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Interaction diagrams for shallow footings on sand

Diagrammes d'interaction pour les fondations superficielles dans les sables

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SYNOPSIS: This study presents the results of an experimental study into the behaviour of shallow footings on sand under inclined loads. The experimental programme was carried out using a new tridimensional small scale physical model designed and built at the University of Padua, which presents the particular characteristics of allowing the preparation of foundation beds with a very high degree of reproducibility. The research also verifies an approach based on diagrams interacting between horizontal and vertical loads as an alternative to traditional bearing capacity solutions.

1 INTRODUCTION

The calculation of a foundation allowable load is an important stage of geotechnical design.

Recently, this fact gave rise to a very large number of solutions to the problem. Neverthless, soil and foundation mechanical characteristics being equal, failure load values which can be determined by means of such solutions are substantially different. This casts great doubts as to the choise of the design calculation value to be assumed. This doubt is especially greater whenever the safety level of a shallow footing subjected to the stress of an eccentric and inclined load resultant is to be estimated.

With regard to this, the experimental study of those problems dealings with the interaction between the soil and footing is especially important. This also because the experimental results often supply with good insights for the formulation of rational analytical approaches.

One of the most interesting ways to solve the problem is based on the use of small scale physical models. In fact, physical models have several advantages:

- they are usually quite economical compared to large scale experiments;
- they can be easily driven to failure;
- they allow the predetermination of soil characteristics and of all the parameters involved in the problem taken into consideration;
- they allow an easy repetition of the experiment so as to have a large amount of data at one's disposal also for statistical consideration.

The University of Padua has been recently moving in this direction which, however much studied in the past, still allows for further research.

For this purpose, a tridimensional small scale physical foundation model has been entirely designed and carried out. The model is extremely versatile and reliable and will be later briefly explained.

This study presents the results of a large number of experimental tests carried out with shallow footings subjected to vertical and inclined loads. It also deals with their arrangement according to an alternative theoretical approach to the traditional solutions found in the literature on the subject.

2 INTERACTION DIAGRAMS

The most used approach to solve bearing capacity problems of shallow strip footings is based on the well known Terzaghi's equation:

$$V = (cN_c + \gamma DN_q + 1/2\gamma\, BN_\gamma) \qquad (1)$$

in which V is the vertical central load on a continuous footing posed at depth D in a soil with unit weight γ and shear strength parameters c', ϕ' and N are semiempirical bearing capacity factors.

If the footing is loaded with a general load R, eccentric by e inclined by α the vertical component of failure load can be determined multiplying each N factor by different semiempirical coefficients which take into account the effects of inclinations and eccentricity.

Thus, the foundation allowable load can be predicted dividing the result of the above bearing capacity calculation by the relative factor of safety.

This way of calculating allowable bearing capacity is quite laborious and does not take into account the non-linear effects due to the presence of horizontal or eccentric loads in the safety factor determination (Georgiadis, 1985).

In this case, an alternative approach to the bearing capacity prediction, based on interaction diagrams between the components of the applied load, can be used (Butterfield, 1981).

2.1 The H-V interaction diagram

Any general load system (R) acting on a shallow strip footing can be transformed into three components V, H and M.

The problem is then to determine what combinations of loading (V,H,M,) will cause the foundation to fail.

Restricting the analysis to a very shallow footing on sand subjected to vertical and horizontal loads only, it can be possible to define the interaction diagram between vertical and horizontal loads in a H-V plane, observing that (fig. 1):

- the horizontal line $V_{max} = 1/2\ \gamma B N_\gamma$ define the failure envelope for plunging failure under vertical load only;
- the inclined line $H = V \tan\delta$ defines the sliding condition along the ground surface when V approaches zero (δ = soilfoundation interface angle).

This line together with the V = Vmax defines a triangular region in fig. 1 within which a limit curve exists which envelops all admissible loads for a surface footing on sand.

The problem is now to determine, on experimental bases, the expression of such a limit curve envelope.

