Generation of seismic boundary values for industrial stress conserving scale models

Génération des conditions limites pour une similation sismique industrielle qui conserve les contraints

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SYNOPSIS: The paper deals with design criteria for seismic simulators.

The study of earthquake effects is a principal justification for the large expenditures concerning centrifuges. Earthquake equipment may consume a large part of the pay-load while echoes dictate the use of large soil bodies. Therefore, centrifuges should be designed for seismic simulation, which is not the case at present. For engineering purposes the design follows closely the search for measurable signals suitable for processing algorithms.

Within the Framework of Laboratoire de Mecanique des Solides - Ecole Polytechnique, France (LMS), the author developed, starting in 1977, the earthquake simulator of the 10m arm centrifuge of Centre d'Etudes Scientifiques et Techniques d'Aquitaine (CESTA) of CEA, near Bordeaux. CESTA carried the material part of the project, and the main participants were first B. Devaure and later P. Leguay (also in vibration tests). A laboratory preliminary cell was used for extensive studies of echoes and free field seismic motions; carried out on 1t of sand - pneumatically rigidified. Then a second cell was built for the centrifuge while the first one was equipped with Hydraulic Gradient and installed at LMS. The centrifuge cell uses programmed explosions and the Hydraulic Gradient one shock tube discharges with similar echo chambers and filters, giving similar free fields. (Devaure Mathivet, 1979, Devaure, 1980, Zelikson Devaure Badel, 1981, Zelikson Bergues, 1981, Zelikson Leguay Pascal, 1983, Zelikson Leguay, 1986). The CESTA earthquake (and vibration) simulator has been in service since 1979 and used by many people. It is still the only installation creating real earthquake imitations on adequately large models. Therefore, it seems that some of the design considerations and experience should be presented to the Geotechnical public.

For lack of space formulae are presented in "short-hand". The inputs are called "force" (f), the outputs "displacement" (x) (even if some of them are velocities) which are (column) matrices, function of time, t, or in the sampled case of an index n (e.g. t=nT). Transforms of any kind use capital letters (e.g. $x \leftrightarrow X$). Discretization by finite elements gives a differential equation e.g. $dx/dt = ax+f$. Principally the model should define a, which varies up to failure. a defines the structure-soil system. The structure on its base and the substratum loaded by the base are totally different systems. For illustration, take the problem of defining the force on an electric charge inside an accessible insulated tube, caused by charges outside the tube and inaccesible. Case (a), well separated point charges close to the tube. The system is defined by a small number of field measurements carried out near the charges. This case illustrates the structure. Case (b), charges are diffused. The field has to be measured all along the tube's section of interest. This case illustrates the substratum. The structure as built is (a) and (b) but closer to (b).

Let it be possible to separate the linear and non linear parts of the equation, namely

$$dx/dt = kx+bx+f, \text{ where } k=k(t).$$

The linear part can also be illustrated by case (a) above. In fact, the system is completely defined by the n solutions y_n of

$$dy_n/dt = ky_n \quad \text{with } y^l{}_n(o)=\delta^l{}_n.$$

Writing y for the matrix the columns of which are respectively y_n, a solution of $dx/dt = kx+f$ (starting at rest) is

$$x(t) = y(t)\int_0^t y^{-1}(l)f(l)dl$$

If the system does not change in time, then

$$y(t)y^{-1}(l) = y(t-l) \text{ and } x(t) = \int_0^t y(t-l)f(l)dl$$

i.e $x = y*f$ (convolution). Therefore, the transforms are related by $X = YF$, and this is the reason for work in the transform or "frequency" domain. Y is the flexibility matrix, and $Y^{-1} \cong K$ the rigidity one. The algorithms for the experimental extraction of K must be suitable for the centrifuge and take in account its limitations (e.g. echoes). They are different according to the structural case (a) or the substratum (b) or both. (A "cousin" of convolution is correlation $c_{xf} = x(t+l)f(l)dl$, for which a similar law gives $C = X\bar{F}$; no distinction will be made between their uses). In order to find the role of the free field signal let the structure's motion be given by x_a for inner points and x_b for the base, and that of the substratum by the same x_b and by x_c which are on a fictious cut which separates it from far away regions.

