Centrifuge modeling of excavations in dry sand

La modélisation en centrifuge des excavations dans les sables secs

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SYNOPSIS: Free-standing and strutted rigid walls were studied using model tests. The strength of the sand was backfigured from the embedment required for equilibrium of free-standing walls and then used to predict the collapse of strutted walls. Results emphasize the dependence of friction angle upon level of stress and the sensitivity of collapse to the post-buckling behavior of struts.

1 BACKGROUND

The study described in this paper began as a demonstration experiment, with an arrangement similar to that shown in Fig. 1, to inaugurate the small geotechnical centrifuge at MIT. All staff and students were invited to predict the centrifugal acceleration at which the model retaining walls would collapse, and were provided in advance with dimensions and results of standard tests upon the sand. Nearly all, including the writers of this paper, rather badly underestimated the load capacity.

The follow-on experiments, undertaken to explain and understand this result, revealed just how difficult it is to predict the load capacity for this rather simple arrangement, and emphasized anew the shortcomings of small-scale model experiments in normal gravity.

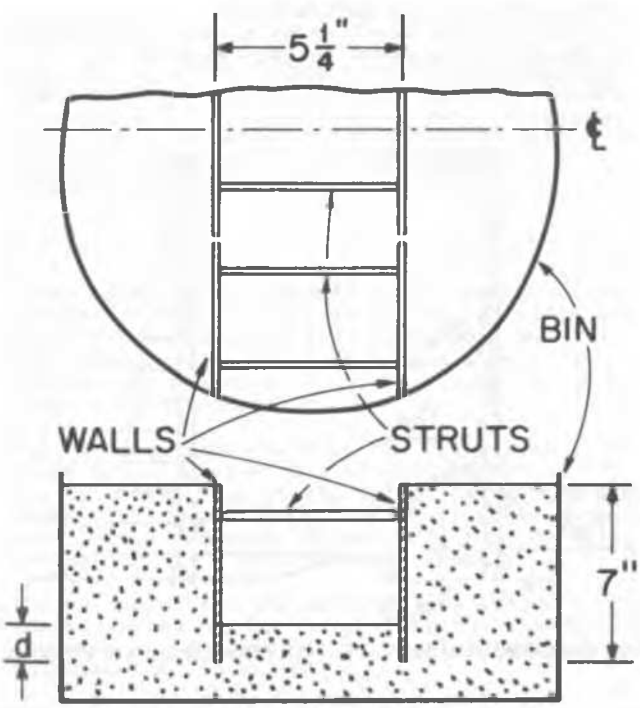

Fig. 1 Schematic of Test

2 EXPERIMENTAL ARRANGEMENTS AND TEST PROGRAM

As shown in Fig. 1, the experiment involved a pair of walls supported by a single row of struts. Viewed from above, each wall consisted of three segments, the idea being that the center sections would be more or less free of the influence of the container sides. A circular bin was used in most tests, although some tests were within a rectangular bin. The walls were made thick (1/4 in.) and rigid so that bending and failure would not be an issue. In all tests the center sections were 6 in. long; the side sections were somewhat shorter, as required for fitting into the bins. Thin tape lubricated with graphite covered the joints between sections and between the end sections and the walls of the bin.

Struts were machined from aluminum, with dimensions (1/16 in. by 5/16 in.) chosen to give a predicted elastic buckling load of about 25 lbs. The ends of the struts were rounded, so as to fit loosely into dimples cut into the wall sections. Struts were placed so that the long edges were parallel to the acceleration field.

The soil in these experiments was air-dried quartz sand, angular and uniform with D_{50} = 0.4 mm. It was placed by raining slowing from a height of about 2 ft. resulting in an average unit weight of 105 pcf, which is essentially 100% relative density. Direct shear tests at this unit weight and a normal stress of 1 tsf gave a peak friction angle of 42°. First one inch of sand was placed and leveled. Then the wall, held together by a framework, was positioned and the remaining sand rained into place. Excess sand beyond the desired depth was removed by vacuuming. The framework was removed immediately before the test.

Tests were performed using the MIT geotechnical centrifuge, which is manufactured by the Genisco Corporation and has a radius of 51 inches in swing-up position. The circular bin had an internal diameter of 18 in., while the rectangular bin was 18 in. in the circumferential

Table I
SUMMARY OF TEST PROGRAM

Series	Bin/Orientation	Condition
H1		Unstrutted Strutted (7/8, 3-1/2) Strutted (7/8) w/sg
H2		Unstrutted Strutted (7/8) Strutted (7/8) w/sg
S		Unstrutted Strutted (7/8)
B		Strutted (7/8,3 1/2, 5)

w/sg denotes strain-gaged struts
Numbers in ( ) are elevation of struts below top of wall in inches.

direction by 16 in. transversely. In a few tests struts were strain-gaged to measure axial force. Otherwise there was no instrumentation and failure was observed via a video camera. Table 1 summarizes the test program. The several series were carried out at various times by different experimenters. Although the basic procedure was the same in each case, there were doubtless variations in experimental techniques.

