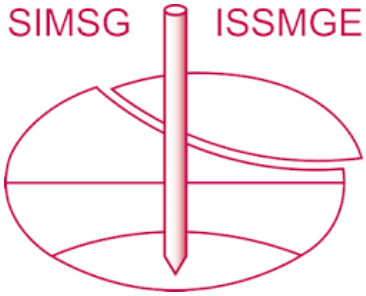
INTERNATIONAL SOCIETY FOR
SOIL MECHANICS AND
GEOTECHNICAL ENGINEERING



This paper was downloaded from the Online Library of the International Society for Soil Mechanics and Geotechnical Engineering (ISSMGE). The library is available here:

https://www.issmge.org/publications/online-library

This is an open-access database that archives thousands of papers published under the Auspices of the ISSMGE and maintained by the Innovation and Development Committee of ISSMGE.

Coupling solutions of bearing capacity - A case study

Solutions couplées de capacité portante - Une étude de cas

A.ASAOKA, Professor, Department of Civil Engineering, Nagoya University, Nagoya, Japan
S.OHTSUKA, Research associate, Department of Civil Engineering, Nagoya University, Nagoya, Japan

SYNOPSIS Bearing capacity problems of saturated soils are solved by making distinction between undrained, partially drained and fully drained conditions. In all cases, the plastic flow of soil foundation, and the distributions both of effective stresses and of excess pore pressure at failure are simultaneously determined by solving limiting equilibrium equations with the use of (i) stress-strain rate relationship at critical state and (ii) a coupling equation between soil skeleton and pore water. In the latter of the present study, the assumptions employed in the conventional $\phi_u = 0$ total stress analysis are also investigated through the case study, which clearly shows potential and limitations of the use of conventional unconfined compression strength of clays.

INTRODUCTION

In saturated soils, external forces are in the state of equilibrium with total stresses, while deformation-strength characteristics of soils are described in terms of effective stresses. Soil problems are, therefore, generally solved with the help of the soil-water coupling equation for determining an additional field variable, i.e., a pore water pressure. This is commonly accepted in the theory of consolidation, and the same should also be introduced to the limit equilibrium problems. A long lasting discussion on "Effective versus total stress analysis" in the stability of soils has been concerned partly with the practical method of determining the excess pore pressure distribution at failure (e.g., Rochell and Marsal, 1981). The difficulty of this kind will be resolved, only when the limit analysis of soils is newly interpreted as a coupling problem just like consolidation. The main objective of the present study is to derive the soil-water coupling solutions for some simple yet fundamental bearing capacity problems.

The volume change of soil skeleton is closely associated with pore water, and the coupling equation should, therefore, take the various forms depending on the drainage conditions, i.e., undrained, partially drained, and fully drained conditions. The typical numerical solutions for undrained and fully drained conditions are discussed first, and the case study of embanking on soft clay under partially drained condition is next illustrated making comparison with the conventional total stress analysis.

BEARING CAPACITY AND EFFECTIVE STRESSES

Limiting equilibrium equations

On the basis of the upper bound theorem in plasticity, the limiting equilibrium equations with necessary constraint conditions are summarized as follows:

$$\int \mathbf{B}^t \mathbf{s}\, dv + \mathbf{L}^t \boldsymbol{\lambda} = \mu \mathbf{F} \tag{1}$$

