Bearing capacity of double layer base, its probability estimation

La capacité portante dans une base à double couche, l'estimation de sa probabilité

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SYNOPSIS A solution is suggestion that enables to determine the bearing capacity of a foundation formed by horizontal soil layers with various strength characteristics on the limit stress condition theory for soils. The solution is based upon the results obtained by the authors earlier for the case of a homogeneous foundation. The first part of the article deals with a deterministic solution represented by dimensionless coefficients while in the second part the strength characteristics and load are treated as random values.

1. When analysing engineering and geological conditions of a site we often come across layers of soil. The existing standards spesified the methods for calculating bearing capacities of the homogeneous foundations. It means that the same problem for layers of foundation has not been solved to the extent required. The known solutions by J.P.Jiroud, Tran-vo-Nhiem and J.P.Obin (1973) have either restrictions or simplifications, thereby their use in engineering practice is limited. F.M.Shikhiev and P.I.Yakovlev (1977) obtained a solution for the determination of the bearing capacity of a homogeneous foundation based upon the technical theory of the limit stress condition of soils. Later this solution has been generalized (P.I.Yakovlev, 1978,1986) so as to include layers of foundation for the case of change of one of the shear resistance parameters. Analysed below is a problem with the bearing capacity of layers of foundation characterized by the layers of various strength characteristics φ_i - internal friction angles, C_i - cohesions and specific weights γ_i . In doing so, known prerequisities of the limit stress condition theory for soils are preserved. Cohesion is to be accounted for on the basis of A.Caquot's theorem for each layer individually. The problem is treated as plane.

Unlike homogeneous foundation - here it is necessary to analyse the identification part of the problem so as to select the particular case out of the range of possible cases. The identification for the double layer foundation case should be conducted in respect of four probable calculation patterns of foundation failure. Table I shows their peculiarities and contains identification criteria. The adopted notations is:

h_1 - thickness of the upper layer,

B - width of the load strip, while

F_1 and F_2 are to be determined from the function below:

$$F_1 = \sin \vartheta_1 \sin \alpha_1 / \cos \varphi_1 ; \tag{1}$$

$$F_2 = \left[\sin \vartheta_1 \exp(\theta_1 tg \varphi_1)/\cos \varphi_1\right] \sin\left(\frac{\pi}{4} - \frac{\varphi_1}{2}\right); \tag{2}$$

where

$$\vartheta_1 = \frac{1}{2}\left[\arccos \frac{\sin \delta_1}{\sin \varphi_1} - \delta' + \varphi_1\right], \tag{3}$$

$$\theta_1 = \frac{\pi}{4} + \vartheta_1 - \frac{\varphi_1}{2}, \tag{4}$$

$$\alpha_1 = \frac{\pi}{2} + \varphi_1 - \vartheta_1 ; \tag{5}$$

and δ' - inclination angle of the resultant with the vertical allowing for cohesion (Fig.1) The identification of the kinematic diagram version is to be conducted in accordance with Table I. When

$$\mu = \frac{h_1}{B} = \sin \vartheta_1 \exp(\vartheta_1 tg \varphi_1)$$

the enveloping slip surfase passes within the limits of the top layer, therefore the calculation is to be performed on the basis of the formulae submitted by P.I.Yakovlev (1978).

Table I

Calculation case	Identification criteria	Peculiarities of kinematic diagram
1	$\mu < F_1$; $\mu < F_2$;	Zones of minimum and maximum stress condition and Prandtel's zone found partially within the limits of the bottom layer
2	$\mu > F_1$; $\mu > F_2$;	Prandtel's zone found partially within the limits of the bottom layer
3	$\mu > F_1$; $\mu < F_2$;	Zone of maximum stress condition and Prandtel's zone found partially within the limits of the bottom layer
4	$\mu < F_1$; $\mu > F_2$	Zone of minimum stress condition and Prandtel's zone found partially within the limits of the bottom layer

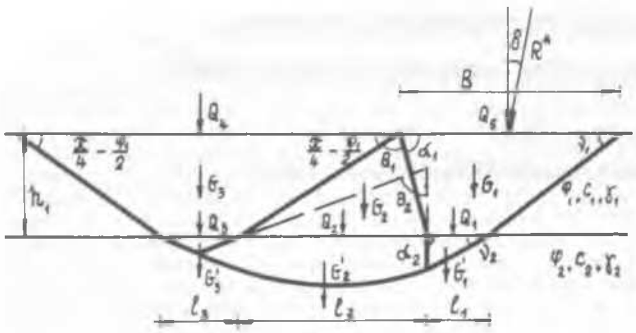

Fig. 1. Design diagram.

