Improved model for progressive failure analysis of slope stability

Un modèle amélioré pour l'analyse de la rupture progressive des pentes

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SYNOPSIS: In the conventional slip-surface-ideal-plastic-failure approach in slope stability analysis only one single strength parameter - the shear strength of the soil - is used. Deformations within the sliding soil body, non-linearity of the stress-strain relationships and their effects on stress distribution along the potential slip surface are disregarded. Time enters into the analysis only in so far as we discern between drained and undrained conditions.
In references Bernander et al (1978-1984) it is shown qualitatively and quantitatively how a minor disturbance acting in a slope of strain-softening material may trigger a progressive failure which eventually propagates into vast areas of - may be - even horizontal ground. The present report deals with a further development of the progressive failure model, where the following improvements have been made: a) The soil volume is discretized not only in the downslope x-direction but also in the perpendicular z-direction thus permitting modelling of the deformations in the failure zone.
b) Along this zone the constitutive relationship is separated into two stages I and II, simulating conditions prior to, respectively after the formation of a slip surface. c) Time dependency of the modulus of elasticity is delt with. d) External arbitrary vertical or downslope loads can be accomodated in the progressive failure analysis. A computer program based on the new formulations has been established. Examples of results of the analysis are presented. The analysis is particularly relevant for markedly strainsoftening residual or sedimentary clays.

1 INTRODUCTION

Many of the extensive landslides which have occurred in Scandinavia, e.g. the Tuve slide, cannot be readily understood by means of conventional analysis based on the concept of ideal plastic failure. In many cases slides have been triggered by man-made operations of unknown intensity (piling, fills etc), but even in hindsight it has often turned out to be very unrewarding to attempt to explain the slide - and in particular the actual extent of a slide - by back-analysis in accordance with current procedures.

Bernander and Olofsson (1981, 1982) have e.g. shown that the Tuve slide may be phenomenologically understood with a progressive failure model. It was also demonstrated that - applying the current ideal-plastic failure analysis - it is possible to prove with a considerable margin of 'safety' (F = 3,5) that what actually happened in the Tuve slide should not have taken place. This fact disqualifies analysis based on ideal-plastic failure in long slopes in sensitive clays - especially for predicting the extent and degree of catastrophy of a potential slide.

The analytical approach demonstrated by Bernander et al (1981, 1982) has been further developed in a contribution to the Toronto symposium on landslides 1984. See references.

A landslide such as that at Tuve starts as a gradual acceleration of an ongoing creep deformation. The final phase takes place within one or a few minutes, which gives the timing of the events that govern the development, propagation and final extent and morphology of a slide. It is therefore the response of the soil at the actual strain rates that is relevant and only laboratory tests and analysis related to the actual timing of a slide can tell us

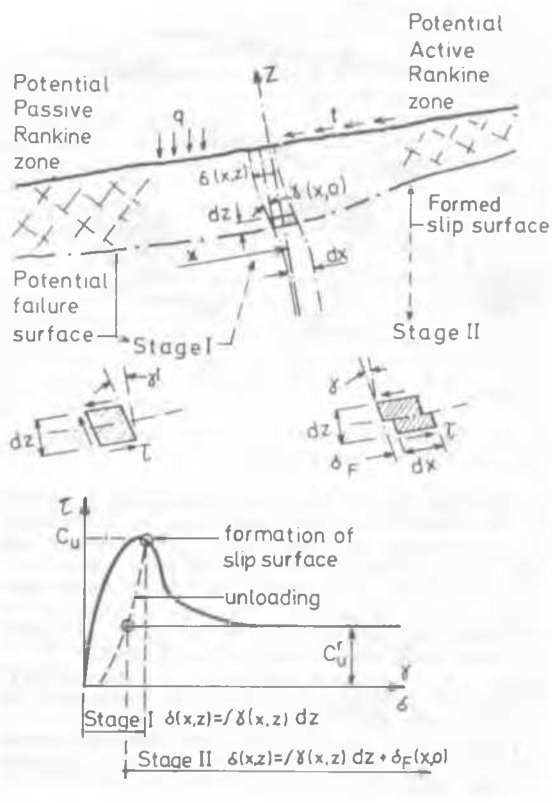

Figure 1. Basic principles for the progressive failure analysis. Notations.

something about the consequences of e.g. a local failure in a major slope.

Thus when analyzing rapid failure mechanisms, failure propagation and the ultimate extent (the degree of disaster) of potential slides in soft clays it is usually necessary to use undrained parameters which are relevant to the strain rates and displacements at stake.