$$\mathbf{F}^t \dot{\mathbf{u}} = 1 \tag{2}$$

$$\mathbf{L}\, \dot{\mathbf{u}} = \mathbf{0} \tag{3}$$

Eq.(1) represents the limit equilibrium state of forces in a weak form, in which $\mathbf{s}$ and $\boldsymbol{\lambda}$ denote deviatoric stresses and mean total stresses in all elements, respectively and μ, the critical load factor that gives ultimate bearing capacity $\mu\mathbf{F}$ where $\mathbf{F}$ expresses the shape of external forces with unit magnitude. The other notations of the symbols in this equation should follow the usual finite element discretization formulation, i.e., $\mathbf{B}$ and $\mathbf{L}$ are the matrices defined such as $\dot{\boldsymbol{\epsilon}}=\mathbf{B}\dot{\mathbf{u}}$, and $\dot{\mathbf{v}}=\mathbf{L}\dot{\mathbf{u}}$, respectively where $\dot{\boldsymbol{\epsilon}}$ and $\dot{\mathbf{u}}$, the vector of strain rates of all elements and the vector of all nodal velocities, while $\dot{\mathbf{v}}$, the rates of volume change of all elements. Eq.(2) defines the unit of the indeterminate magnitude of the velocity field $\dot{\mathbf{u}}$, under a flexible footing, while Eq.(3) reflects non-dilatant characteristics of soils received at critical state, which is commonly observed in normally consolidated and lightly overconsolidated clays, and the same, in loose sand.

Constitutive equations at critical state

The set of Eqs.(1)-(3) are valid for statically indeterminate limiting equilibrium problems, and are solved with the help of constitutive equations which represent stress-strain rate relationships at critical state. For normally but anisotropically consolidated clays, following Sekiguchi and Ohta's model(1977), one gets

$$\dot{\epsilon}^p_{ij} = \left(\sqrt{\frac{3}{2}}\frac{\dot{e}}{\eta^*}\right)(\eta_{ij}-\eta_{ij0}),\quad \eta^* = \sqrt{\frac{3}{2}(\eta_{ij}-\eta_{ij0})(\eta_{ij}-\eta_{ij0})} \tag{4}$$

in which η_{ij} denotes the stress ratio, s_{ij}/p' at critical state where p' represents mean effective stress at failure. Anisotropically consolidated initial stress state is introduced through η_{ij0}, while $\dot{e}$ in the equation just represents a so-called equivalent plastic strain rate. The critical state condition in terms of effective stresses for Eq.(4) is expressed in the following equation:

$$M - \frac{3}{2\eta^*}\eta_{ij}(\eta_{ij} - \eta_{ij0}) = 0 \qquad (5)$$

On the other hand, for remoulded and/or isotropically consolidated clay, one has

$$\dot{\varepsilon}^p_{ij} = \sqrt{\frac{3}{2}}\frac{\dot{e}}{M}\eta_{ij} \ , \qquad (6)$$

which are also applicable to loose sands at failure as suggested by Schofield and Wroth(1968).

Coupling equation at limit equilibrium state

Eq.(4) and Eq.(6) include one additional unkown field variable p', which indentifies the requirement for a coupling equation. When mean effective stress p' at failure is determined, the pore pressure is calculated by the difference between mean total stress and mean effective stress. The coupling equations are introduced here both for (i) undrained loading and for (ii) fully drained loading, with typical numerical solutions of bearing capacity problems.

(i) Undrained loading
When undrained condition is considered throughout the whole procedure of loading, the mean effective stress at failure is re-evaluated from strain rate at critical state by the following equation(Asaoka and Ohtsuka, 1987):

$$(p')_f = p'_0 \exp\{\Lambda(-1 + \sqrt{\frac{3}{2}}\frac{\dot{\varepsilon}^p_{ij}\eta_{ij0}}{M\ \dot{e}})\} \qquad (7)$$

which completes the additional field equation i.e., the coupling equation. In Eq.(7) Λ denotes the irreversibility ratio defined by $1 - \kappa/\lambda$ where λ and κ are slopes of consolidation and swelling lines, respectively.

Fig.1 illustrates the bearing capacity problem of anisotropically consolidated soft clay under the axi-symmetric flexible footing with radius of 12m which simulates the basement of oil storage tank.

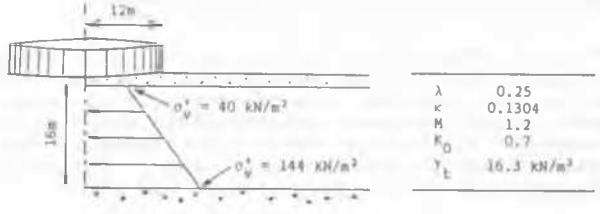


Fig.1a Initial stress and parameters

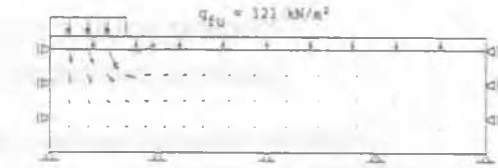


Fig.1b Velocity field (Axi-symmetric)