The free field earthquake is x_b^f, x_c^f. The earthquake creates base forces only, f_b. For a free field $f_b=0$. The rigidity matrices for the structure K, for the soil Z, and they are properly partitioned giving in the linear case:

$$K_{aa}X_a + K_{ab}X_b = 0;\ K_{ba}X_a + K_{bb}X_b = F_b$$

(structure)

$$Z_{bb}X_b + Z_{bc}X_c = -F_b;\ \text{also } Z_{bb}X_b^f + Z_{bc}X_c^f = 0.$$

It is assumed that as points c become far from the structure, the latter's influence disappears, giving $X_c^f \cong X_c$ so that $Z_{bb}X_b^f + Z_{bc}X_c \cong 0$ giving $Z_{bb}X_b - Z_{bb}X_b^f = -F_b$.

The equations for the structure as built become

$$K_{aa}X_a + K_{ab}X_b = 0;\quad K_{ba}X_a + (K_{bb} + Z_{bb})X_b = Z_{bb}X_b^f.$$

The key elements are: 1) Z, the rigidity matrix of the base relative to a soil which is at rest in far regions, 2) x_b^f, the free field motion of the base and eventually, 3) f_b given by $f_b = Z_{bb} * X_b^f$, i.e. the force at the base.

In any event, the outer part of the soil will alway behave elastically, and b points can be placed at its boundary, thus including part of the soil in the structure. In the case of piles, for example, those points might be at some depth. The structure as built than can behave inelastically subject to the external forces X_b^f and internal forces $-Z_{bb}*X_b$. X_b are part of the unknowns of the differential equation which must be solved in the time domain. Let the differential equation be written in the form:

$m\ddot{x} + c\dot{x} + kx = f$ and let $c\dot{u} + ku = f$.

That gives $m(x-u)'' + c(x-u)' + k(x-u) = -m\ddot{u}$ where $-m\ddot{u}$ are the inertial ("volume") forces relative to u. If the structure is simply supported then x_b generates in it only rigid body motions. In that case $u_b = x_b^f$ and the inertial forces follow as if the structures were excited on a shaking table. This simple case applies e.g. to earth dams on rock. Normally the base points b will be under the superstructure, and in the non-linear case an $f_b - x_b$ relation will be needed when the earthquake motions X_c are acting. In can be argued that X_c could be taken as zero. However, this argument depends on the size of the soil body.

An earthquake generator is thus seen to be that which creates given free field motions at points b. An actuator generates a given signal at its end by a closed loop control. There is no way an actuator can be controlled (automatically) to give free field motions at several points. This can be achieved by several actuators controlled by a computer.

The remaining solution is to apply "tailored" signals at one or several points. Those are pre-arranged signals without automatic control, which have been found by trial to best give the free field motions.

