Power spectral density methods for site motions

Les méthodes puissantes de densité spectrale pour les mouvements des sites

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SYNOPSIS: Site-dependent ground motions often are established by combining the response spectra of a suite of strong motion records from earthquakes of the same magnitude recorded for similar ground conditions. Different results are obtained when different suites of records are selected, but the statistics of the spectra can be combined by the technique described in this paper. Several techniques based on the theory of random vibrations have been developed to describe the motion in the frequency domain by power spectral density functions. Most require that a prescribed response spectrum be recaptured, which can be done best for spectra based on actual records instead of arbitrarily smoothed shapes. Defining motion at high frequencies and interpreting and understanding the results require further study.

1 INTRODUCTION

Recent developments in earthquake engineering have emphasized the use of design ground motions that represent the conditions at the site of the facility in preference to generalized motions that are intended to apply to a range of conditions. Such site-dependent ground motions are usually represented by response spectra, but suites of time histories of acceleration also have been used, with arguments existing for each approach. The major disadvantage of the former is that engineers have not agreed as to how the effects are to be transmitted through multiple-degree-of-freedom systems, especially when the damping varies within the structure, while the major disadvantage of the latter is that the phasing of the component frequencies is fixed for each time history and, therefore, each time history gives unique results in a dynamic analysis. Most recently, the methods of random vibration theory, especially in the form of the power spectral density function, have given considerable promise of resolving these problems. This paper presents statistical methods for combining different estimates of site-dependent response spectra and reviews techniques for developing corresponding power spectral density functions.

2 METHODS OF STATISTICAL COMBINATION

Several approaches can be taken to develop a suite of recorded accelerograms from which the site-dependent response spectrum will be computed. Hunt et al. (1986) describe a site at which about 35 m of sands and gravels overlie very competent rock. Records could be selected from sites that match this geometry as closely as possible, or one could choose recordings made on rock sites and perform amplification calculations to obtain the motions at the surface of the site profile. While either method gives a reasonable response spectrum, the mean spectral acceleration is different in the two cases. Similarly, different estimates of the site-dependent spectrum can be obtained by different models of the seismicity of the region, of the soil profile, or of other parameters. How then should the best estimate of the site-dependent spectrum be obtained?

A proposal by K. F. Reinschmidt (1985) for combining two estimates can be extended to an arbitrary number of estimates by the following procedure. It is assumed that the ordinates of the response spectra making up any one suite at each frequency are log-normally distributed; this can be verified easily from the data themselves. If there are n_i records in the ith suite and the mean and standard deviation of the logarithms of the ordinates at a given frequency are x_i and s_i, respectively, then the estimate of the mean of the combination of the suites is

$$\bar{x} = \sum w_i x_i \tag{1}$$

where w_i is the weight to be used for the mean of the ith suite and the summation is understood to be done over all the suites. The variance of the estimate of the mean is the combination of the variances of the means for the suites:

$$s^2 = \sum w_i^2 s_i^2/n_i \tag{2}$$

The weights can be found by minimizing s^2 with respect to the individual weights under the constraint that the sum of the weights must be unity. The easiest approach is to define the constraint by a function:

$$\phi = \sum w_i - 1 = 0 \tag{3}$$

With the introduction of the Lagrangian multiplier λ, the function to be made stationary becomes $s^2 - 2\lambda\phi$. This leads to

$$w_i s_i^2/n_i - \lambda = 0 \tag{4}$$

and, as the sum of the weights is unity,

$$\lambda = 1/\left(\sum n_i/s_i^2\right) \tag{5}$$

and

$$w_i = (n_i / s_i^2) / \left(\sum n_i / s_i^2\right) \quad (6)$$

The variance of the combination of the suites is found by recognizing that the variance of a normally distributed functions is χ^2 distributed with the number of degrees of freedom one less than the number of samples. If σ^2 is the variance of the population, the variance of n samples is

$$s^2 = \chi^2 \sigma^2 / (n-1) \quad (7)$$

The results of m independent samplings from the same population, each χ^2 distributed, is the sum of the individual χ^2:

$$\chi^2 = \sum (n_i - 1)\, s_i^2 / \sigma^2 \quad (8)$$

As the best estimator for σ^2 is

$$s^2 = \sigma^2 \chi^2 / \left(\sum n_i - m\right) \quad (9)$$

it follows that

$$s^2 = \sum [(n_i - 1)\, s_i^2] / \sum (n_i - 1) \quad (10)$$

Figure 1 shows the 5 percent damped response spectrum developed by Hunt et al. (1986) from the two suites of records combined by the above process. The mean and the mean plus one standard deviation are plotted together with a previously established design spectrum.

