Determination of the load transfer parameters of pile foundations

Détermination des paramètres de transfert de charge dans les fondations sur pieux

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SYNOPSIS In reference (1), many factors affecting the general mechanism model of the pile-cap -soil interaction were pointed out. Now, through a comprehensive consideration of these factors, the basic theoretical equations for calculating the pile-soil displacement by elasticity theory are proposed, so that it makes the concrete analysis of the stability and settlement calculation of a low-cap piled foundation by load transfer theory possible. By this proposed method, the load transfer of non-linear characteristics between pile and soil on the pile shaft and under the pile tip can be considered and the effect of all factors mentioned in reference (1) on the portions of the total load carried by the pile tip, the pile shaft and the soil under the bottom of the pile cap can also be estimated. Finally, the load-settlement curve can be plotted.

I. INTRODUCTION

Since the available methods for calculating the bearing capacity and load-settlement curve of piled foundation have their severe limitations nowadays, as a consequance, the basic concept of piled foundation load transfer theory was developed in order to establish a more rational calculating model. The aim of this paper is to find a way to calculate the displacement of a pile-soil system with the basic equations of the theory of elasticity with giving a comprehensive consideration to all factors mentioned in Ref. (1), so as to make the practical usage of the calculation of the stability, settlement of a piled foundation by load transfer theory possible.

II. ESTABLISHMENT OF CALCULATION METHOD

Since the problem is rather complicat, it is necessary to stress on the main contradiction i.e. the non-linearity of load transfer in a pile-soil system. Therefore following assumptions must be made:

1. Only the final settlement and not the process of consolidation is to be considered.
2. The additional stress and settlement of every point in subsoil under the cap are calculated by the thoery of elasticity.
3. For simplification, the cap is considered to be absolutely rigid.
4. Loading on the foundation and the overburden pressure of subsoil are transfered through the cap to piles and to subsoil at on time.

In establishing the basic equations, several methods was proposed in Ref.(1). In which following main points can be summarized: 1. The relative displacement between pile and soil is caused by the difference between the settlement of the soil at every points and the forced settlement of the pile. 2. The unit friction is an unique function of the relative displacement. 3. In considering the installation of piles, the geological and geometrical conditions, the resistance and settlement of the pile tip are expresed by multinomial functions. 4. For frictional soild, strengthening effect of the pile cap pressure to the pile tip resistance and the friction on the pile shaft should be considered. 5. To a certain depth from the bottom of the cap the downward movement of the pile and the soil are actually synchronous, so that the friction within this depth is weakened. As for rigid piles, its "Barrier effect" will render the movement of the soil within the pile group being close to be vertical, therefore the one-dimensional compression theory can be used directly in the calculation of the settlement of the soil within pile group.

The procedures of the application of these methods will be illustrated here through the following example. The derivation of the basic equation is as follows:

1. Portion of load carried by the cap Per(%) and the expression of P-S curve.

From Ref. (1), it is known,

$$P_{er}(\%)=\frac{P_c}{P}\times 100(\%)=\frac{P_c}{nP_b+nP_s+P_c}\times 100(\%) \qquad (1)$$

In which P, P_c-- total load on foundation and the load carried by the cap respectively

P_b, P_s-- pile tip resistance and shaft resistance of each pile in foundation respectively

n-- number of piles

All terms in both sides of Eq.(1) are depended upon settlement, so Eq.(1) can be rewritten as:

$$P_{er}(\%)=\frac{F_2(S_g)}{nF_3(S_b)+n\pi D F[\Delta S_{(z)},\sigma_{x(z)}]+F_2(S_g)}\times 100(\%) \qquad (2)$$

In which

$F_2(S_g)$ equals to P_c, the total load acting on the bottom of the cap, a function of settlement of the ground surface S_g.

$F_3(S_b)$ equals to P_b, the tip resistance of each pile, a function of the displacement of pile tip.

$\pi D F(\Delta S_{(z)},\sigma_{x(z)})$ equals to P_s, the shaft friction of each pile, it is a function of the relative displacement of pile-soil

$\Delta S(z)$ and the effective normal pressure on the pile-soil boundary $\sigma_x(z)$ at depth Z.

