circular plate with a diameter d, installed in depth in a soil mass characterized by a modulus E and a Poisson's coefficient of μ, loaded by a force P can therefore be expressed by:

$$\Phi_b=\frac{P\cdot(1-\mu^2)}{2\cdot d\cdot E} \tag{20}$$

where, following the elastic theory:

$$\mu=\frac{K_o}{1+K_o} \tag{21}$$

At each level along the shaft z, the settlement Φ_z is equal to the cumulative of the elastic shortenings dl_i of the elements n to i, plus the tip settlement Φ_b, namely;

$$\delta_s = \sum_{k=i}^{n} \delta_k + \Phi_b \qquad (22)$$

6. DEVELOPMENT OF THE ELASTOPLASTIC MODEL

The model developed is based on an iterative cycle that re-evaluates the capacity and demand for the elements of the shaft by taking into account the displacements. These initial values of capacity and demand are derived from the rigid-plastic method using peak values of the shearing resistance for each soil layer.

The analyses performed with this model have shown that the iterative algorithm converges generally well, but as with most iterative methods, some convergence problems can occur given particular situations.

6.1 Choice of the stress-strain relationship

Since the model can be developed independently from the definition of the stress-strain relationship and that this definition was not the major concern in this research, the selection of the bi- and tri-linear models already developed in the literature (Murff,1980), was made taking into account the simplicity as a major criterion. These models were then transposed into probabilistic terms (Figure 3).

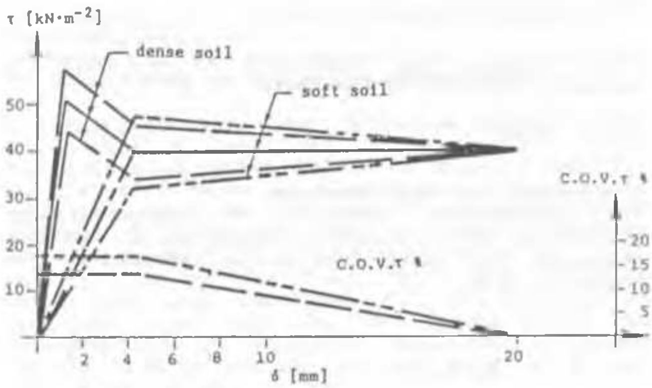

Figure 3. Transposition of Murff's bi- and tri-linear models into probabilistic terms.

A three point estimate method useful for the evaluation of the moments of functions of stochastic variables was developed for this study because it can be shown that the discretization by two point estimates can yield erroneous results when applied to relationships similar to the stress-strain function of dense soils. The three point estimate method for symmetrical independent variables has been presented by the author for publication.

6.2 Analyses of results

Some of the first analyses using the elastoplastic model have shown that the probabilistic approach smoothens the bi- and tri-linear relationships and that if the displacements are very scattered a significant decrease of the average shear resistance can be observed for a wide range of average displacements.

Moreover, it was remarked that the elastoplastic method yields greater displacements than the rigid-plastic method and that an increase of the scatter of the elastic modulus of the soils under the tip results in an augmentation of the expected settlements at the shaft head.

A comparison carried out between the proposed method and the elastic method of Poulos & Davis,1980 showed good agreement of the results.

7. CONVERGENCE ANALYSIS

Extensive analyses have been carried out in order to determine the influence of the discretization on results yielded by the developed method. It is observed that the model has a sensitivity comparable to other classical methods routinely used in geotechnical engineering, that is, as long as a minimum 15 to 20 elements are used, the results do not vary.

8. SAFETY QUALIFIERS AND DESIGN

As a result of the developed models, it is possible to make some comments relative to the qualification of the safety for bored shafts.

As previously pointed out, the lower portion of a long shaft can be stressed by negligible average transfer forces at the level of the service load. In these cases the classical concept of factor of safety obtained by combining tip and shaft resistance under the implicit assumption of their independence and dividing this value by the service load can lead to fallacious interpretations.