The free field signals are part of the regulating codes. In many cases in the past actual earthquake motions (e.g. El Centro) have been used. However, information given by accelerometers (in built areas) is incomplete, in the same way that giving the acceleration at the head of a pile is incomplete without the force. The power is p.v (p traction, v velocity), p=s:n (s stress, n normal vector), so that the power flux vector is s:v. In the case of an advancing plane wave its magnitude is $(dc)v^2$ (d density, c phase velocity). For example, let a model be excited by p acting on an 0.5m by 0.5m (0.25 m^2) area of its boundary, for a velocity of 1 ms^{-1} amplitude, and acceleration of 100x6.28 = 628 ms^{-1} at 100 Hz, (corresponding at 100 times g to 1 Hz, 0.6 times g in situ). Let c=300 ms^{-1}; d=1500 kg/m^3 so that p=0.45 MPa and the mean power flux 1/2x450,000 W/m^2 or 58 KW on the area. An earthquake of 20 cycles will require 58,000x0.2 $\cong$ 12,000 J corresponding e.g. to 6g explosives, or an equivalent volume of compressed air. In standing waves there is no power flux through the soil at all. In order to translate the signals at an observation station to another site X_c^f must be calculated from X_b^f using $Z_{bb}X_b^f + Z_{bc}X_o^f = 0$ by "deconvolution". But Z_{bb}, Z_{bc} are normally unknown. To resume, based on actually earth-quake signals, the motion at a given site can only be guessed. For calculations and models a signal has to be chosen. For this purpose signals are first processed to give a pseudo velocity [v], and pseudo acceleration [a], displacement [x], or the "shock spectrum". The signal is used as an input to a single degree of freedom resonator h and the maxima of the outputs $\ddot{x}^f * h$ used for the spectrum as the frequencies and dampings of the resonator are varied. In the "structural" case the system is defined by a small number of resonating modes, for each of which a value of [v] can be chosen and the results combined and averaged. The structure on its base is <u>not</u> such a system, and experiments to define its modes usually have poor yield either in situ or on centrifuge models where 2-3 modes can be distinguished with difficulty. (Much material is given in the proceedings of K2 section of Chicago SMIRT 1983 - where one American discusser said: "if you go into the wood looking for people ("modes") and find only gorillas - that is because no people are there in the first place"). This fact is recognized by the calculators who use the free field to calculate the shock spectra at the floor level which gives another one for the installations, etc. It is proposed to rationalize as follows: A simplified structure and soil system is taken which includes: horizontal and rotational base motions, 1-2 degrees of freedom for the structure (cantilever in shear or bending, etc.). For such a system the springs and frictions are time dependent to represent deterioration. The basic solutions y_n give the matrix y (which has y_n as columns), the inputs f are processed by

$$x(t) = y(t)\int_0^t y^{-1}(l)f(l)dl$$

and pertinent items (base motion maxima, accelerations at the top, stresses,...) taken as criterions. The y systems can be defined by the codes.

The boundaries of the cell must provide the correct initial stresses at rest, and the motions of the earthquakes. Incorrect motions send echoes back to the structure and may result in cellular modes inexistent in situ. It is sometimes argued that protective layers eliminate the echoes; e.g. if mud is used to support the walls or if the soil is sloping as an embankment. However, it is seen that the initial conditions are incorrect. Also a substantial amount of pay-load is lost (take e.g. the "embankment" case) because according to theory the waves do not "see" obstacles less than a quarter wave length and the boundary has a large area (for some materials like Duxseel introduced by Prevost the thickness is halved, but that does not change the situation). The conclusion is that the cell must be large relative to the structure. How best to use a cell? A ball thrown at the back of a receeding car will fall limply to the ground. Likewise active boundaries can absorb echoes, as in Hydraulic laboratories. From (Slater): "Experiments aimed at generating power from sea waves used models which sometimes reflected waves back to the wave-maker. The difficulty was avoided by changing the wave maker control system so that it could absorb reflections. A wide tank using a bank of absorbing wave makers is being used..." Another example is the current use of cells the side of which move "in step" with the base during shaking table seismic simulations. Another way is the transformation of echoes to noise. By definition the correlation between the useful signal and the noise is zero. Therefore, signals radiated from the base, converted to noise by the boundaries and dissipated inside the cell should cause the same base-damping as if radiations were to infinity. Such a conversion is carried out by non linearities in the boundary region. It is usually enough for large cells. One way to do it is by randomly moving geometric changes at the boundary. An industrial model should be qualified for echoes. Let the echo system h^e be linearised: inputs f_b at the base give output echoes x_b^e according to $x_b^e = h^e * f_b$; $X_b^e = H^e F_b$. If f_b is small enough, the direct interaction is also linear $x_b^d = h^d * f_b$. h^d can be found from the part of signals before the first echo arrives. In large cells this echo-free part can be made larger by using smaller models of the model base. If the direct signal plus the echo is $x_b = h * f_b$, then $h^e = h - h^d$. If h_e, H^e are known than during an earthquake simulation the generator controls should in principle be able to eliminate the echoes using the f_b signals in real time, like in the sea waves example above. In any event, they can be taken in account for tailoring the earthquake signal. If a calculation is carried out, the echoes can be filtered out. In some cases, H^e was separated from F_b by taking the complex logarithm $\log X_b^e = \log H^e + \log F_b$. The procedure used in speech and seismology signals processing is described in details in (Oppenheim-Shafer). The effect of echoes can best be measured by "inter" and "auto" correlations, i.e. by inner products of two signals, one of which is time shifted relative to the other. This was one of the procedures used to qualify the CESTA cell. The signal was a sharp blow of a mass falling on a (0.08m diameter) plate. Fig. 1 shows the signals at 0.1m and 0.3m (0.1m from the wall) from the centre of impact; and at a depth of 0.1m. The influence of the wall is clearly seen, while the middle of the cell is clear of echoes. Figure 2 gives the auto-correlation curves which lead to the same conclusion. The tests qualified a nuclear power station model the base of which was 0.15m radius.