Test series H2 was designed to indicate the effect, if any, of the geometry of the bin containing the wall and sand. Test series S was carried out to investigate the effect of the orientation of the wall relative to the direction of movement at the end of the centrifuge arm; in this series the walls were parallel to the tangential motion, while in all other series the walls were perpendicular to this motion.

3 UNSTRUTTED TESTS

The results of these tests, with centrifugal acceleration (Ng) causing failure vs. depth of embedment on the inside of the walls, are summarized in Fig. 2. Here g is the acceleration of gravity, and N is an acceleration coefficient. Results plotted above zero N are from tests at normal gravity (1g).

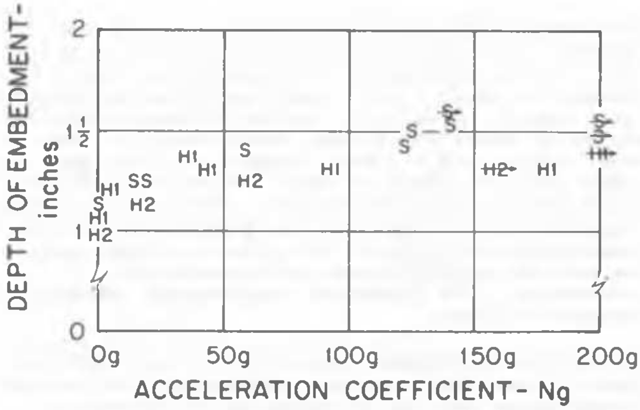

Fig. 2 Effect of Embedment for Unstrutted Walls

Tests at 1g began with an embedment greater than 2 in. The sand within the walls was then gradually reduced by vacuuming until failure-defined as a sudden inward tilting of one or both walls-was observed. Thus the plotted points represent the smallest depth of embedment for which the walls are stable in normal gravity. Actually, each point represents an average from at least 3 tests, and the difference in the plotted results arises primarily from variations in experimental technique. On the average, the depth required for stability is 1-1/16 inches.

Next, tests were prepared with depths of embedment greater than 1-1/16 inches, and the models were placed on the centrifuge. N was increased slowly until failure of one wall - defined as a sudden inward tilting as seen on the videoscreen - occurred. Data points with a superimposed arrow indicate that there was no failure at the maximum N. On the average, the depth of embedment required for stability at N>100 is about 1-7/16 inches.

The trend for the required depth of embedment to increase with N is attributed to the influence of stress level upon friction angle -although it is recognized that apparent cohesion may have influenced the results obtained at normal gravity.

Analysis of Results

Fig. 3 portrays the forces that are typically assumed to act on a free-standing wall. Given that a certain depth of embedment d is required for the threshold of equilibrium, the friction angle for the sand can be back-calculated by trial and error. The steps are as follows:

* Assume friction angle ϕ and find the components of the active force.
* Taking moments about point A, find the normal component of passive thrust required for equilibrium.

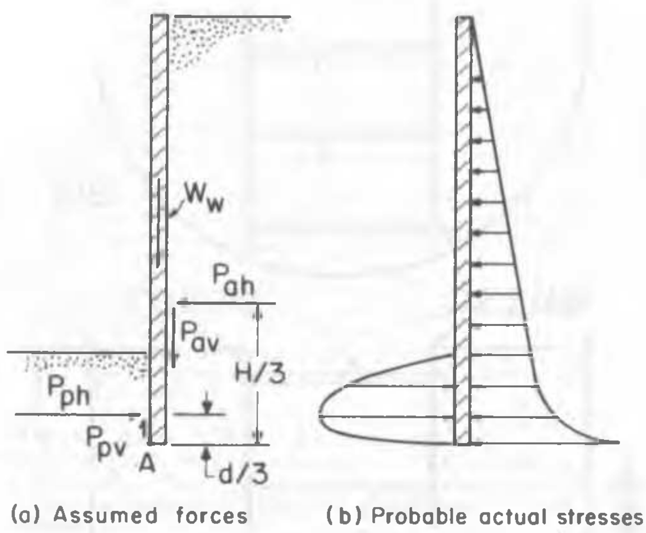

Fig. 3 Forces and Stresses on Free Standing Wall

* Considering the wall friction on the passive side ($P_{pv} = P_{av} + W_w$), find the friction angle corresponding to the passive thrust.
* Repeat until the assumed and computed friction angles agree.