In these cases it would be more correct to introduce the notion of Supplementary Load Factor (SLF) defined as the smallest value between the values $F_b=P^*_b/P_s$ (the structural factor of safety) and F_{pa}=the ratio of the loading giving a failure probability p at the tip and P_s.

where:

P^*_b = ultimate load of the reinforced concrete section of the shaft
P_s = service load of the shaft

As a supplement to the SLF, the effective length L^* concept can be introduced. This parameter defines the length of the shaft that is stressed by non-negligible forces when the head of the shaft is loaded with the service load. If $L>L^*$ then the shaft can be considered as long.

One can then define another indicator called the factor of effectiveness $F_e=L^*/L$ of the shaft. In an economic design the F_e should be kept as near as possible to unity, being understood that the settlement requirements have to be met.

As a quick review of the design criteria that can be developed by using the above mentionned qualifiers one can cite the following:

a) $F_e \approx 1$
b) $F_b \geq$ limiting value for the reinforced concrete section
c) $F_{pa} \approx 1$

The design can therefore be carried out following the methodology defined hereafter:

i) Determination of the service load P_a.
ii) Determination of the minimal cross section necessary to meet structural and installation requirements.
iii) Determination of the theoretical load transfer along the axis of the shaft into a multi-layered system representing the actual soils.
iv) Determination of the length of the shaft so that $F_p \approx 1$, which implies that $L \approx L^*$.

9. SIMULATION OF A LOADING TEST

In order to explore the performances of the proposed models, a loading test was simulated. The geotechnical parameters and the shaft parameters are synthetized in Table 1.

Table 1. Loading test characteristics.

Shaft

length of shaft	= 32 m
number of elements	= 8
length of elements	= 4 m
diameter of shaft	= 1 m
concrete modulus E	= $2.1 \cdot 10^7$ KN·m⁻²

Soils

	μ_x	σ_x	x_{min}	x_{max}
γ (KN·m⁻³)	20	0.5	10	26
OCR ---	1	0.1		
φ'(degrees)	25	2.5	18	33
c'(KN·m⁻²)	20	10	0	50
E (KN·m⁻²)	10000	3500		

Water

	μ_x	σ_x	x_{min}	x_{max}
h_w (m)	1	0.3	0	2

Stress-Strain Relationship

displacement for peak	3 mm
softening	20 % to 40%
displacement for full softening	6 mm

Loadings

Variable with constant C.O.V = 10%

The P_o/Φ_o diagrams in Figure 4 present the classical curvilinear shape of shaft loading tests.

The load-deflection relationship defined by the simulation could be idealized by a tri-linear diagram with the breaking points at approximately P_o=1900 KN and 6000 KN. For the loading stage of P_o=6000 KN, the computed expected value of load transferred to the tip yielded by the rigid-plastic method is 760 KN. It is at loading stages of this magnitude (i.e. 6000 KN for this example) that stresses at the tip are developed in a non-negligible way.

In order to show how the shearing resistance is gradually mobilized along the shaft, Table 2 gives for each loading stage the probability p_{fi} of exceeding the maximum shearing resistance along each shaft element. The results shown were obtained by using the rigid-plastic model.

Table 2. Variation of the probabilities of failure p_{fi}[%] in function of the applied load.

		P_o [KN]							
S	z-z	100	500	1000	2000	3000	4000	5000	6000
1	0-4	2	85	100	100	100	100	100	100
2	4-8	0	3	80	100	100	100	100	100
3	8-12		0	4	97	100	100	100	100
4	12-16			0	30	99	100	100	100
5	16-20				0	45	98	100	100
6	20-24					0	50	95	100
7	24-28						0	40	90
8	28-32							0	60

S = number of the considered element
z-z = depth at the end of the S segment [m]