Often a linearised system has to be defined: to find either y or Y in respectively $x = y * f$ or $X = YF$. Periodogramme techniques do it directly, and the noise appears in x, which is not so important for "structures" where "zooming" near resonance can be used. In correlogramme techniques the noise does not appear, as its correlation with the signals is zero. c_{xf}, c_{ff} are calculated, then C_{xf}, C_{ff} giving: $C_{xf} = X\bar{F}$, $X = YF$ $C_{xf} = Y(F\bar{F})$ or $C_{xf} = Y\, C_{ff}$. In both cases it is better to use the echo-free part of the signal. The standard deviation is proportional to $\sqrt{N^{-1}}$, N being the number of oscillations taken in account, which need not form a single train of vibrations.

The usefulness of the frequency domain drops considerably in the nonlinear case. Stoker wrote a book about the equation $m\ddot{x} + f(x, \dot{x}) = A \cos wt$. (f "almost" linear). He resumes in p.117: "General solution not known, but there exist many types of periodic motions (subharmonics, etc.)". For an excitation $A_1 \cos w_1 t + A_2 \cos w_2 t$: "General motion unknown... Mathematical difficulty of small divisors occurs..." Volterra generalised the convolution as follows:

$$x(t) = \int_0^t y_1(t-l) f(l)\,dl + \int_0^t y_2(t-l, t-k) f(l) f(k)\,dl\,dk + \ldots$$

y_1 - first order approximation, y_2 - second order, etc., which can be extracted if the system is excited by programmed series of pulses, without echoes. A more suitable way is to back calculate the f-x relations from the Lineard spirals (Lineard 1928, Stoker), (which show that in the case of Coulomb friction x can diverge to infinity for a steady f, in contrast to the viscous case). An X-F Bode diagramme is given in Fig. 3; the hysteresis loops of which are e.g. Fig. 4. They were obtained from steady state shaking of a 4 piles group by an electromagnetic shaker. A perturbing resonance is clearly seen in Fig. 3, resulting from a proximity to the wall. In problems like the group of piles, what is required is the behaviour at the end of a (limited) number of cycles. Therefore, a shock could be applied after a sequence of low frequency mechanical shaking. Electromagnetic shakers which have no constant forces are unsuitable. Motors with small masses have been used in CESTA. They have very low (parasitic) masses and perform well, but are suitable for steady state only. Precision excitation has yet to be fully developed.