For simplification and by considering this research has its practical meaning only for the low-cap piled foundation of rather whort piles. The elastic compression of piles can be neglected, therfore $S_b=S_g$, Eq.(2) can be rewritten as:

$$P_{er}(\%)=\frac{F_2(S_g)}{nF_3(S_g)+n\pi DF[\Delta S_{(z)},\sigma_{x(z)}]+F_2(S_g)}\times 100\ (\%)$$

In which n is number of piles, F_2 and F_3 are function of S_g, only F is not an unique function of S_g, but in the final analysis $\Delta S(z)$ and $\sigma_{x}(z)$ are both depended on the development of S_g.
The expression of $F_2(S_g)$ and $F_3(S_g)$ were given in Ref.(1). In here, the derivation of functions between F and S_g is emphasized. Once they can be done, the whole problem can be solved.
The meaning of symbols used in Eq.(2) are shown in Fig.1 and 2. In Fig.2 the settlement of soil at depth z is expressed as:

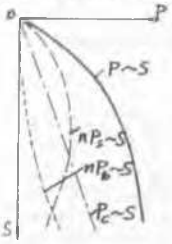

Fig.1 P-S Curve

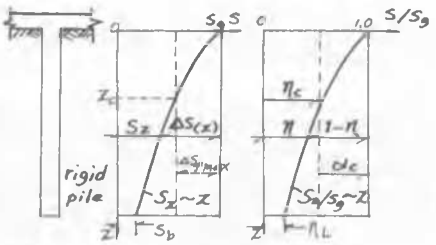

Fig.2 Settlement of soil vs depth

$$S_z=\eta S_g\ (\eta<1.0) \tag{3}$$

and the pile-soil relative displacement at depth z is

$$\Delta S_{(z)}=(1-\eta)S_g \tag{4}$$

Eq.4 is simplified because the elastic compression is neglected. It can also be rewritten as:

$$\Delta S_{(z)}/S_g=(1-\eta) \tag{5}$$

Similarly, for certain types of soils and pile materials, the maximum relative displacement corresponding to the ultimate shaft friction on pile-soil boundary is a constant without much fluctuation, so it can be expressed by a coefficient α, then

$$\alpha=\Delta S_{fmax}/S_g \tag{6}$$

α varies from $\infty\to 0$, corresponding to S_g varies from $0\to\infty$. From Eq.5 and Eq. 6 we obtain,

$$\Delta S_{(z)}/\Delta S_{fmax}=(1-\eta)/\alpha \tag{7}$$

Furthermore, the function F in Eq.2 and Eq.2 itself has already contained the factor of "Strengthening effect". If it is neglected, F will become $F[\Delta S_{(z)}]$.

2. The derivation of function $F[\Delta S_{(z)},\sigma_{x(z)}]$
Function F contained two factors i.e. the "Weakening effect" and the "Strengthening effect". In order to find out finally the relation between $F[\Delta S_{(z)},\sigma_{x(z)}]$ and the settlement of ground surface S_g, two coefficients ζ_1 and ζ_2 are adopted, then

$$F[\Delta S_{(z)},\sigma_{x(z)}]=\zeta_1\zeta_2 F_1(S_g)\cdot L \tag{8}$$

In which ζ_1 --modified coefficient for "Weakening effect"
ζ_2 --modified coefficient for "Strengthening effect"
can be defined as

$$\zeta_1=\frac{n\pi D\int_0^L F_1(\Delta S_{(z)})\,dz}{n\pi D\int_0^L F_1(S_g)\,dz}=\frac{\int_0^L F_1(\Delta S_{(z)})\,dz}{\int_0^L F_1(S_g)\,dz} \tag{9}$$