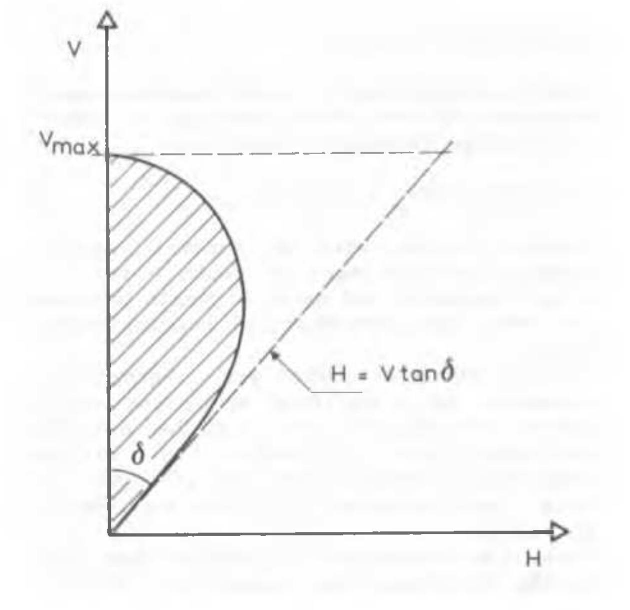

Fig. 1 - (H,V) interaction diagram for surface strip footing on sand.

2.2 The experimental work in the past

The earliest investigation on the effect of load inclination is reported by Meyerhof (1953); he proposed the relationship

$$V/V_{max} = (1 - \alpha / \Phi)^2 \qquad (2)$$

which allows the determination of bearing capacity for a surface footing supporting a load with inclination $\alpha = \mathrm{atan}\,(H/V)$.

After Meyerhof, experimental studies on the subject using small scale physical models have been performed by several researches (Jumikis, 1961; Zaharescu, 1961; Kezdi, 1961; Saran et al., 1971; Tennekoon, 1977; Head, 1977; Ticof, 1977); the effects of load inclination was also studied on a large scale by Giraudet (1965) and Muhs & Weiss (1973).

Hansen (1957) proposed a semiempirical expression for calculating the decrease in the vertical failure load due to load inclination

$$V/V_{max} = (1 - \beta_1\ H/V)^{\beta_2} \qquad (3)$$

where $\beta_1 = 0.7$ and $\beta_2 = 5$ (Hansen, 1970). The same expression was used by Muhs & Weiss (1973), who indicated for β_1 and β_2 the values 1 and 2 respectively.

In general, the tests performed in the past considered surface strip footings subjected to loads inclined in the direction of foundation width.

When a load is inclined in the direction of footing length, the effect of load inclination on the ultimate bearing capacity cannot be evaluated by using the previous expressions.

In this case, the decrease of bearing capacity can be calculated by using the relation proposed recently by Meyerhof and Koumoto (1987); this relation, for surface strip footing on cohesionless soil, takes the form:

$$V/V_{max} = \cos\alpha' \,(1 - \sin\alpha'/\sin\Phi) \qquad (4)$$

where α' is the inclination of a centreline continuous load, inclined with respect to the footing length.

All the expressions (2), (3) and (4), proposed by the researchers as reduction factors, define in a H-V plane interaction diagrams between horizontal and vertical components of load; this will be clearly seen in the next paragraphs together with the experimental work carried out at the University of Padua.

3 EXPERIMENTAL EQUIPMENT OF PADUA

The small scale 3D-shallow footing model built in Padua is composed essentially of five main parts (fig. 2):

- a mobile crane;
- a hopper;
- a test tank;