The simulation of earthquakes by controlled shaking of parts of the boundary of very large cells can be carried out on Hydraulic Gradient models. On the centrifuge, rotating buckets protect the arm from the (ordinary) vertical vibrations. Correspondingly, a large reaction mass has to be provided. (If momentum $\pm p$ is acting on m_1, m_2 the energy is as $1/m_1$ to $1/m_2$). Logically that can only be the mass of the cell itself, i.e. systems like the CESTA one, excited also from below. There, the frequency of tailored signals depends on the size of the filters and those can be studied using the similarity with waves in shallow water ("water table"). For fixed buckets (Cambridge. ISMES) motion must be in the flight direction (to protect the arm), but the heavy arm can be used for reaction and for installation of most of the earthquake generation equipment (Coriolis parasites are less than 10%), so that actuators are possible.

To conclude, design can start once one knows what he wishes to do with the signals - and looks for suitable inputs according to the type of structures, and the regulating codes. Earthquake and vibration equipment should be designed with the centrifuge.

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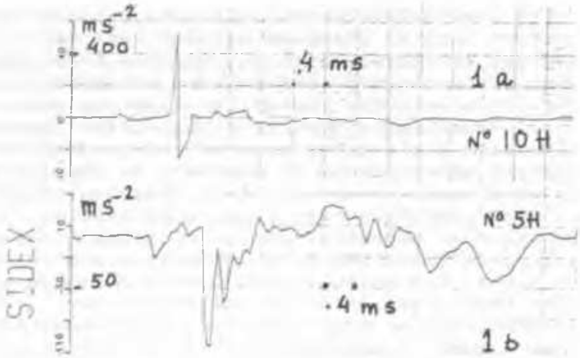

Figure 1. Acceleration signals.

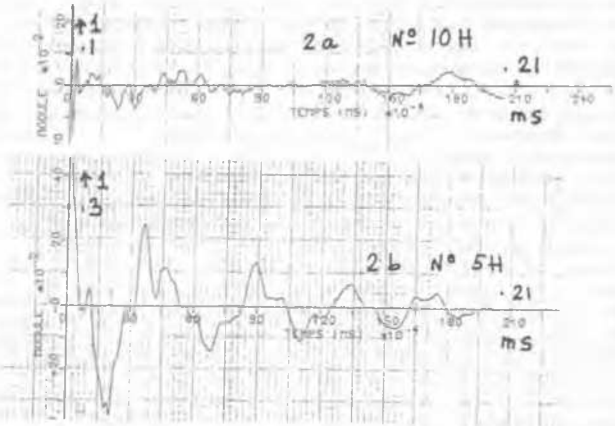

Figure 2. Autocorrelation curves.

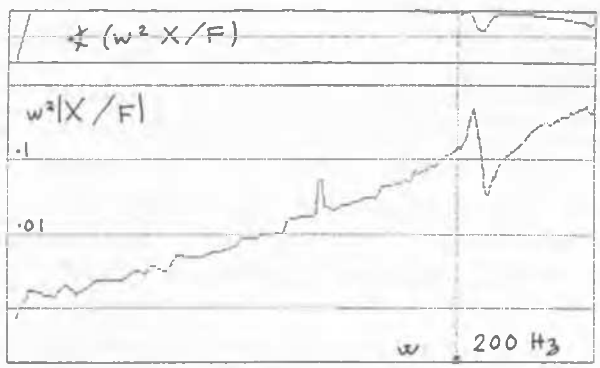

Figure 3. Bode diagramme.

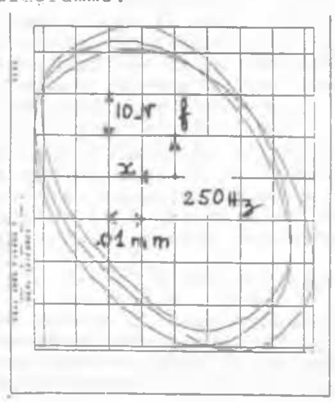

Figure 4. Hysteresis loops of force-displacement.