This calculation has been made using values of K_A and Kp from Caquot and Kerisel (1949). On the active side, ϕ_w was taken as 2/3ϕ. Results are as follows:

Depth d(in.)	ϕ
1-1/16	53°
1-3/16	50°
1-7/16	46°

In tests at normal gravity the level of stresses within the sand is very low. Even at 100g, the average vertical stress within the passive wedge is only about 0.2 tsf. Hence high effective friction angles are to be expected.

This calculation requires, in order to satisfy horizontal force equilibrium, a rather large force at the base of the wall on the active side. Presumably this force represents large pressures developed as a reaction against rotation of the bottom of the wall back against the backfill in this area.

Some Complications

One factor ignored in the foregoing analysis is the "horizontal" component of centrifugal acceleration that acts when the walls are oriented as in test series H1 and H2. This effect arises because each wall is off the "vertical" centerline of the model so that the centrifugal acceleration is inclined slightly. This situation creates a restoring moment, thus reducing the depth of embedment required for equilibrium. The results of test series S, where this effect is not present, are presumably more correct.

Test series H2 performed in a rectangular bin gave somewhat smaller required depth of embedment than series H1 in the circular bin. Presumably the difference reflects the effect of side friction, but exactly how this effect is manifested is not clear.

4 STRUTTED TESTS

These tests were prepared in the same manner as unstrutted tests. The sand inside the walls was smaller than that required to support an unstrutted wall. N was increased slowly until failure was observed. Typically such failure was sudden and catastrophic.

Fig. 4 summarizes the data for N required to cause failure, when struts were positioned 7/8 in. below the tops of the walls, versus depth of embedment. Buckling of the struts could be seen well before collapse; at N=60-85, more or less independently of the embedment depth.

Fig. 5 summarizes the data for the influence of the elevation of the struts. As the elevation decreased, the N at initial buckling and at collapse both decreased.

Buckling Strength of Struts

Five tests were performed using a standard compression testing machine to establish the buckling load of the struts: peak forces were 25.9 ± 1.3 lbs. Buckling occurred at an axial shortening of about 0.01 to 0.02 in, with resistance then dropping slowly as further compression was applied. The predicted elastic buckling load was 26.8 lbs.

In several tests on the centrifuge, two or more struts were strain-gaged to measure axial force. Results showed a peak force when N=70, with the measured force then remaining constant or dropping somewhat as N increased further. These peak forces ranged from 21 lbs to 36 lbs, with a mean of 28.0 lbs. Various factures may be responsible for the differences from the expected peak loads: eccentricities introduced during construction of the models, greater restraint at the ends of struts, and inconsistencies of calibration relating strain gage output to axial force.

Analysis of Results

Calculations were performed using the forces shown in Fig. 3, but now with strut forces included. A friction angle of 45° was assumed as representative for stress levels associated with large N together with lateral force coefficients from Caquot and Kerisel. The N required to cause failure was computed for several different assumed strengths of the pair of struts. Results are given in Table 2.

Also given in Table 2 are predictions for the N at which buckling of the struts should begin. These values were estimated by the same calculation, but omitting passive resistance, wall friction on the active side and the weight of the wall-on the basis that until strut buckling begins there is not enough wall movement to mobilize wall friction or passive resistance. This hypothesis is supported by the data indicating that buckling begins well before a collapse condition is reached.

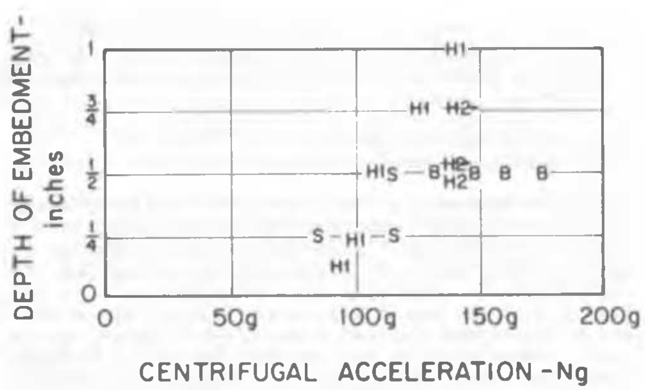

Fig. 4 Effect of Embedment for Struts 7/8 in. Below Top of Walls

Table II
COMPUTED CENTRIFUGAL ACCELERATION COEFFICIENTS REQUIRED TO CAUSE COLLAPSE (INITIAL BUCKLING IN PARENTHESES) ASSUMING ϕ = 45°