When $\varphi_i=\varphi_{i+1}, c_i\neq c_{i+1}, \gamma_i\neq\gamma_{i+1}$ the kinematic part of the problem has no peculiarities in comparison with the solution obtained at $c_i=c_{i+1}, \gamma_i=\gamma_{i+1}$, therefore it can be solved using the results that had been obtained earlier. The same cannot be true for the statistical part of the problem. The change in a jump γ_i results in the modification of the algorithm for calculating their own weights of the limit stress condition zones. A similar change of c_i, in line with A.Caquot's theorem, accounts for additional resultants of cohesion pressures in the layer boundaries wich are to be determined in accordance with the expression

$$Q_{i,i+1}=\ell_\kappa(c_{i+1}\operatorname{ctg}\varphi_{i+1}-c_i\operatorname{ctg}\varphi_i), \tag{6}$$

where ℓ_κ – length of a boundary section i and $i+1$ of soil layers.

Generally, when the characteristics of soil layers are defined, certain pequliarities appear not only in the statistical part, but also in the kinematic part of the problem. The entire solution becomes too clumsy. Therefore, for practical reasons it is abvisable to present the results analytically using dimensionless coefficients of bearing capacity.

The influence of heterogenity of the layers is most substantially pronounced in the first of the variants shown in Table I. For this reason the following functions are consistent with this calculation case of the kinematic diagram. The bearing capacity of the foundation is to be calculated in conformity with the formula

$$R=B^2\left[\gamma_1\mu(\mu N_{\gamma_1}+N'_{\gamma_1})+\gamma_2(\mu^2N_{\gamma_2}+N'_{\gamma_2})\right] + B\left[c_1(\mu N_{c_1}+N'_{c_1})+c_2(\mu N_{c_2}+N'_{c_2})\right]. \tag{7}$$

The coefficients $N_{\gamma_1}, N'_{\gamma_1}, N_{\gamma_2}, N'_{\gamma_2}, N_{c_1}, N'_{c_1}, N_{c_2}, N'_{c_2}$ are dimensionless and are to be determined by the following formulae:

$$N_{\gamma_1}=-p_8p_1-0{,}5u_0p_5-p_2\operatorname{tg}u_3, \tag{8}$$

$$N'_{\gamma_1}=-2K_0p_3\cos u_2-0{,}5(1+f_0)p_8; \tag{9}$$

$$N_{\gamma_2}=0{,}5u_0^2p_5\sin u_2\sin\alpha_2/\sin\theta_2; \tag{10}$$

$$N'_{\gamma_2}=\frac{-0{,}5K_0^2p_3\cos\varphi_2-0{,}5f_0^2\sin\alpha_2\sin\vartheta_2p_4}{\cos\varphi_2-0{,}25d_0^2(R_2^2-1)p_5\operatorname{ctg}\varphi_2}; \tag{11}$$

$$N_{c_1}=u_0p_5\operatorname{ctg}\varphi_1-p_1p_9-2\operatorname{ctg}\varphi_1\operatorname{tg}u_3p_2; \tag{12}$$

$$N'_{c_1}=(f_0p_4-p_6)\operatorname{ctg}\varphi_1; \tag{13}$$

$$N_{c_2}=-u_0p_5\operatorname{ctg}\varphi_2; \tag{14}$$

$$N'_{c_2}=-(2K_0p_3\cos u_2+f_0p_4)\operatorname{ctg}\varphi_2 \tag{15}$$

In the expression (8–15) the following notations is adopted:

$$p_0=\sin(\delta+\vartheta_2-\varphi_2)\sin(\beta_2-\alpha_2-\varphi_2-\frac{\pi}{2}),$$

$$p_1=\cos\varphi_1/\sin(\delta+\vartheta_1-\varphi_1)-\sin(\beta_2-\alpha_1+\varphi_1-\frac{\pi}{2})\cos\varphi_2/p_0,$$