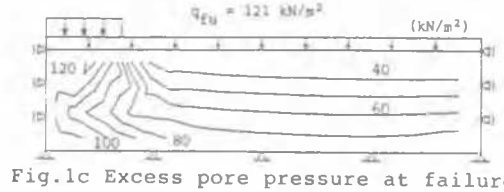


Fig.1c Excess pore pressure at failure

As shown in Figs.1b and 1c, plastic flow of soil skeleton and excess pore pressure at failure are simultaneously determined in the soil foundation concerned.

(ii) Fully drained loading
The typical stability problem under fully drained condition is observed at the cases of foundation failure of loose sand due to seepage forces under and along sheetpiles. When high permeability is assumed in the problem, seepage potential and pore pressure can be determined from usual steady state flow analysis. Unknown mean effective stresses, p' should, therefore, be solved iteratively using the following equation as a coupling equation:

$$p' = p - u \qquad (8)$$

in which p is the mean total stress at failure that should be solved as an element of vector $\mathbf{\lambda}$ in Eq.(1) and u, the pore pressure determined from the steady state flow. In solving Eqs.(1)-(3) with Eq.(6), p' is successively corrected till Eq.(8) is satisfied.

Fig.2 illustrates two dimensional plane strain failure mode of loose sand due to seepage forces. The typical plastic flow of saturated sand at failure is obtained as shown in Fig.2a, while the mean effective stresses at failure, in Fig.2b. The ultimate water head on the left hand side of the sheetpile is dependent of the critical state parameter M as suggested in Eq.(6), while the estimate from the conventional quick sand condition is independent of the strength parameter M. This is because the concept of "critical gradient" in conventional quick condition analysis accounts for limit

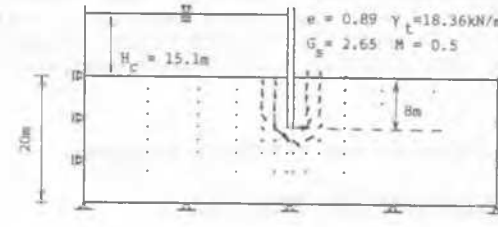


Fig.2a Typical velocity field at failure due to seepage force

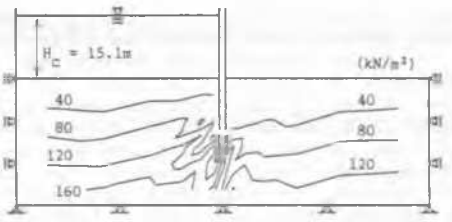


Fig.2b Typical distribution of mean effective stress at failure

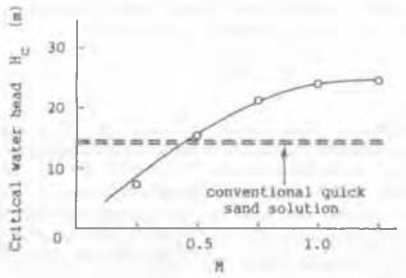


Fig.2c $H_{critical}$ versus M

equilibrium only in vertical direction and does not concern with shear resistance resulted from inclined body force due to multi-dimensional seepage. Comparison between the quick sand solution and that of the developed procedure is made as shown in Fig.2c taking M as variable, which suggests that, when M is large, the developed procedure could evaluate general shear failure of the soil foundation as a whole better than the conventional quick sand solution that only evaluates point-wise local failure in the vertical direction sticking to a sheetpile.