Assumed strut resist.	Embedment and strut depth-in.			
	d = 1/2			d=3/4
lbs.	7/8	3 1/2	5	7/8
45	109(94)	62(54)	36(31)	120
50	121(105)	69(60)	39(34)	133
55	133(115)	76(66)	43(38)	147
60	145(125)	83(72)	47(41)	160
65	157(136)	90(78)	51(44)	174
Observed	106-175 (±60-80)	108(~60)	72 (40-60)	114->140

The forces giving moment equilibrium in these calculations do not in general satisfy horizontal force equilibrium. If only force equilibrium is considered, collapse is computed to occur at centrifugal accelerations of from 69g to 100g, for strut strengths from 45lb to 65lb, respectively.

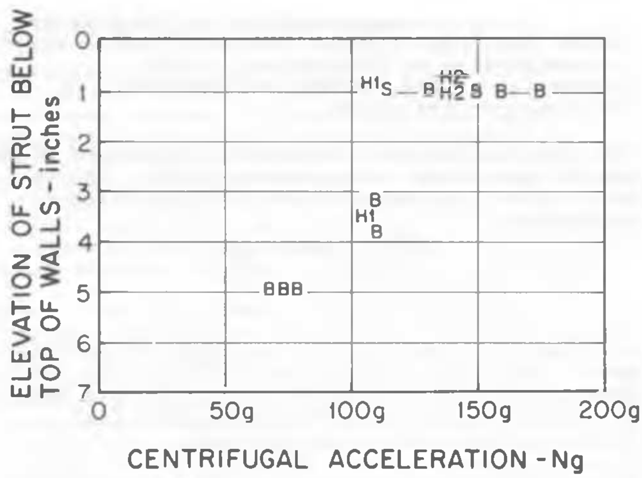

Fig. 5 Effect of Elevation of Strut for d=1/2 in.

5 INTERPRETATION OF RESULTS

The foregoing comparisons between theory and observed results may be summarized as follows:

* The calculation procedure for unstrutted excavation provides a reasonable explanation for the results. While the required ϕ are somewhat high, the decrease in ϕ as a function of g-level is reasonable. The results imply large active-side pressures near the base.
* For strutted walls, the adopted calculation method produced the following predictions for N at buckling or failure, compared to the the observed values:

Struts at	7/8 in.	3 1/2 in.	5 in.
Buckling	Overest.	OK	Underest.
Collapse	OK	Underest.	Underest.

With struts at 7/8 in. depth, force equilibrium is violated at the N predicted from moment equilibrium. For 3 1/2 in. depth, both force and moment equilibrium can be satisfied simultaneously. The other pertinent observation is that there was considerable scatter in the N at which apparently similar models, constructed by different investigators, failed.

The following hypotheses are offered in explanation of these observations:

(1) As N increases, the initially stiff struts restrain the wall and prevent development of active conditions. Once the struts begin to buckle, the lateral stresses drop to active values and at the same time the passive resistance at the toe is mobilized. The N at which collapse occurs is sensitive to the post-peak resistance of the struts, which is in turn affected by machining and installation of the struts.

(2) The available passive resistance of the sand is greater than predicted, because the effective ϕ is larger than assumed or because the soil between the walls is squeezed to a greater than initial thickness before full passive resistance was mobilized. A 40% increase in passive resistance suffices to explain many of the inconsistencies between calculations and observations. In the unstrutted case, this hypothesis permits a greater moment of passive resistance for a given resultant passive force (see Fig. 3b). In the strutted case, the hypothesis implies a larger potential passive resistance which contributes to equilibrium of horizontal forces and - with the deeper struts - contributes more significantly to resisting the overturning moment.

6 FINAL COMMENTS

The results generally confirm previously-observed considerations to the behavior of strutted excavations and emphasizes again the need for conservatism when designing a support system employing compression members.

The tests also demonstrate once again the sensitivity of friction angle to stress level, and that results from small-scale tests in normaly gravity can be misleading quantitatively (and perhaps qualitatively). This difficulty can be overcome by centrifuge testing, thus allowing investigators to realize the advantage of small-scale tests that can be performed rapidly and duplicated readily. Disadvantages of centrifuge tests lie in the difficulty of instrumentation at that scale and in the effects of a non-uniform gravity field.

ACKNOWLEDGEMENTS

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REFERENCE

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