$$p_2=\sin(\beta_2-u_1)\cos\varphi_2/(2p_0\cos u_4);$$

$$p_3=\sin(\beta_2-u_2)\cos\varphi_2/(2p_0\cos u_4);$$

$$p_4=\sin(\vartheta_2-\varphi_2)/\sin(\delta+\vartheta_2-\varphi_2);$$

$$p_5=\sin(\beta_2-\frac{\pi}{2})\cos\varphi_2/p_0;$$

$$p_6=\sin(\vartheta_1-\varphi_1)/\sin(\delta+\vartheta_1-\varphi_1);$$

$$p_7=\cos u_3/\sin(\beta_1-\alpha_1+\varphi_1-90);$$

$$p_8=\frac{1}{2}\left[\sin(\beta_1-u_1)\operatorname{ctg}u_1/p_7+u_0\sin(\beta_1-\frac{\pi}{2})/\sin(\beta_1-\frac{\pi}{2}-\alpha_1+\varphi_1)\right],$$

$$p_9=\sin(\beta_1-u_1)\operatorname{ctg}\varphi_1\operatorname{ctg}u_1/p_7;$$

$$u_0=\operatorname{ctg}u_1+\operatorname{ctg}\alpha_1; \quad u_1=\frac{\pi}{4}-\frac{\varphi_1}{2}; \quad u_2=\frac{\pi}{4}-\frac{\varphi_2}{2};$$

$$u_3=\frac{\pi}{4}+\frac{\varphi_1}{2}; \quad u_4=\frac{\pi}{4}+\frac{\varphi_2}{2}; \quad R_2=\exp(\theta_2\operatorname{tg}\varphi_2);$$

$$f_0=1-\mu(\operatorname{ctg}\alpha_1+\operatorname{ctg}\vartheta_1);$$

$$d_0=\mu u_0\sin u_2/\sin\theta_2+f_0\sin\vartheta_2/\cos\varphi_2;$$

$$K_0=d_0R_2-\mu u_0\sin\alpha_2\sin\theta_2.$$

For calculations by the submitted formalae, it is required to find $\vartheta_2, \alpha_2, \theta_2$ which formulae is identical to (3–5), in these formulae φ_1 should be replaced by φ_2. The solutions for the variants of the kinematic diagrams 2–4 (Table I) are accomplished in the same way.

2. Numerous researth showed that the foundation soils are characterized by a considerable statistical variability of properties, which can be directly accounted for by assuming a probabilistic approah to the analysed problem. The theory of the limit stress condition in its probabilistic aspect (A.V.Shkola, 1975) has been considered before. The analysis of the results obtained when solving the problem for the homogeneous foundations has proved that the bearing capacity aquires a considerable scale under real variability of the foundation soils. The results that support this condition are depicted in Fig.2. Moreover, it is established that the means for the solution must be the method of statistical testing (Monte-Carlo method) as far as the linearization method, as a rule, results in considerable inaccuracy when estimating the reliability level of a base.

Table II

Variant	$\bar{\varphi}^\circ_1$	$\sigma^\circ_{\varphi_1}$	$\bar{\varphi}^\circ_2$	$\sigma^\circ_{\varphi_2}$	δ°	σ°_δ	$\bar{\gamma}_1$ kN/m³	σ_{γ_1} kN/m³	$\bar{\gamma}_2$ kN/m³	σ_{γ_2} kN/m³	$\bar{c}_1$ kPa	σ_{c_1} kPa	$\bar{c}_2$ kPa	σ_{c_2} kPa
1	30	3,0	25	2,0	15	1,0	10	-	10	-	-	-	-	-
2	35	3,5	20	2,0	15	1,0	10	-	10	-	-	-	-	-
3	30	3,0	25	2,0	15	1,5	12	0,6	7	0,35	6	0,6	37	3,7
4	20	2,0	25	2,5	10	1,5	10	0,5	11	0,55	10	1,0	20	2,0
5	30	3,0	30	3,0	15	1,5	10	0,5	10	0,50	-	-	-	-