The meaning of ζ_1 in mechanics is as follows: The relative displacement between pile and soil of a rigid single pile changes from a uniform value of S_g into $\Delta S_{(z)}$ of each pile shaft in the pile group under a low-cap at different depth z. The value of ΔS_{fmax} above the critical depth z_c is smaller than ΔS_{fmax} (Fig.2,a), therefore at a certain depth below the cap, the mobilization of the shaft friction is limited and consequently weakenened.
Evidently, the value of ζ_1 depends on three factors: S_g, $\Delta S_{(z)}$ and ΔS_{fmax}. In above equation, function F_1 itself is a function of the mobilized shaft friction f_s and $\Delta S_{(z)}$ of the corresponding point.
The definition of ζ_2 can be expressed by,

$$\zeta_2=\frac{\int_0^L[F_1(\Delta S_{(z)})+\Delta F_1(\Delta S_{(z)})]\,dz}{\int_0^L F_1(\Delta S_{(z)})\,dz} \tag{10}$$

In which $F_1(\Delta S_{(z)})$ defined as before, and $\Delta F_1(\Delta S_{(z)})$ is a function of the increment of the shaft friction induced by the cap bottom pressure.

Therefore

$$F[\Delta S_{(z)},\sigma_{x(z)}]=\zeta_1\zeta_2\int_0^L F_1(S_g)\,dz$$
$$=\int_0^L[F_1(\Delta S_{(z)})+\Delta F_1(\Delta S_{(z)})]\,dz \tag{11}$$

as $\Delta F_1(\Delta S_{(z)})$ is related to the horizontal stress increment in the soil inside the pile group, so it is possible to obtain by Boussinesq equations and then find out the $\Delta\sigma_x$ to obtain $\Delta F_1(\Delta S_{(z)})$ directly or it also can be obtained indirectly by $\Delta\sigma_z$ times K_0(earth pressure coefficient at rest) to obtain $\Delta\sigma_x$. In this paper $K_0\Delta\sigma_z$ is used in which the "Barrier effect" of the soil inside the pile group is considered.
From Eq.8 or 11, Eq. 2 can be finally rewritten as

$$P_{er}(\%)=\frac{F_2(S_g)}{nF_3(S_g)+n\pi DL\zeta_1\zeta_2F_1(S_g)+F_2(S_g)}\times 100\ (\%) \tag{12}$$

In the right side of the above equation, except the two coefficients ζ_1, ζ_2 which should be calculated according to aspecific engineering and geological condition, the other three functions F_1, F_2 and F_3 are mainly determined by the properties of soil and the characteristics of the boundary materials. They can be approximately predetermined and consequently optimizing selection of geometrical elememts of the piled foundation can be carried out.

III. DETERMINATION OF COEFFICIENTS ζ_1 AND ζ_2

Although F_1, F_2 and F_3 can also be called partial load transfer functions of a low-cap piled foundation. They can also be obtained simplely by pileload test or by plate load test. The F_2 even can be computed by the one-dimensional layerwise summation method with the result of the laboratory tests. Therefore for a low-cap piled foundation, the load transfer parameters to be determined in the light of specific geometrical and subsoil conditions. The derivation of ζ_1 and ζ_2 for a low-cap piled foundation with four piles is illustrated as follows:

(1). Determination of ζ_1

Assume the modulus of deformation is E and the Poisson's ratio equals to 0.5 for saturated soil and 0.3 for sand. Let the width of a square cap be B (or its equivalent radius for a circular cap). In according to the Boussinesq equations, the settlement at depth z below the bottom of the cap is:

for $\mu=0.5$

$$S_z=\int_z^\infty \varepsilon_z dz=\frac{1.5P_cR}{E}\frac{1}{[1+(z/R)^2]^{1/2}}=\frac{S_g}{[1+(z/R)^2]^{1/2}}=\eta S_g \quad (13)$$

for $\mu=0.3$

$$S_z=\int_z^\infty \varepsilon_z dz=\frac{1.82P_cR}{E}\left[\frac{1+0.286(z/R)^2}{\sqrt{1+(z/R)^2}}-0.286\left(\frac{z}{R}\right)\right] \quad (13')$$

$$=S_g\left[\frac{1+0.286(z/R)^2}{\sqrt{1+(z/R)^2}}-0.286\left(\frac{z}{R}\right)\right]=S_g\eta$$

For the function F_1, several calculation models have been proposed by different authers. For simplification, one of the models is adopted here as follows:

When $0\leq S_{(z)}\leq S_{fmax}$, the shaft friction is:

$$f_{s(z)}=\frac{\Delta S_{(z)}}{\Delta S_{fmax}}f_{smax}$$

when $\Delta S_{(z)}>\Delta S_{fmax}$,

$$f_{s(z)}=f_{smax}$$

The above relationship can be seen in Fig.3.