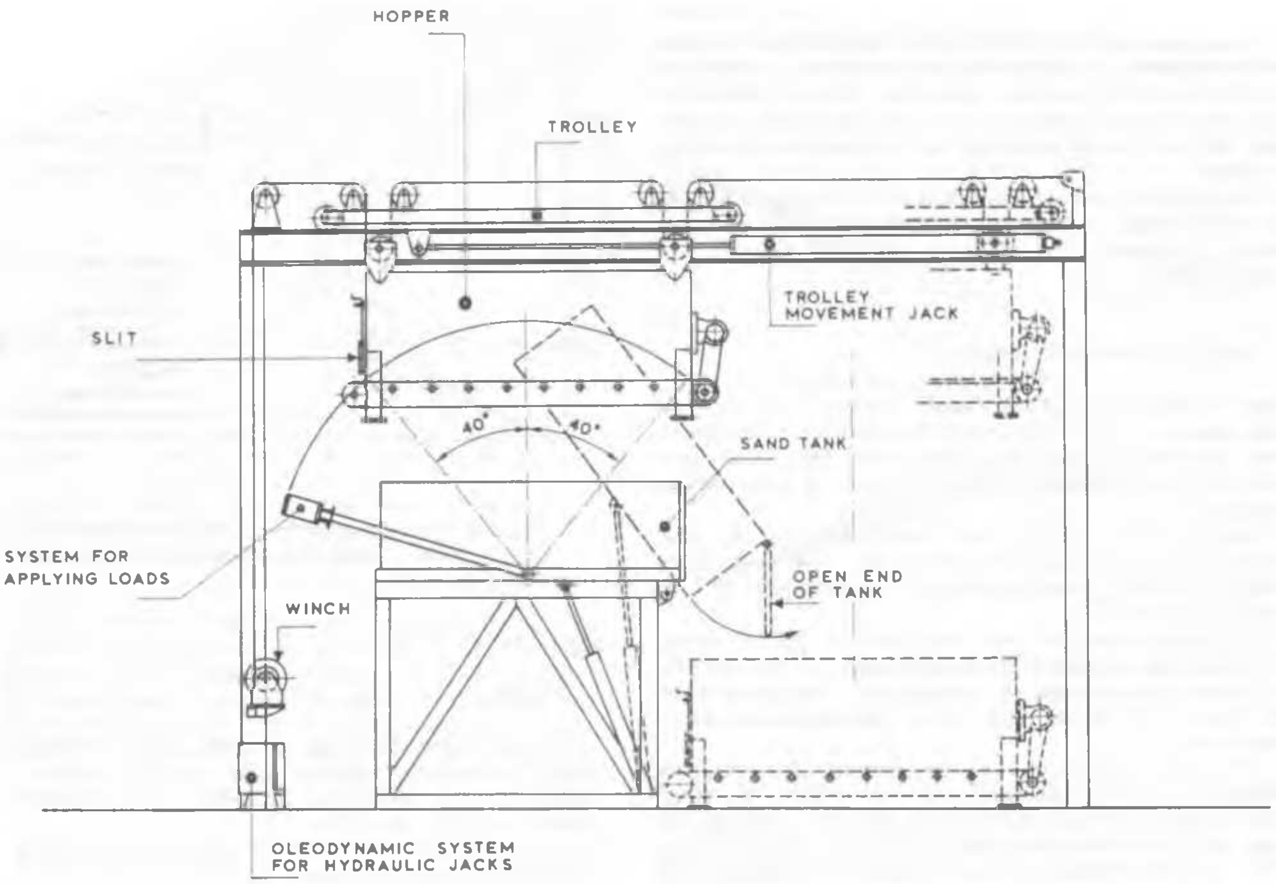

Fig. 2 - Experimental set up at the University of Padua.

- a loading device;
- a hydraulic generator.

The mobile crane is driven by four pulleys. It supports a hopper containing sand for preparing foundation beds by pluvial deposition. The bottom of the hopper is composed of a conveyor belt which transports the sand, letting it fall through an adjustable slit at the front, as wide as the hopper itself.

While the sand falls, the hopper is moved in a fully automatic way by the mobile crane, alternatively backwards and forwards, so as to cover the whole area of the test tank, in which the bearing capacity tests are carried out. The translation speed of the mobile crane is 2.4 m/min.

As the tank is gradually filled, the hopper is raised so that the height from which the sand falls is kept constant.

When the test tank (1800x1800x700 mm deep) is full a small footing is placed on the sand bed.

The loading device is composed of an arch supporting a servocontrolled hydraulic jack. The arch is being moved so that an inclination to the chosen load direction can be easily given. The jack transmits the load through a spheric joint to the foundation plate. The bearing capacity tests may thus be carried out.

At the end of each test, the hopper is placed in front of the test tank so that the sand can be easily unloaded into the hopper again. The equipment is thus ready for preparing the next foundation bed.

4 THE FOUNDATION BEDS

The foundation beds are prepared by pluvial deposition of the sand. The sand is a fine uniform sand coming from the mouth of Adige River, already used at the University of Padua. The sand has a mean grain size D_{50} = 0.35 mm and a coefficient of uniformity of 2.

According to the ASTM Standard procedure No. D-2049, the minimum and maximum unit weights have been determined and the following values have been found: 13.58 and 16.52 kN/m^3.

The parameters controlling the obtainable relative density are height of fall (i.e. height of the hopper), aperture of the slit located in front of the hopper, and conveyor belt speed, which is kept constant in all the tests at 0.40

m/min.

More details of the pluvial deposition system can be found in Favaretti and Simonini (1987).

In all the bearing capacity tests, the slit was kept at an aperture of 5 mm and fall height was 80 cm, corresponding to a relative density of 78%.

Foundation beds reproducibility turned out to be very high, measured differences in relative density between one test and another being less than 0.5%.

5 BEARING CAPACITY TESTS

The foundation plate was chosen so as to reproduce a stiff strip of 100x500 mm. The plate was placed on the sand bed and the test was carried out, taking the soil-footing complex to failure.

During the tests, the load applied to the plate with the jack was measured together with the vertical displacements corresponding to every pressure.