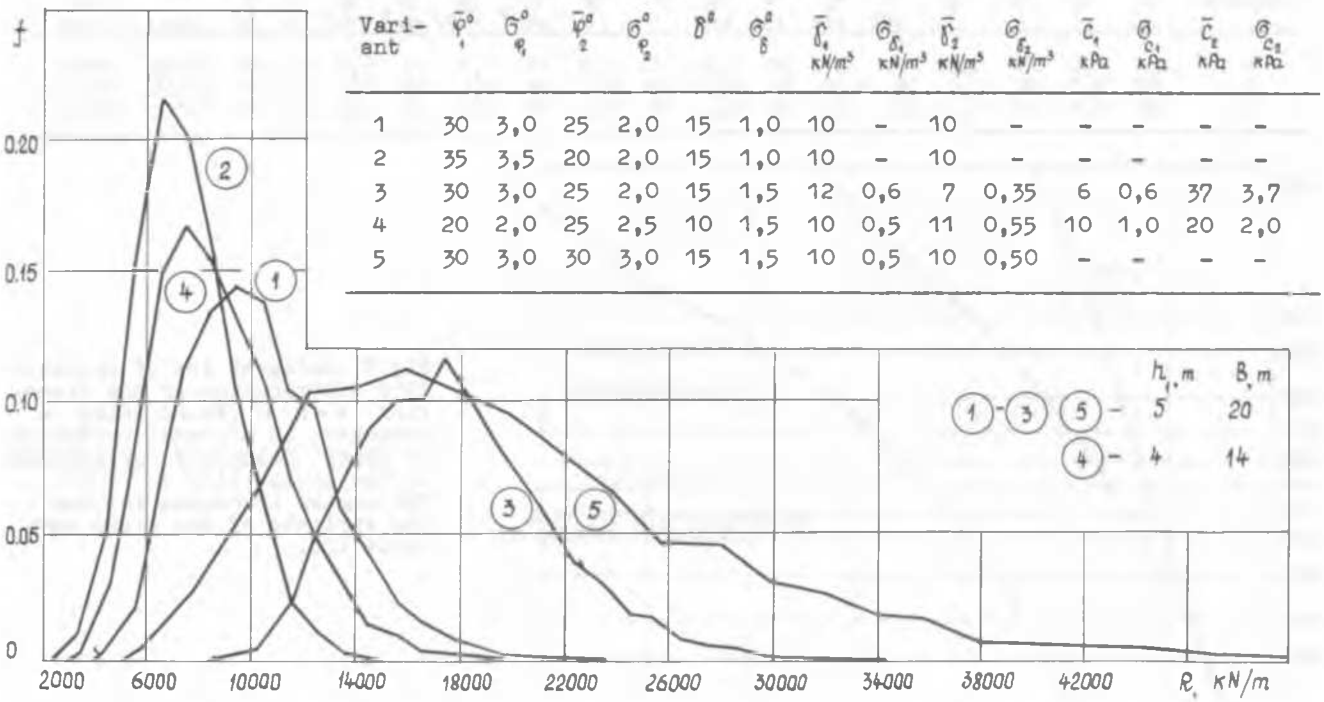

Fig.2. Frequency probability distribution of foundation bearing capacity computed by statistical test-method. Curves correspond to sets of initial data given in Table II.

Therefore, this method is not advisable for calculating the structures of importance (A.V.Shkola, 1980).

The obtained results give room for generalizing the probability solution for the case of layers of foundation. The above described algorithm is used, as a deterministic model. The angles of internal friction φ_i , cohesions c_i and specific weights γ_i , cagers of foundation as well as the angle of load orientation δ relative to the vertical are considered as random.

The algorithm for solving the problem by Monte-Carlo-method comprised the following main calculation steps:

- organisation of prendorandom numbers;
- simulation of rated particular values of random quantities (according to specified probability distribution functions) for the K-th probabilistic situation;
- revisins of rater particular values of random quantities to the specified original probabilistic characteristics of the random parameters;
- calculation of the bearing capacity of the foundation by the formulae of the deterministic model for the set of random parameters that correspond to K-th probabilistic situation;
- breaking down of the calculation outputs (bearing capacity of foundation) into quantization intervals;
- calculation of the probabilistic characteristics of the probability distribution function of the bearing capacity of layers of base.

To realize this method in practical work, it is necessary to apply a relatively high-speed computer, because the number of statistical tests must be at least $(2\text{-}3)\cdot 10^3$, if one wants to achieve the high-quality simulation of the probabilistic distribution laws with the inaccuracy not exceeding 2-3%. The results of the calculation of the initial data presented in Table 2, are shown in Fig.2. Every random parameter has been simulated by means of the subordinate Gaussian function. The volume of statistical testing equals $3\cdot 10^3$.

The analysis proves, that the foundation bearing capacity distribution function is monomodal and asymmetrically that is in line with the results obtained for a homogeneous foundation and confirms the substantial inaccuracy of the linearization method.

Estimation of the reliability level of layers foundation is to be conducted simmilar to the procedure applied to a homogeneous foundation by finding the probability of $P(\Psi = R - R^* > 0)$, where R^* - acting load (resultant along the load strip) that is generally presented by the probabilistic distribution law.

