Modelling the dynamic response of piles in dry sand

La modélisation de la réponse dynamique des pieux dans le sable sec

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Data of the response of piles to base shaking obtained from centrifuge model tests have been analysed to assess the relevance of pseudostatic calculations. The lateral response of the pile is found to be critically dependent on the natural frequency of the soil-pile system; methods to predict the natural frequency are presented and compared with experimental data. An equivalent statical load method is used to analyse the data from all available single pile tests.

INTRODUCTION

The dynamic loading of a soil-pile system by base shaking or earthquake loading generates a response which depends on the fundamental frequency of the system. Interpreting this behaviour using analyses for the static lateral loading of piles is difficult unless the interaction between the pile cap inertia and the ground movements is taken into account. A series of dynamic model tests were undertaken using the Cambridge Geotechnical Centrifuge to generate a data-base of the behaviour of piles embedded in dry sand and subjected to base shaking. These facilities and the scaling laws for centrifuge modelling have been described in detail elsewhere (Schofield, 1980,1981).

MODEL TESTS

Tests AM-1, AM-2 and AM-9 were carried out at Cambridge and AM-1 and AM-2 are described in detail by Maheetharan and Steedman,1987. The basic geometry and soil and pile data for the model tests reported here is given in Table I. AM-1 and AM-9 represented the same prototype structure but used a different construction technique. Test AM-2 investigated the effect of a light pile cap. In the analysis, data is also used from a model called Test 41, which was carried out as part of a independent study reported by Finn and Gohl,1987. Their pile was made from stainless steel and represented a third type of prototype structure. It was tested using the centrifuge at the California Institute of Technology.

As an example, the general arrangement of model test AM-1 is shown (in cross-section) in Fig.1. A pile made from dural tube supported a heavy pile cap. The pile was instrumented with strain gauges to measure bending or axial compression and with accelerometers. The Leighton Buzzard sand used in the Cambridge tests was a fine sub-rounded sand which passed/was retained on B.S. sieves Nos. 52/100 respectively. Test 41 used a fine Nevada sand. With the pile temporarily fixed in position dry sand was poured around the

TABLE I

Centrifuge Model Summary

Model		AM-1	AM-2	AM-9	41
m (pile cap mass)	Kg	1.08	0.09	0.77	0.25
h (height cent'd)	mm	40	35	40	32
ρ (pile density)	g/mm	0.173	0.173	0.173	0.055
r_o (pile radius)	mm	4.76	4.76	4.76	4.76
J (pile cap)	Kgmm^2	369	6	262	40
EI (pile) x10^6	Nmm^2	30.3	30.3	30.3	14
l_c (observed)	mm	100	120	100	100
l_c (predicted)	mm	77	77	77	65
e		0.6	0.6	0.61	0.57
nom. grain size	mm	0.23	0.23	0.23	0.13
g level		80	80	80	60
f (driving freq.)	Hz	120	120	120	30
f_n (predicted)	Hz	59	180	70	92

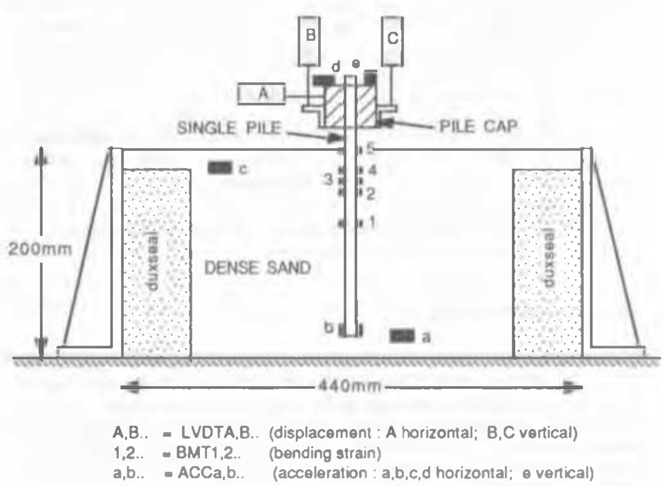

Fig.1 Cross-section through Model AM-1

pile to form a dense foundation. By contrast, the construction of model AM-9 and test 41 involved the pushing of the pile into a dense dry sand bed at 1g.