Soil-footing failure was marked by a sharp increase in vertical displacements of the plate; at the same time a reduction in the load necessary to keep the plate moving was also measured.

The soil surface clearly showed the failure surface. Later, the pressure necessary to keep the foundation moving was constant, indicating that the soil had reached a critical state along the failure line, characterized by strain at constant stress and no change in volume.

The above results, together with the high relative density reached by the foundation beds, presume that general failure of the soil has always occurred (Vesic, 1973).

Over fourty bearing capacity tests have been carried out with vertical load or inclined load both in transversal and longitudinal direction at 5, 10 and 20°.

The sliding angle δ has been evaluated in approximately 43°.

6 TEST RESULTS ANALYSIS

The results obtained from the bearing capacity tests have been directly diagrammed on the H/Vmax plane vs. V/Vmax, in which the variables H and V are dimensionless with regard to the value of maximum failure load determined for a vertically loaded foundation.

In fig. 3, the values of the ratios H/Vmax and V/Vmax are reported which were obtained from tests carried out with load inclination in a transversal direction.

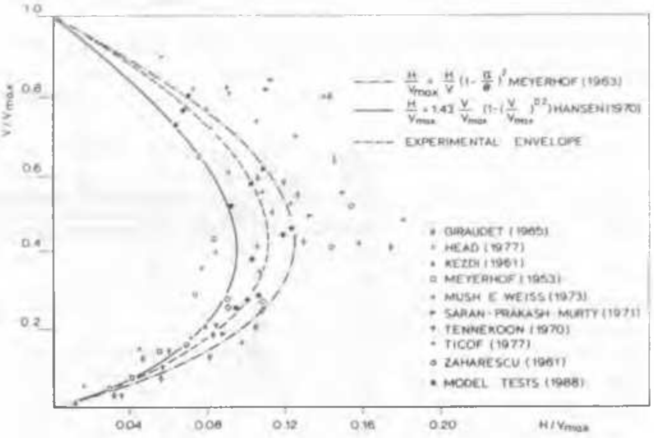

Fig. 3 - Experimental failure envelope for loads inclined in the direction of footing width.

They have been interpreted according to a dimensionless envelope of the type:

$$H/Vmax = \alpha_1 \, V/Vmax \, (1 - (V/Vmax)^{\alpha_2}) \qquad (5)$$

which can also be expressed in the more traditional form:

$$V/Vmax = (1 - \beta_1 \, H/V)^{\beta_2} \qquad (6)$$

In the examined case α_1 and α_2 values have been obtained equal to 0.93 and 0.4 respectively, whereas β_1 and β_2 values are equal to 1.07 and 2.5.

The value of the maximum foundation allowable horizontal load is equal to

$$Hmax = 0.11 \, Vmax \qquad (7)$$

and is slightly superior to that determined by Georgiadis & Butterfield (1988).

On the same diagram of fig. 3, the curves characterized by equations (2) and (3) proposed by Meyerhof (1953) and Hansen (1970) are reported. These curves can be considered as superior and inferior limits whithin which all the values determined through model tests are contained. In fact, the pattern obtained through experimentation places itself in an intermediate position between the two above mentioned and close to that of equation (2).

Other experimental data found in the literature on the subject have also been included in the diagram.

As it can noticed, there is a remarkable dispersion of experimental values. These discrepancies can probably be attributed to differences in the particular experimental apparatus used by each investigator. For example, Saran et al. (1971) and Muhs & Weiss (1973) used equipment which imposed kinematic restraints on the footings; they were allowed to translate along the loading path only at a fixed angle. Such kinematic constraints results in higher failure loads in the case where the footing is free to translate (Ticof, 1977).

The effect of load inclination in a longi-

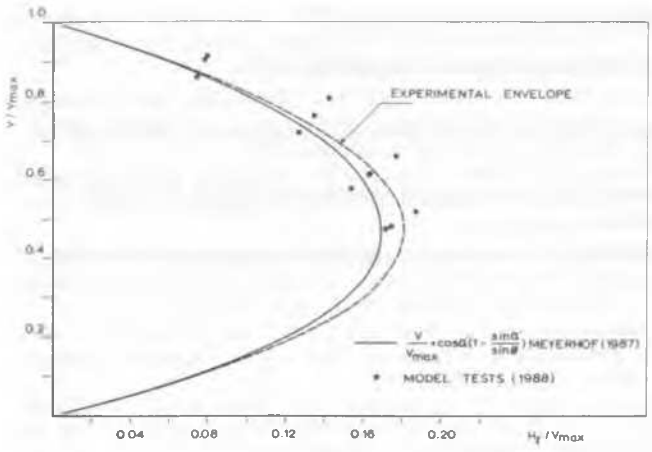

Fig. 5 - (H,H_ℓ- V) interaction diagrams for surface strip footing subjected to loads inclined in any direction.