PARTIALLY DRAINED BEARING CAPACITY OF SOFT CLAY, A CASE STUDY

Soil data and conventional total stress analysis

Embankment stability on an alluvial clay deposit was examined at the site selected from an area investigated in connection with To-mei Express way in Aichi, Japan. Undrained shear strength c_u was determined by unconfined compression test using cylindrical soil specimens obtained by thin-walled samplers. The soil profile with depth is shown in Fig.3, from which the strength distribution for analysis was assumed to follow a linear regression equation:

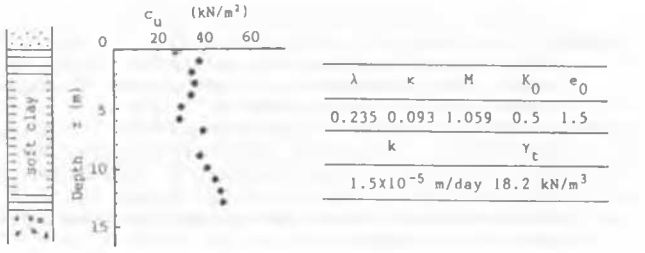


Fig.3 Strength distribution and representative soil parameters

$$c_u(z) = 2.49(z + 9.36)\ \text{kN/m}^2 \quad (0 \le z \le 13.5\text{m}) \qquad (9)$$

The conventional $\phi_u = 0$ stability analysis was performed with the following assumptions:

(i) Embanking is made instantaneously,
(ii) Undrained strength is isotropic, and
(iii) The strength determined under axi-symmetric condition is also applicable to plane strain condition,

from which ultimate load intensity for embankment of 12m wide (an access for machinery to the construction site) was calculated as

$$q_{fu} = 157\ \text{kN/m}^2 . \qquad (10)$$

Next, the assumptions and soil profile employed in this total stress analysis are to be examined one by one, in which the partially drained condition during loading is also newly introduced.

The use of plane strain strength

For a detailed analysis, the initial stress distribution and boundary conditions were assumed as shown in Figs.4 and 5, which are consistent to the strength distribution given in Eq.(9).

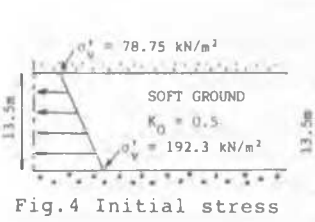


Fig.4 Initial stress

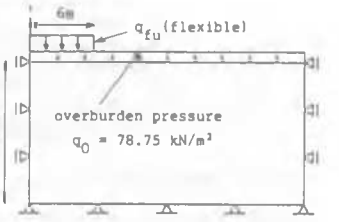


Fig.5 Boundary condition

Undrained strength in axi-symmetric condition was first converted into the strength in plane strain condition, still assuming strength isotropy. Isotropic undrained bearing capacity analysis under plane strain condition yields the ultimate load intensity about 15% larger than that by the conventional analysis:

$$q_{fu} = 182\ \text{kN/m}^2 \qquad (11)$$

Strength anisotropy

Natural clay deposits, anisotropically and normally consolidated, tend to exhibit anisotropic characteristics in both deformation and strength behaviour, and soil specimens sampled in vertical direction should produce strengths greater than the average ones along a potential slip surface. In order to take into account the effect of the anisotropy in undrained strength on the reduction of bearing capacity, constitutive relations at critical state given in Eq.(4) with K_0 = 0.5 was newly employed instead of Eq.(6). Anisotropic undrained analysis under plane strain condition is summarized in Figs.6a and 6b, yielding

$$q_{fu} = 126\ \text{kN/m}^2 , \qquad (12)$$

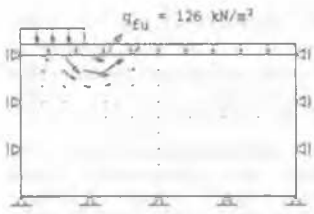


Fig.6a Anisotropic undrained analysis

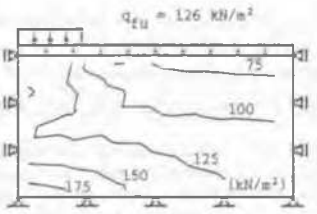


Fig.6b Corresponding excess pore pressure

which becomes, now, rather small as compared with the former by conventional isotropic analyses.