The following report deals with an improvement of the soil model given according to Bernander et al (1981). The 1981 model was chosen for reasons of simplicity - the object of the analysis being of a piloting nature, i.e. to find out whether progressive failures in soft clays were phenomena likely to occur at all. The very instructive and encouraging results gotten from this progressive failure model have justified an improvement of the physical model. This new model was first presented at a poster session at ICSMFE 85, San Fransisco.

2 NEW SOIL MODEL

The improved model is defined in fig.2. The soil volume is discretized into vertical slices of thickness Δx as before. However, in the new model each vertical slice is subdivided into a number of elements in the z-direction thus permitting modelling of the shear deformations within the soil profile and more particularly the shear deformations in the potential failure zone - before as well as after the slip surface has been formed.

$$\delta(x,z) = \int \gamma(x,z)dz + \delta_F(x,o) \qquad (1)$$

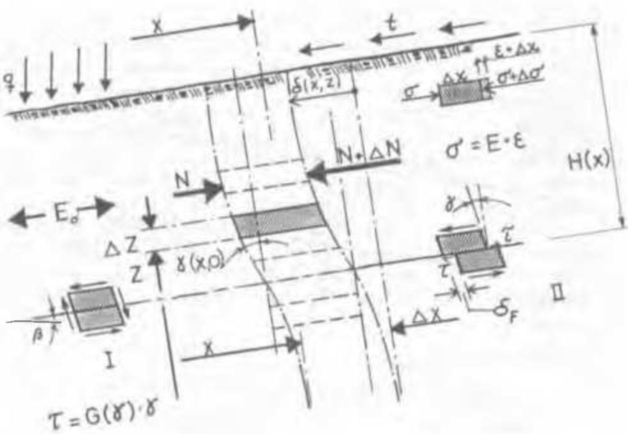

Figure 2. Improved model for the progressive failure analysis. Notations.

The problems with the time dependency of the moduli of elasticity are overcome by assuming that the modulus of elasticity (E) and the shear modulus (G) in the potential failure zone vary with time in a similar way such that the ratio of G/E is constant. The justification of this assumption may of course be queried but it is an often used expedient in structural analysis - e.g. in the analysis of statically indeterminate concrete structures. However, the inaccuracy that may ensue from this simplification is believed to be insignificant compared to the errors involved in the current use of the ideal-plastic failure approach on potentially extensive landslides in strain softening soils.

A third improvement of the progressive failure model is the possibility to accomodate additional arbitrary vertical and horizontal loads anywhere on the slope.

The formulation of the equations considers equilibrium critera as well as the compatibility of shear deformations and downslope axial compression in the soil volume.

a) Stage I - before slip surface formation. (Note x is positive in upslope direction $\cos\beta \simeq 1$. Note: In stage 1 $\delta_F = 0$).

$$\Delta N = \tau(x,o)\cdot b(x)\cdot\Delta x - \left[\sum_{o}^{H(x)} \rho(z)\cdot\Delta z\right] g\cdot b(x)\cdot\sin\beta(x)\cdot\Delta x - \Delta E_o(x) - q(x)\cdot b(x)\cdot\sin\beta(x)\cdot\Delta x - t(x)\cdot b(x)\cdot\Delta x \qquad (2)$$

$$\Delta\delta_{ave} = \frac{\Delta x}{E\cdot H(x)\cdot b(x)}\left(N + \frac{\Delta N}{2}\right) \qquad (3)$$

$$\delta_{ave} = \sum_{o}^{\alpha H(x)} \frac{\tau(x,z) - \tau_o(x,z)}{G(\gamma)}\Delta z + \delta_F(x,o) \qquad (1a)$$

$$\tau(x,z) = \phi(\gamma, \delta_F) \quad \text{(cf fig.3)} \qquad (4)$$

$$\tau_o(x,o) = \left[\sum_{o}^{H(x)} \rho(z)\Delta z\right] g\cdot\sin\beta(x) + \frac{\Delta E_o(x)}{\Delta x\cdot b(x)} \qquad (5)$$

where

- δ_{ave} = average downslope, displacement of the soil above the potential slip surface
- $\alpha H(x)$ = level at which average downslope displacement is valid
- $E_o(x)$ = in situ earth pressure
- $N(x)$ = additional axial downslope force (earth pressure)
- $E(x)$ = $E_o(x) + N(x)$
- $\tau_o(x)$ = in situ shear stress
- γ = deviatoric strain
- $q(x)$ = additional vertical load
- $t(x)$ = additional down slope load.
- $b(x)$ = width of element

The shear stress τ is a function (equ.4) of the deviatoric strain γ and the displacement δ_F in the slip surface. If this function is known the differential equations can be integrated numerically yielding the states of stress, strains and displacements for any chosen modes of mobilizing the resistance to failure propagation - and that for any selected portion of the slope. Fig. 4.