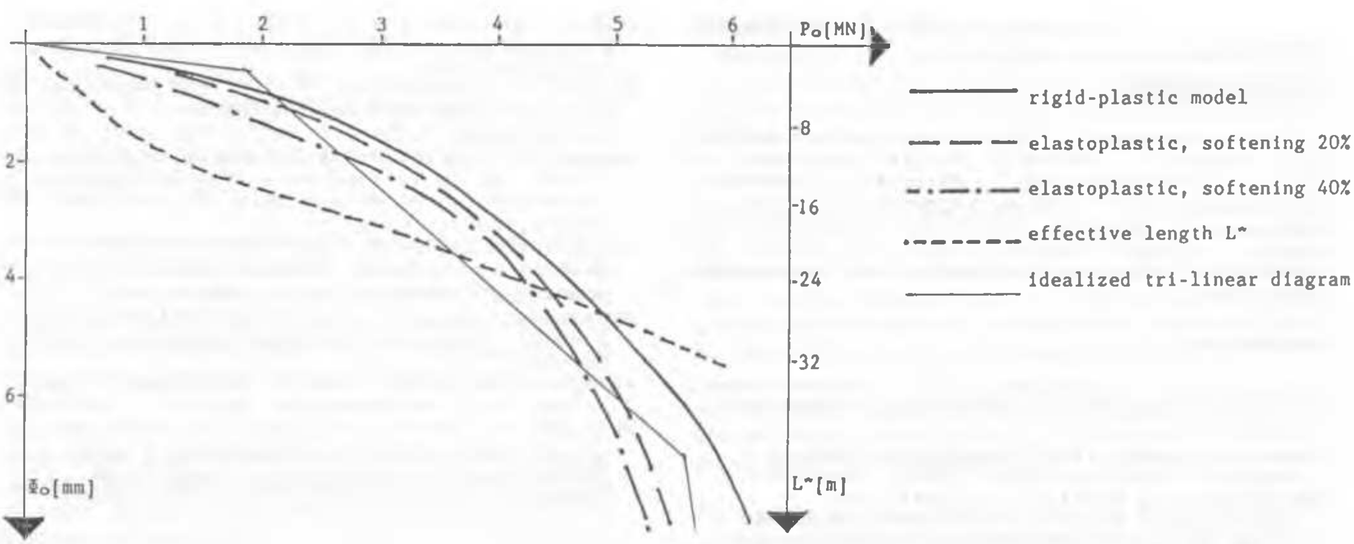

Figure 4. Simulation of a loading test of an idealized bored shaft (see Table 1 for data).

It is interesting to note that when the applied load P_o increases from 5000 to 6000 KN, the probability of failure of the 8th element dramatically increases from ≈0% to 60%.

As a result of analyses carried out by the author (Oboni,1988) it can be inferred that for the majority of the shafts installed in soft soils (stress-strain relationship without peak) and having typically standard proportions and rigidity, the rigid-plastic method of analysis can offer sufficient precision, being understood that, particularly in the lower part of the shaft, the assumptions diverge from reality. It is precisely in this lower region of the shaft that the deformations can be inferior to those necessary to the mobilization of the maximum shear resistance.

If the considered soils exhibit stress-strain relationships characterized by peak values and residual values obtained after large displacements, then the use of the rigid-plastic method should be avoided because of the uncertainty involved in the choice of the shear resistance for each soil at each level of the shaft.

10. CONCLUSIONS

The presented probabilistic models allow to forecast the behaviour of single, axially loaded, bored shafts embedded in a layered soil system. The scatter or the spatial variability of the pertinent parameters can be taken into account so that the influence of the increasing level of information acquired on a particular site can be shown. Moreover, new rational qualifiers of the safety have been defined.

A similar approach has already been presented in the area of slope stability by Oboni & Bourdeau (1983) and Oboni, Bourdeau & Russo (1984).

The proposed models constitute an extension of these previous studies since the compressibility of the element is taken into account and consequently the deformations can be evaluated. It would be possible in the near future to integrate the results of this study with the slope stability analysis thereby increasing its potential.

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