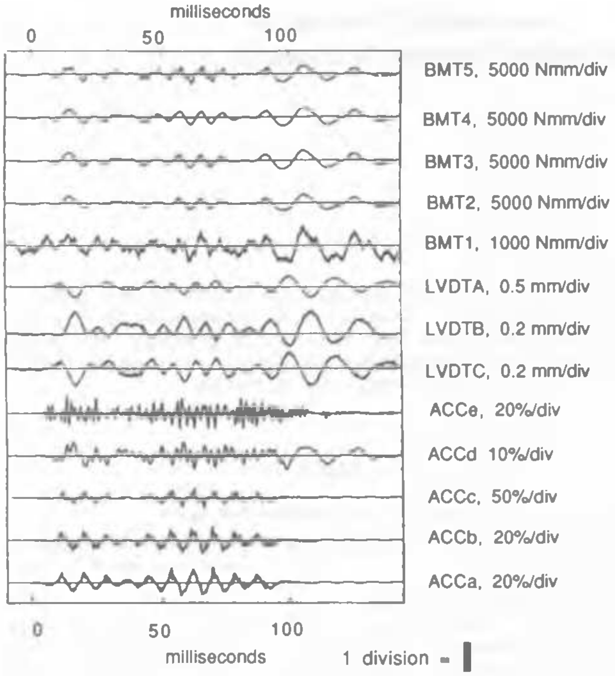

Fig.2 Data from AM-1, earthquake 2

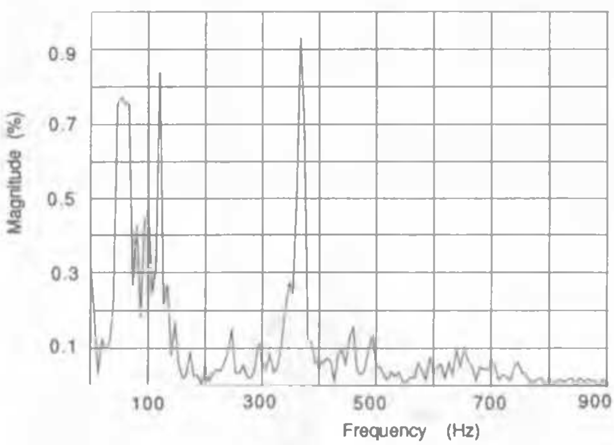

Fig.3 Fourier spectrum of ACCd, pile cap (horizontal)

DATA PRESENTATION

A portion of the dataset obtained from a typical base shaking applied to model AM-1 is shown in Fig.2. Accelerometer ACCc, at the ground surface, shows a slight amplification and phase lag (about 6°) compared to accelerometer ACCa near the base of the sand. Although the pile tip accelerometer (ACCb) shows motion similar in magnitude and phase to the surrounding sand bed (ACCa), the pile cap acceleration is out of phase (about 160°) and reduced in magnitude to about 34% of the surface soil motion (ACCd on the pile cap, ACCc at the soil surface). The fourier spectrum of the pile cap acceleration, Fig. 3, shows that the natural frequency of the soil-pile system is in the range 55-60 Hz. This can also be deduced from the post-earthquake free vibration of the pile cap (ACCd) in Fig.2.

However, there is also a strong peak in the response of the pile cap at a frequency of 360Hz. This frequency, which is clearly identifiable in the time history of the horizontal accelerometer ACCd, is dominant in the vertically mounted accelerometer ACCe and it may be concluded that this is a frequency associated with torsional vibration or rocking of the pile cap. It coincides with the third harmonic of the 120Hz horizontal base shaking.