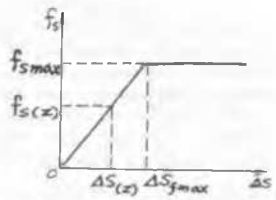

Fig.3 f_s vs ΔS

From Eq.9, $F_1(S_{(z)})$ respresents the mobilization of the unit shaft friction at z, While $F_1(S_g)$ represents the mobilization of f_s at z=0 when the settlement of the ground surface equals to S_g.

Therefore, for the consideration of the degree of the mobilization of the f_s in the entire length of the pile, the development of S_g can be divided into three stages.

a. First stage, $\alpha>1.0$

This implies S_g is very small, no failure occurs along the boundary, i.e. $S_z<S_g<\Delta S_{fmax}$

$$\zeta_1=\frac{\int_0^L\frac{\Delta S_{(z)}}{\Delta S_{fmax}}f_{smax}dz}{\int_0^L\frac{S_g}{\Delta S_{fmax}}f_{smax}dz}=\frac{\int_0^{\sqrt{\pi}t}\frac{1}{\alpha}(1-\eta)\frac{B}{\sqrt{\pi}}d\xi}{\int_0^{\sqrt{\pi}t}\frac{1}{\alpha}\frac{B}{\sqrt{\pi}}d\xi}$$

$$=1-\frac{1}{\sqrt{\pi}t}\ln(\sqrt{\pi}t+\sqrt{1+\pi t^2}) \quad (14)$$

all symbols are shown in Fig.4.

b. Second stage, $1.0>\alpha\geq\alpha_L=1-1/\sqrt{1+\pi t^2}$

This implies S_g is already larger than S_{fmax}, but point C in Fig.4 is below ξ_L, the entire length of ξ_L need not to be divided into two sections in calculation. Same as before, we get:

$$\zeta_1=\frac{1}{\alpha}\left[1-\frac{1}{\sqrt{\pi}t}\ln(\sqrt{\pi}t+\sqrt{1+\pi t^2})\right] \quad (15)$$

c. Third stage, $\alpha\leq\alpha_L=1-1/\sqrt{1+\pi t^2}$

In this case, S_g becomes considerable large, not

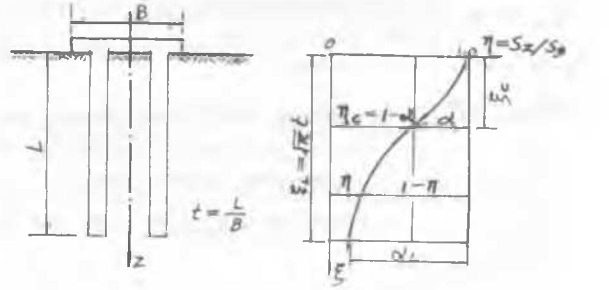

Fig.4 Relationship between η and ξ

only $S_g>\Delta S_{fmax}$ point C in Fig.4 is located in the interval of ξ_L. Therefore the entire length of ξ_L should be divided into $0\to\xi_c$ and $\xi_c\to\xi_L$ two sections in calculation. Same as before, we get:

$$\zeta_1=\frac{1}{\sqrt{\pi}t}\left[\int_0^{\xi_c}\frac{1}{\alpha}(1-\eta)d\xi+\int_{\xi_c}^{\sqrt{\pi}t}d\xi\right]$$

where $\eta=S_z/S_g$

$$1-\eta=\frac{S_g-S_z}{S_g}=\frac{\Delta S_{(z)}}{S_g}$$

$$\frac{1}{\alpha}(1-\eta)=\frac{\Delta S_{(z)}}{\Delta S_{fmax}}$$

$$\eta=1/(1+\xi^2)$$

$$\Delta S_{(z)}/S_g=1-1/(1+\xi^2)$$

From $\eta_c=1-\alpha$, so $\xi_c=\sqrt{2\alpha-\alpha^2}/(1-\alpha)$

$$\therefore\ \zeta_1=1-\frac{1}{\alpha\sqrt{\pi}t}\left[\ln\left(\frac{1+\sqrt{2\alpha-\alpha^2}}{1-\alpha}\right)-\sqrt{2\alpha-\alpha^2}\right] \quad (16)$$

Eqs. 14, 15 and 16 are valid for $\mu=0.5$.