The following step-by-step method may be used for manual as well as computer analysis:

Step 1) Beginning at some point (x_o) down slope the shear stress is increased by $\Delta\tau$ such that $\tau = \tau_o + \Delta\tau$

2) Equ. (1a) and the relationship (4) render the corresponding values of the shear strain γ and the displacement δ_{ave}.

3) Δx_o is then obtained by solving equ.(1a) and (3) with respect to Δx.

4) ΔN is then computed from equ. (2).

5) The analysed section is then advanced a distance Δx_o.

6) From this point onward the calculation is continued by advancing in steps of suitably chosen fixed values for Δx.

b) Stage II - after slip surface formation (Equation 2 is substituted by 2a).

$$\Delta N = c_r(x)\cdot b(x)\cdot\Delta x - \Delta E_o(x) - \left[\sum_0^{H(x)} b(x)\rho(z)\cdot\Delta z\right] g\cdot\sin\beta(x)\cdot\Delta x - q(x)\cdot b(x)\cdot\sin\beta(x)\cdot\Delta x - t(x)\cdot b(x)\cdot\Delta x \quad (2a)$$

where $c_r(x)$ is the large strain - and/or high strain rate - strength of the sensitive clay.

3 RESULTS

As mentioned the numerical solution of the differential equations (1) to (5) yield the additional earth pressures (N), the shear stresses (τ) in the failure zone and the downslope displacement (δ) in the slope due to changes of loading conditions or - which is important - to changes of the response of the soil as a result of e.g. pore water pressure build up and strain softening.

Fig. 3 shows a typical nonlinear relationship between deviatoric stress and strain (stage I) as well as stress and displacement (stage II). This relationship forms the basic input (equ. 4) regarding soil behaviour in the analysis. The figure also demonstrates the apalling scale effect on brittleness associated with the difference in size between the laboratory test specimen and the real dimensions of the soil structure.

In fig. 4 and 5 the principal parameters derived from the analysis are shown. N(x) denotes the maximum additional earth pressure that may be applied at section (x) lest the leading displacements - associated with N(x) and $\tau(x)$ - propagate beyond the starting point (x_o) for the analysis where $\delta(x_o) \approx 0$.

N(x) will thus duly assume different values depending on to what extent - as defined by the point of reference (x_o) - the resistance of the slope is mobilized. Hence, any chosen part of the slope can be analyzed.

The point where N(x) = 0 defines the limiting or critical length ($L_{cr} = x(N{=}0)-x_o$) and for x-values beyond this point equilibrium is no longer possible for the studied stress - strain relationship. Fig. 4.

Another important critical value N(x)= Nmax signifies the maximum earth pressure that can be applied in section (x) lest a progressive failure be initiated. If $E(x) = E_o(x)+N(x)$ - due to changes of loading or stress-strain properties - exceeds the passive Rankine pressure (Ep) the progressive failure develops into a catastrophic slide with heave in the down slope passive zone and cavity formation in the up slope active Rankine zone. The reduced tangent modulus of elasticity E_t at impending Rankine failure in the down slope passive zone will (acc. to the analysis) greatly enhance progressive failure propagation over the valley floor.

Vital criteria for landslide disaster are thus:

$$E_o(x) + N(x) \leq E_p(x)/F \quad (6)$$

$$N_{actual} \leq N(x)max/F \quad (7)$$

where E_p = the passive Rankine earth pressure.
F = safety factor.

By applying the analysis to different parts of the slope, alternative potential failure zones and different time dependent stress-strain relationships the response characteristics of any slope can be studied - even with regard to the effects of time.

However, an exhaustive discussion of the applications of this progressive failure analysis is not possible within the scope of this article. Reference is therefore made to papers by Bernander et al (1978-85).

It should, however, be emphasized that the potential of the analysis ranges from the evaluation of in situ long term stresses in the slope to the prediction of the probable extent and appearence of a potential slide initiated by some local disturbance.

4 COMPUTER PROGRAM

Hand calculations according to the described numerical integration procedure are rather laborious as several iterative processes are involved. A computer program following the outlines given above has been made at the Department for design and engineering of SKANSKA (Gothenburg). Several interesting studies of existing and failed slopes have been performed.

5 CONCLUSIONS

The proposed improved model for progressive failure analysis of slope stability has proved to be a useful tool for evaluating in-situ stresses and the additional effects of superimposed loading cases. In each case the stress-strain response in the soil may be related to the current timing of that particular load. By means of this approach the load and stress history can be accounted for when studying the possible critical effect of a short term load. Thus the factor of time can be introduced into the analysis. Bernander & Gustås (1984).