Bending strain transducers BMT2-5 show strong cyclic moments at the driving frequency during the base shaking and at the natural frequency of the soil-pile system after the shaking has died away. Only BMT5, located just below the ground surface, also shows evidence of the rocking mode superimposed on the dominant horizontal cycling.

FUNDAMENTAL FREQUENCY OF THE SOIL-PILE SYSTEM

The attenuation and phase lag of the pile cap may be explained by noting that the ratio of the driving frequency to the measured fundamental frequency of horizontal shaking is $f/f_n \approx 2.1$. An idealisation of the soil-pile system as a single degree of freedom system under damped forced vibration, see for example Sandor, 1983, would predict an attenuation of 3.3 and a phase lag of 175° for a damping of 7.5%. This value for damping was based on the decay of the post-earthquake vibration of the pile cap. These predictions are close to the observed values although the response of the actual soil-pile system is complicated by the superposition of the torsional mode of vibration described above.

<u>Rayleigh's Method for an equivalent cantilever</u>
For the free vibration of an undamped single degree of freedom system, an estimate of the fundamental frequency may be obtained by equating the maximum potential energy to the maximum kinetic energy.

Fig.4 Equivalent cantilever

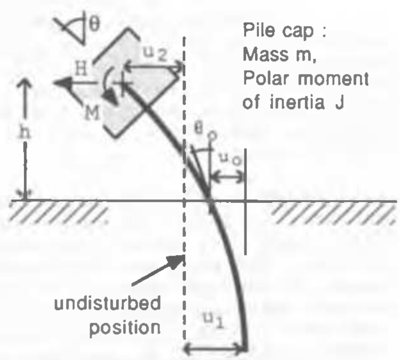

Fig.5 Pile subject to base shaking

The potential energy of the system is stored in both the pile shaft and in the surrounding soil. To simplify the prediction of the fundamental frequency, an equivalent cantilever length, L, was calculated to model the soil-pile system, Fig.4. Firstly, the shear modulus, G, of the soil was assumed to vary linearly with depth such that G = mz. Following Randolph, 1981, a value of the average characteristic shear modulus G_c was deduced using the observed critical length l_c. G_c may be thought of as representative of the actual level of soil strains around the pile. Randolph's expression for the surface deflection of a laterally loaded pile was modified to incorporate the free length of the pile above the ground and used to deduce the length of the equivalent cantilever by equating the lateral stiffness at pile cap level.

This analytical idealisation reduces the number of terms to be evaluated to four. Firstly, the maximum potential energy of the pile shaft may be expressed as

$$PE_{sh} = \frac{1}{2} EI \int_0^L \left(\frac{d^2y}{dx^2} \right)^2 dx \qquad (1)$$

where y is the lateral displacement, x is measured along the pile and EI the bending stiffness. The lateral deflection profile of the pile was approximated by calculating the profile of the pile when subject to a uniform static lateral acceleration, producing a combination of a point load at pile cap level and a uniform load distributed along the pile.

The kinetic energy of the pile cap is derived from two components, firstly from the transverse (horizontal) velocity :

$$KE_{ct} = \frac{1}{2} m \, (y_c \omega)^2 \qquad (2)$$

where m is the pile cap mass and y_c the lateral deflection at pile cap level. The frequency of vibration is denoted by ω. The second contribution to the kinetic energy is from the rotational velocity of the pile cap :

$$KE_{cr} = \frac{1}{2} J \left(\frac{d\theta}{dt} \right)^2 \qquad (3)$$

where J is the polar moment of inertia of the pile cap and θ the rotation at pile cap level. The final term is the maximum kinetic energy of the pile shaft which may be calculated from :

$$KE_{sh} = \frac{1}{2} \int_0^L \rho \, (y\omega)^2 \, dx \qquad (4)$$

where ρ is the mass per unit length. For model AM-1 the estimated natural frequency was 59Hz, within the range of observed values of 55-60Hz. Furthermore, a subgrade reaction calculation was solved numerically and gave fn = 56Hz, which provided an independent confirmation of the analysis described above.