For cohesionless soils $\mu=0.3$ we get:

$$\zeta_{1(0.3)}=\zeta_{1(0.5)}+0.143\left[\frac{1}{\sqrt{\pi}t}\ln(\sqrt{\pi}t+\sqrt{1+\pi t^2})+\sqrt{\pi}t-\sqrt{1-\pi t}\right] \quad (17)$$

(2). Derivation of ζ_2

On the basis of the above mentioned solution, an additional coefficient ζ_2 is used to take the "Strengthening effect" into account. The definition of ζ_2 is:

$$\zeta_2=\frac{nP_s+n\Delta P_s}{nP_s}=\frac{P_s+\Delta P_s}{P_s} \quad (18)$$

where P_s--shaft friction of each pile without canaidering the "Strenthening effect".

ΔP_s--the increment of P_s caused by the contact pressure on the bottom of pilecap.

Evidently, $\zeta_2>1.0$ is mainly valid for cohesionless soils only. In short, the copmlicat factors such as the original stress condition of the subsoil, the way of pile installation are disregarded at first. Let the normal stress induced by the self-weight of the subsoil on the pile shaft is equal to $K_o\gamma z$, so

$$P_s=\int_0^L K_o\gamma z\,tg\delta\,F_1(\Delta S_{(z)})dz \quad (19)$$

in where $tg\delta$--external friction coefficient on the boundary of pile shaft.

γ--the unit weight of subsoil.

From Ref.(1), there is linear relation between P_c and S_g, that is

$$P_c=CS_g \quad (20)$$

The increment of P_s induced by P_c is

$$\Delta P_s=\int_0^L K_o\,tg\delta\,\Delta\sigma_{z(z)}\,F_1(\Delta S_{(z)})dz \quad (21)$$

Furthermore, from Eqs.18, 19 and 20 we get:

$$\zeta_z = 1 + \frac{\int_0^L \Delta\sigma_{z(z)} F_1(\Delta S_{(z)}) dz}{\int_0^L \gamma z F_1(\Delta S_{(z)}) dz} \quad (22)$$

From Ref.(2), it is known,

$$\Delta\sigma_{z(z)} = P_c\left[1 - \frac{z^3}{\sqrt{(z^2+R^2)^3}}\right] = P_c\left[1 - \frac{\xi^3}{\sqrt{(1+\xi^2)^3}}\right] \quad (23)$$

$$\therefore \zeta_z = 1 + \frac{\int_0^L P_c\left[1 - \frac{\xi^3}{\sqrt{(1+\xi^2)^3}}\right] F_1(\Delta S_{(z)}) dz}{\int_0^L \gamma z F_1(\Delta S_{(z)}) dz} \quad (22')$$

Similar to Eq.14,

$$F_1(\Delta S_{(z)}) = \frac{\Delta S_{(z)}}{\Delta S_{fmax}} f_{smax} = f_{smax} \cdot \frac{1}{\alpha}(1-\eta) \quad (24)$$

Substitute into Eq. 22, finally, we get, when $S_g \leq \Delta S_{fmax}$

$$\zeta_z = 1 + \frac{\sqrt{\pi} c S_g}{\gamma B} \frac{\int_0^{\sqrt{\pi}t}\left[1 - \frac{\xi^3}{\sqrt{(1+\xi^2)^3}}\right]\left[1 - \frac{1}{\sqrt{1+\xi^2}}\right] d\xi}{\int_0^{\sqrt{\pi}t} \xi\left[1 + \frac{1}{\sqrt{1+\xi^2}}\right] d\xi} \quad (25)$$