Two important failure criteria are given in equ. 6 and 7. Equ. 6 implies that large slopes tend to fail primarily due to a global collapse of the soil structure in passive Rankine failure rather than as monolitic integral blocks of soil sliding down slope according to the laws of the conventional slip-surface-ideal-plastic-failure model. Equ. 7 is the criterion with regard to initiation of progressive failure due to some local load effect.

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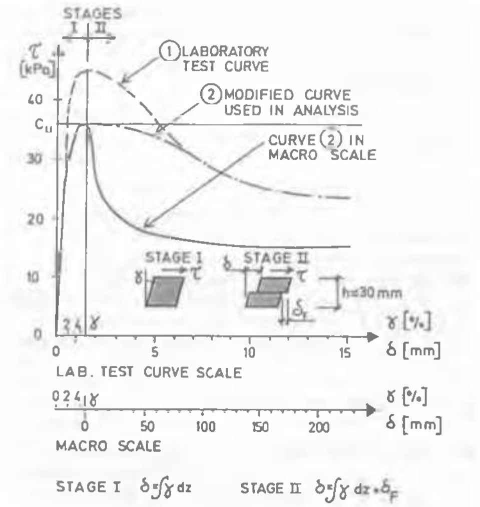

Figure 3. Time dependent stress-strain relationship $\tau = \phi(\gamma, \delta_F)$. Laboratory test curve compared with the same curve translated to the real dimensions of the soil structure. Note the apparent difference in brittleness.

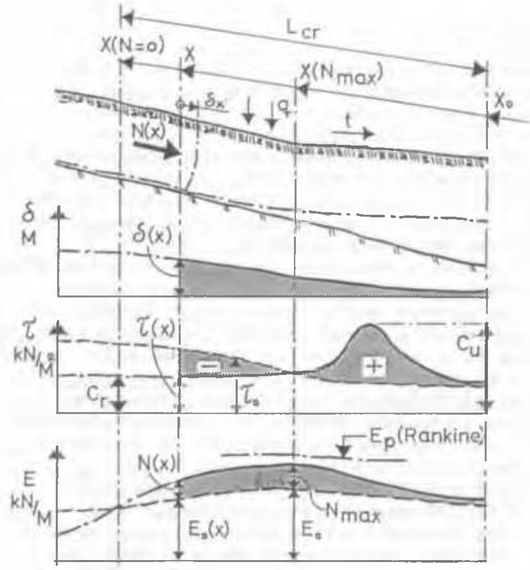

Figure 4. Principal results of the progressive failure analysis - Notations.

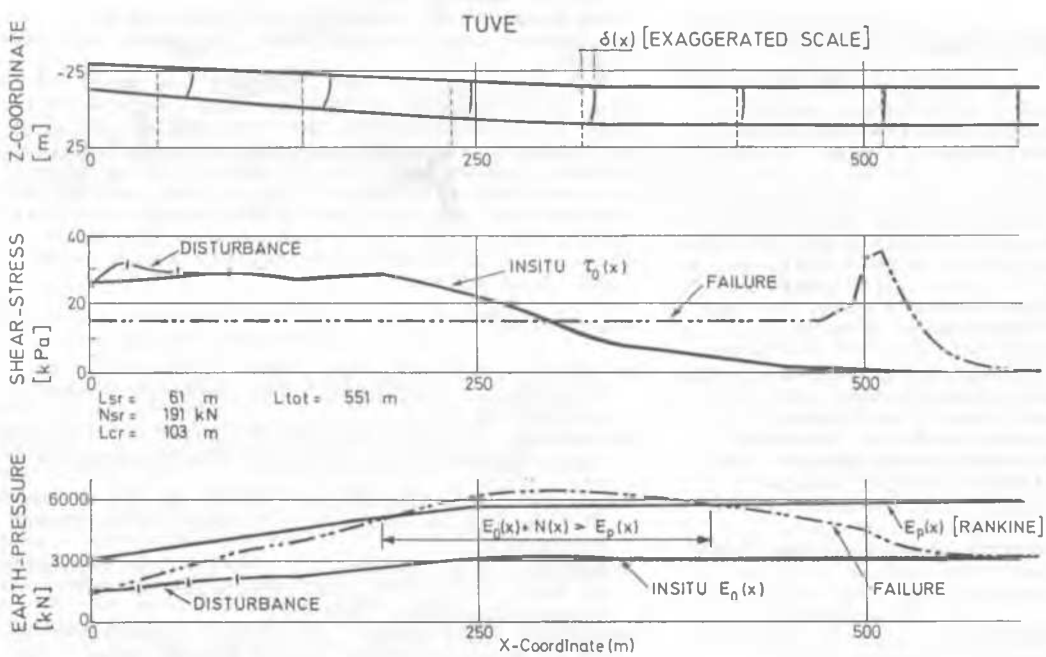

Figure 5. Graphic display from the computer analysis of progressive failure.