PREDICTION OF MAXIMUM SURFACE BENDING MOMENT

Equivalent statical load

The response of a pile with a natural frequency higher than the driving frequency will be markedly different to the response of a pile with a natural frequency lower than the driving frequency. Leaving aside systems which are close to resonance, the former will be amplified in its response, and remain broadly in phase with the base shaking, and the latter will be attenuated in amplitude and out of phase. The four available centrifuge model tests cover this range of behaviour, with two models having fundamental frequencies higher than the driving frequency and two models having fundamental frequencies lower, as shown in Table I.

To normalise the behaviour of all four pile systems, an analysis was developed which incorporates this different response. Fig.5 shows a pile cap moving out of phase with the base of the pile. If the maximum deflection of the pile cap is u_2 then the horizontal inertia force at the pile cap, H, is simply

$$H = m \, u_2 \, \omega^2 \qquad (5)$$

The moment, M, induced by a rotation, θ, of the pile cap is

$$M = J \, \theta \omega^2 \qquad (6)$$

If the centroid of the pile cap in Fig.5 is subject to a lateral force H and moment M and is a height h above ground level, then

$$u_1 + u_2 = \frac{1}{EI} \left[\frac{M h^2}{2} + \frac{H h^3}{3} \right] + \theta_0 h + u_0 \qquad (7)$$

$$\theta = \frac{1}{EI} \left[M h + \frac{H h^2}{2} \right] + \theta_0 \qquad (8)$$

Equations 7 and 8 may then be written as a function of l_c, EI, h and ρ_c, where ρ_c is a parameter describing the homogeneity of the soil stiffness as defined by Randolph. In this analysis a shear modulus proportional to depth was assumed (ρ_c = 0.5).

Using the observed values of critical length l_c and substituting the appropriate pile parameters and values for J and ω the pile cap moment M and lateral force H may be determined as functions of the amplitude of base acceleration of the pile, determined by the base displacement u_1 and ω. The surface shear force H_0 = H and the surface moment, M_0, is simply $M_0 = M + Hh$.

This analysis is valid for piles shaken away from resonance either above or below, with M_0 turning out to be either positive or negative depending on whether f/f_n is less than or greater than 1 respectively.

All the data of peak bending moments from the centrifuge model tests was normalised using this method and is presented in Fig.6. To allow for amplification through the soil and to reduce errors from the accelerometers in the models an average acceleration was used over the upper portion of the pile. The normalisation of the depth of the transducers was achieved by dividing by the observed critical pile length.