Lightly overconsolidated zone at the top of clay

The top of a natural clay deposit is usually in overconsolidated state, which can be seen also in Fig.3. To see the overconsolidation effect on the increase of bearing capacity, initial stress distribution in Fig.4 was modified to a new profile as shown in Fig.7, which is equivalent to assume the strength distribution in the following form:

$$c_u(z)=30.8\,(0\le z\le 3\text{m}),\ c_u(z)=2.5z+23.3\text{kN/m}^2\,(3\le z\le 13.5\text{m}) \quad (13)$$

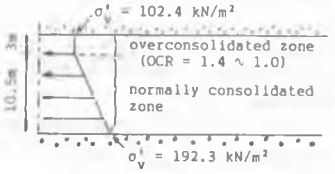


Fig.7 Modified initial stress distribution

Undrained bearing capacity was re-evaluated on the basis of the modified soil profile that evaluates overconsolidated state. The details of the analysis are shown in Figs.8a and 8b, which yields

$$q_{fu} = 142\ \text{kN/m}^2\ . \quad (14)$$

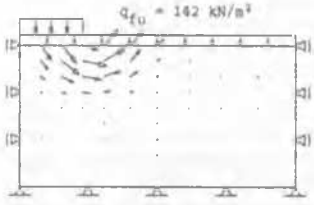


Fig.8a Velocity field (overconsolidated top surface)

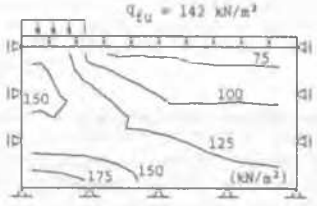


Fig.8b Excess pore pressure

Even considering overconsolidated state, it is found that the result is still smaller than the ultimate load intensity obtained from the conventional analysis, i.e., q_{fu}=157kN/m², see Eq.(10).

Partial drainage effect during loading procedure

Since embanking in-situ is made gradually, hardening of soils during loading procedure is much more accelerated due to consolidation (see Fig.9), and the conventional undrained assumption becomes also controversial. The current effective stress state that proceeds with consolidation deformation can be determined from time to time on the basis of the elasto-plastic consolidation analysis. Therefore, the undrained bearing capacity is also obtained from time to time as consolidation proceeds, from which the partially drained bearing capacity is naturally determined by extrapolation as shown in Fig.10. The analysis made for the case study is also summarized in the figure, yielding

$$q_{fu} = 159\ \text{kN/m}^2\ , \quad (15)$$

which becomes now very close to the result by the conventional total stress analysis.

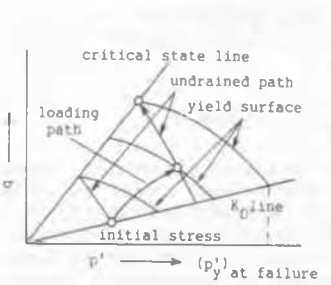


Fig.9 Hardening of soil

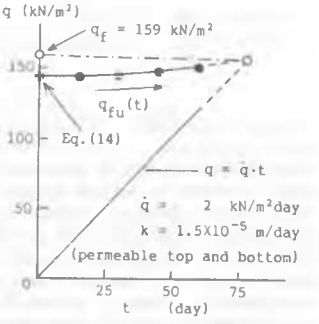


Fig.10 Partial drainage effect

CONCLUSIONS

(i) The coupling solutions can evaluate bearing capacity of saturated soils under various drainage conditions during loading.

(ii) The ϕ_u = 0 conventional total stress analysis should reflect a remarkable "lucky harmony" among errors associated with the assumptions employed in the analysis, and is concluded to be a reliable method in soil engineering practice although two other important factors, at least, appear to have important contribution to this problem: one is sampling disturbance, and the other, the strength reduction due to creep.

REFFERENCES

Asaoka, A. and S. Ohtsuka (1987). Bearing capacity analysis of a normally consolidated clay foundation. Soils and Foundations, vol.27, No.3, 58-70.

Rochelle, P. L. and R. J. Marsal (1981). Slope stability, General Report(Session 11). proc. X ICSMFE, vol.4, 485-508, Stockholm.

Schofield, A. N. and C. P. Wroth (1968). Critical state soil mechanics. McGraww-Hill Book Co., London.

Sekiguchi, H. and H. Ohta (1977). Induced anisotropy and time dependency in clays. proc. of Speciality Session 9, IX ICSMFE, 229-238, Tokyo.