In this case the deterministic model terminates with a definition of the parameter $\Psi = R - R^*$ following the above descibed algorithm (Fig.). Where R^*=const this necessity is no longer required and an expected value is $\bar{\Psi} = \bar{R} - R^*$. In this case the law of distribution Ψ is to be obtained by offsetting of the distribution function R on R^* along the horizontal axis.

Table III

Variant	$\bar{\varphi}_1^o$	$\sigma_{\varphi_1}^o$	$\bar{\varphi}_2^o$	$\sigma_{\varphi_2}^o$	$\bar{c}_1$ kPa	σ_{c_1} kPa	$\bar{c}_2$ kPa	σ_{c_2} kPa	$\bar{\delta}^o$	σ_{δ}^o	$\bar{\gamma}_1$ kN/m^3	σ_{γ_1} kN/m^3	$\bar{\gamma}_2$ kN/m^3	σ_{γ_2} kN/m^3	h_1 m	B m	R^* kN/m	σ_{R^*} kN/m
1	38	3,0	25	2,5	0	0	20	2,0	10	1,0	11	1,1	10	1,0	4	11	12000	1000
2	20	1,0	40	1,4	28	4,0	20	2,0	10	1,0	10	0,5	10	0,5	6	11	12000	1000
3	28	1,4	20	1,0	20	2,0	40	4,0	10	1,0	10	0,5	10	0,5	6	11	12000	1000

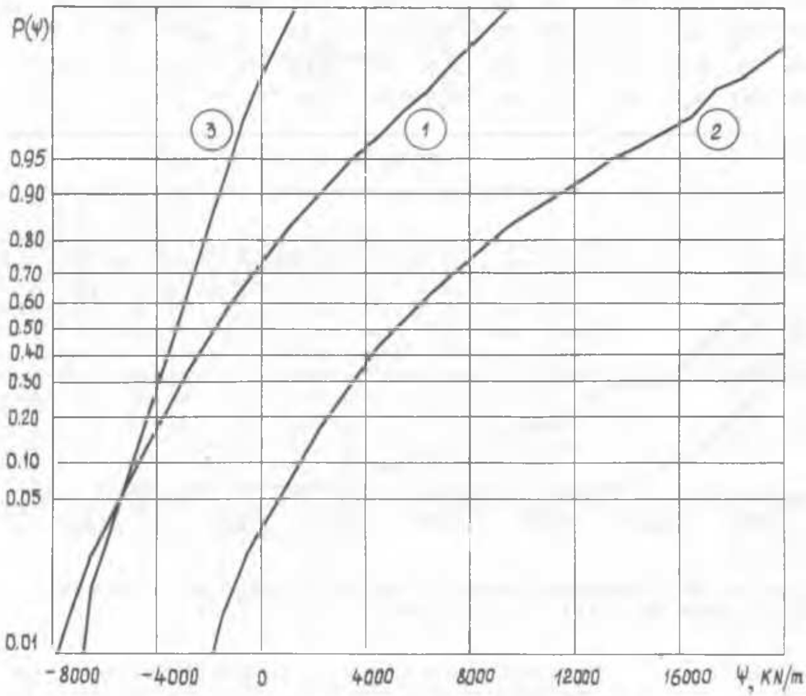

Fig.3. Integral law of probability distribution of the creterium $\Psi = R - R^*$, found using a computer, on a normal probability paper. Reability corresponds to the probability $P = 1 - P(\Psi)$. The number of curves follows the variants of the given data Table III.

SUMMARY A bearing capacity of layers foundation is to considerable extent determined by the ratio h_1/B , therefore the reduction of these foundations to their homogeneous equivalents introduced sufficient inaccuracy into the results. The obtained functions are recomended for use in the standards because their dimensionless expression makes it comparatively easy to tabulate the results over a wide range of the initial data.

An account of variable cohesion values in the foundation layers changes the bearing capacity of the foundations considerably. The solution can be generalized so, as to account for seismic effect using the approach described in the paper by P.I. Yakovlev and A.V.Shcola (1978) and the other factors as well as for a multilayer foundation.

A probabilistic approach to the analysed problem makes it possible to atandon standardization of the initial parametres at the input and reduced the allocation of the reserves to the standardization of the final result, which facilitates the use of the economic criteria in design and results in maximum utilization of the initial information.

The estimation of a probabilistic reliability level for the structures of importance should be performed by Monte-Carlo-method to be implemented with the aid of relatively high-speed computers.

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