tudinal direction can be seen by examining the experimental results in fig. 4. The failure envelope determined through minimum square regression gave constant α_1 and α_2 values for equation (5) equal to 0.93 and 0.7 respectively. β_1 and β_2 values for equation (6) are equal to 1.07 and 1.4. The maximum value for allowable horizontal stress H_ℓ max is

$$H_\ell \max = 0.18 \text{ Vmax} \qquad (8)$$

The basis for the comparison with solutions found in the literature is that supplied by Meyerhof and Koumoto relation (1987). The experimentally determined curve is slightly external to the Meyerhof curves, even if some load tests results of load tests gave values inside the equation's theoretical envelope (6).

7 SOME CONSIDERATIONS

The versatility in the use of the tridimensional model built in Padua allowed the execution of bearing capacity tests with an inclined load both in the direction of footing width and of footing length.

In relation to this, it was possible to obtain a global viewpoint as to the behaviour of a continuous shallow footing under any inclined load. The arrangement was subsequently represented in terms of interaction diagrams.

Figure 5 summarizes the results obtained from tests carried out with inclined loads both in a transversal and a longitudinal direction. In examining the two curves superimposed on the same diagram, it is immediately possible to notice how the inclination in a longitudinal direction affects the maximum allowable failure load in a less sensitive manner compared to transversal inclination. The ratio between maximum horizontal stresses in the two cases is equal to about 1.6 in favour of the foundations with an inclined load in a longitudinal direction.

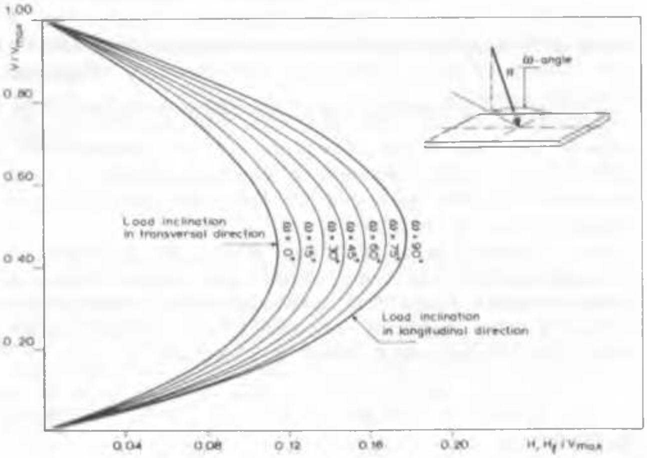

Fig. 4 - Experimental failure envelope for loads inclined in the direction of footing length.

In the same figure, intermediate curves have also been obtained and reported as a function of an angle ω . On the horizontal plane (fig. 5), this angle expresses the direction of the resultant of horizontal stresses (i.e. ω = 0° for inclined load in a transversal direction only; ω = 90° for inclined load in a longitudinal direction only).

The diagrammed curves in the figure allow for an easy definition of the value of continuous foundation failure load under central and inclined loads in any direction, once the direction of the resultant of horizontal loads and the bearing capacity value for foundation under vertical loads are known.

8 CONCLUSIONS

The experimental research performed allowed the definition of a continuous footing failure behaviour under inclined loads. The research consisted in a wide series of bearing capacity tests with model foundations on granular soil.

In particular, the effect of load inclination both in the direction of footing width and of footing length has been estimated.

For this purpose, a tridimensional small scale shallow footing model has been used, which has been recently built at the University of Padua.

The new model allows the preparation of perfectly homogeneous and reproducible foundation beds in a fully automatic manner. In fact, differences in per cent terms inferior to 0.5% have been found for bed densities of each performed test.

Bearing capacity test results are of good experimental quality and confirm the effectiveness of some solutions proposed by the literature. They have been interpreted in terms of interaction diagrams between horizontal and vertical stresses. These diagrams allow for a quick and easy calculation of failure load for a continuous shallow footing subjected to whatever inclined loads.

In particular, it has been noticed that the effect of load inclination in a transversal direction on the maximum allowable failure load is greater than that due to the inclination in a longitudinal direction.

As a consequence of this, when the foundation is subjected to an inclined load in a longitudinal direction, the maximum horizontal stress it can support is about 1.6 times higher than that in a transversal direction.

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