when $S_g > \Delta S_{fmax}$, and $\alpha < \alpha_L = 1 - 1/\sqrt{1+\pi t^2}$

$$\zeta_z = 1 + \frac{\int_0^{\xi_c} c S_g\left[1 - \frac{\xi^3}{\sqrt{(1+\xi^2)^3}}\right]\frac{1}{\alpha}(1-\eta) d\xi + }{\alpha\int_0^{\xi_c} \frac{R\xi}{\alpha}(1-\eta) d\xi + \gamma\int_{\xi_c}^{\sqrt{\pi}t} R\xi d\xi} \; \frac{+\int_{\xi_c}^{\sqrt{\pi}t} c S_g\left[1 - \frac{\xi^3}{(1+\xi^2)^3}\right] d\xi}{(\text{denominator})} \quad (26)$$

Now ζ_z is solved, combining with ζ, then the P-S_g curve and the P_{er}-S_g curve can be obtained from Eq.12

IV. VERIFICATION OF THE CALCULATING MODEL

(1). Verification by a pile model in sand through the measured P-S_g curve and the P_{er}-S_g calculated from the data observed from strain gages sticked on the head of the model pile is quite agreed with that obtained theoretically by the above theory (Fig.5).

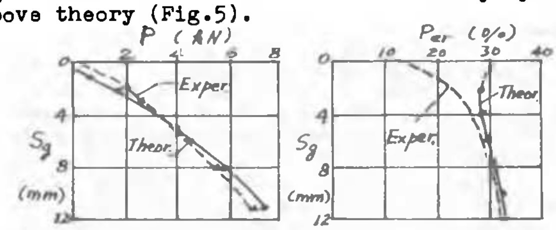

Fig.5 Comparison between theoretical results and experimental results for P-S_g and P_{er}-S_g

The axial load, the shaft friction and the bending moment distribution measured by many strain gages sticked along the entire length of the model pile are also clearly shown that the accuracy of the calculating method based on load transfer theory on piled foundations.

(2). Verification by in situ loading test of piled foundations

A single pile with a square cap is loaded in situ in a construction site in Zhejiang (Ref.3). The result of the test shows its shaft friction is weaker than that of a single pile without cap under the same settlement S_g. The modified coefficient of "Weakening effect" calculated by the result of the in situ test is also agreed closely with the one obtained by above theoretical calculation (see Fig.6)

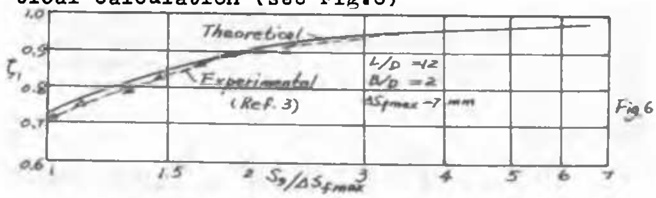

Fig.6 Comparison between theoretical result and experimental result for ζ_1-$S_g/\Delta S_{fmax}$

Another in situ test was carried in Shandong (Ref.4 and 5) of a series of pile groups also definitely indicated that the average shaft friction was affected by the presence of low-cap. The test result also shown that the "Weakening effect" and the "Strengthening effect" are existed.

The magnitude of these effects are depended upon the pile spacing.

V. CONCLUSIONS

(1). The load transfer theory not only can be used for analysis of single pile behaviours, it is vilad to be used in studying the low-cap piled foundations as well. The theory can be used to consider the non-linear charcteristic of stress and strain in the pile-soil boundary, the complicat geological conditions. It can be used to find the P-S_g curve and the corresponding portion of load carried by the cap Per(%) under different settlement S_g also can be find.

(2). The basic calculation formula proposed in this paper summarized the effects of all factors. In using them to solve pratical engineering problems, it is necessary to integrate them with the specific condition, but need not alternating the basic calculation principles.

(3). Pile-soil-cap interaction problems exist in all low-cap piled foundations (including the piled raft foundations). A design neglecting the portion of load can be carried by the soil under the bottom of the cap will be an highly oversafe one, especially for the low-cap piled foundation built on the frictional subsoils. As for other special conditions, such as friction pile groups in saturated soft subsoils (in which negative friction may become significant) and end-bearing pile groups etc., the behaviours of pile groups can also be predicted by using the calculation principles proposed here.

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