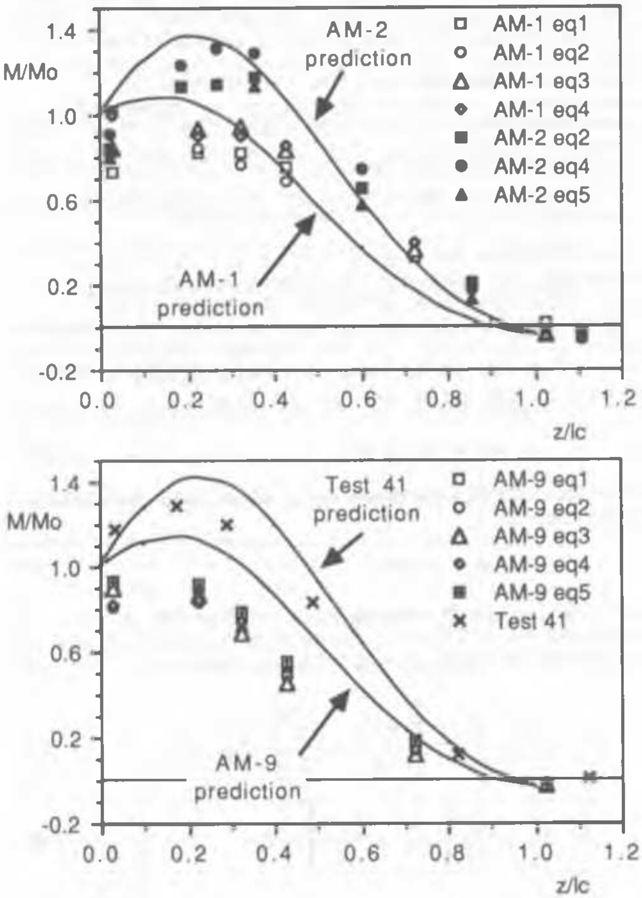

Fig.6 Peak moments plotted vs. depth, (data separated into two graphs for clarity)

The results of this approach are very encouraging, bringing together pile data from four widely different models, with and without pile caps, being shaken above and below their fundamental frequencies. Some scatter is to be expected from instrumentation error but the principal variation in moment distribution with depth is due to the differing proportion of shear force to bending moment at the ground surface.

PREDICTIONS OF MOMENT DISTRIBUTION WITH DEPTH

Following Randolph,1981 predictions of the distribution of bending moment with depth were made by adding contributions from static moment and shear loads applied at the ground surface. These are shown in Fig.6 for each model, and highlight the importance of the proportion of surface shear force to bending moment in the distribution of bending moment along the pile.

Prediction of the critical length

A prediction of the critical pile length for each model was made following Randolph, 1981. Theoretical predictions of the variation of the shear modulus in the free field usually propose a square root variation with depth (Hardin and Black, 1968). However, to make some allowance for the increased strain levels near the pile a linear variation of shear modulus with depth was used which matched the square root function at the critical depth. The critical length is then found by iteration. It may be seen from Table I that this theoretical critical length is shorter than the observed value, which suggests that even a linear variation of shear modulus with depth is insufficient allowance for the high strain levels near the ground surface.

CONCLUSIONS

(1) The importance of the fundamental frequency in determining the reponse of a single pile to base shaking has been demonstrated by centrifuge model tests. An equivalent cantilever model was successfully used to predict the fundamental frequency of each pile.

(2) An equivalent statical load method which accounts for horizontal and rotational inertia of the pile cap has been shown to predict the value of surface moment and to describe the distribution of moment with depth.

(3) The critical pile length estimated using conventional techniques based on static lateral loading was observed to be consistently shorter than the actual length under dynamic loading. High shear strains in the upper portion of the pile are concluded to be causing a large reduction in the insitu shear modulus and a consequent increase in the critical length.

REFERENCES

Finn, W.D.L. and Gohl, W.B. (1987). Centrifuge model studies of piles under simulated earthquake lateral loading. Proc.Spec. Session Geotech.Eng.Div., ASCE Conv. Atlantic City, Geotech Spec.Pub. 11, 21-39.

Hardin, B.O. and Black, W.L. (1968). Vibration modulus of normally consolidated clay. ASCE, 105, EM5, 811-827.

Maheetharan, A. and Steedman, R.S. (1987). Data report of three dynamic centrifuge model tests on piles and pile groups. Int.Report CUED/D-SOILS/TR206, Cambridge Univ.

Randolph, M.F. (1981). The response of flexible piles to lateral loading. Geotechnique 31 (2), 247-259.

Sandor, B.I. (1983). Engineering Mechanics. pp 979-981, Prentice-Hall, N.J.

Schofield, A.N. (1980). Cambridge geotechnical centrifuge operations. Geotechnique 30, (3), 227-268.

Schofield, A.N. (1981). Dynamic and earthquake geotechnical centrifuge modelling. Proc. Int.Conf.Rec.Adv.Geotech.E'quake Eng.Soil Dynamics, April 26-May 3, Univ.Miss.Rolla.