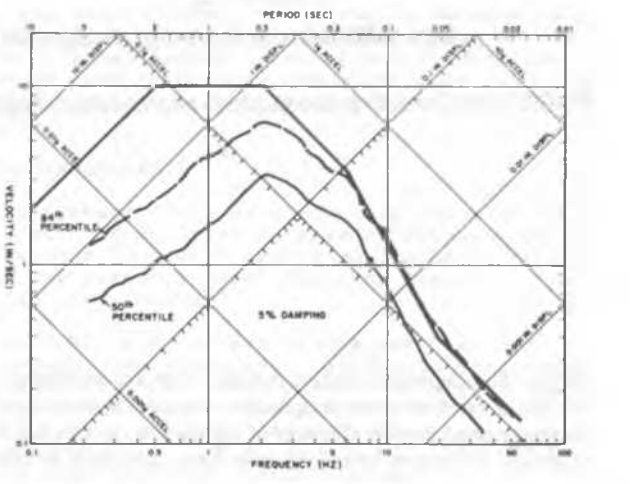

Figure 1. Statistically determined site-dependent response spectrum

3 POWER SPECTRAL DENSITY FUNCTIONS

Random vibration theory has been used for many years in the study of dynamic systems, including both structures and mechanical equipment, but it has not been widely employed in the geotechnical aspects of earthquake engineering. Crandall and Mark (1963) present both an excellent introduction to the subject and descriptions of many of its basic results. Vanmarcke (1976) developed the ideas further to describe ground motions due to earthquakes. These have been elaborated over the last several years by a number of researchers so that there is now a substantial body of experience dealing with random vibration techniques for earthquake engineering.

The concept of the power spectral density function is central to the theory of random vibrations. This can be derived from the Fourier transform $A(\omega)$ of a particular signal or time history $a(t)$. By use of Parseval's theorem, the total power in the signal can be expressed in terms of $A(\omega)$ or $a(t)$:

$$\int_{-\infty}^{\infty} |\, a(t)\, |^2\, dt = \int_{-\infty}^{\infty} |\, A(\omega)\, |^2\, d\omega = power \quad (11)$$

The term under the second integral represents the distribution of the power over the range of frequencies and is equal to the square of the magnitude of the Fourier transform. This is called the power spectral density function. As the integrals of Eq. 11 are symmetrical about the origin for a real function $a(t)$, the power spectral density function can be defined either for positive values of frequency only or for both positive and negative values. For the total energy to remain constant the ordinates in the former case must be twice those of the latter. It is also possible to work with frequency in radians/second (ω) or cycles/second (f). Selecting between the single- or double-sided versions of the function and between ω or f has an effect on the magnitude of the ordinates, and the user of power spectral density methods should be careful to establish which conventions are being applied.

The power spectral density function directly describes the distribution of energy over frequency, whereas the response spectrum describes the response of a particular type of system to the input. The ordinates of the response spectrum at high frequencies necessarily reflect the input of energy at the lower frequencies.

Determining a power spectral density function ψ at some point in a structure or system, if the power spectral density function ϕ at another point is known, requires only that the power spectral density function of the input be multiplied by the square of the magnitude of the complex transfer function between the two points:

$$\psi(\omega) = |\, H(\omega)\, |^2\, \phi(\omega) \quad (12)$$

Because methods of dynamic soil-structure interaction analysis formulated in terms of complex transfer functions have evolved significantly over the past decade, the necessary transfer functions often are readily available, and the calculation is relatively simple. This greatly expedites the evaluation of seismic response in different regions of a structure or soil deposit.

For many applications it is still necessary to have a response spectrum. To compute the response spectrum from a power spectral density, it is first necessary to evaluate the statistics of the response of a single-degree-of-freedom system. Although the techniques for doing this rigorously can become very complicated, several researchers, including Kaul (1978), Sundararajan (1980), and Unruh and Kana (1981), have described simplifications that make the calculations more tractable. These methods require first that the amplification function for the system (with natural frequency Ω and damping β) be

convolved with the power spectral density function to obtain the variance of the response:

$$\sigma\ (\Omega)^2 = \int_{-\infty}^{\infty} \frac{\Omega^4 + 4\beta^2\ \omega^2\ \Omega^2}{(\Omega^2 - \omega^2)^2 + 4\beta^2\ \omega^2\ \Omega^2}\ \phi\ (\omega)\ d\omega \quad (13)$$

The square root of the variance is then multiplied by a peak factor, F, that is based on studies of the statistics of response to white noise. As shown in Eq. 14, this peak factor F is a function of frequency and depends on the ratio, r, of the value and the time derivative of the standard deviation of the response, the duration of the motion, D, and the probability, p, of exceeding the response spectrum when the power spectral density function is known. Christian (1988) describes some of the details of the procedure.

$$F\ (\Omega) = \left[2\ \ln\left\{-\frac{D}{\pi r}\ \frac{1}{\ln\ (1-p)}\right\}\right]^{1/2} \quad (14)$$

Although Kaul (1978) describes a rigorous and accurate method for determining the power spectral density function of an underlying statistical process from a given response spectrum, an iterative procedure, such as the one developed by Unruh and Kana (1981), usually is employed. The iterations start with an estimate of the desired power spectral density function. That function is then used to compute a response spectrum, and the square of the ratio between the computed response spectrum and the given one is used to correct the power spectral density function at each frequency.

Figure 2 shows the comparison between the 84th percentile response spectrum of Figure 1 and the response spectrum calculated from the fifth iteration of the power spectral density function computation. The calculations were done by a microcomputer program called SPEED, which is based on the previously cited work by Kaul (1978), Sundararajan (1980), and Unruh and Kana (1981). The ordinates are spectral acceleration. The two curves match very closely despite the small number of iterations. It was observed that very similar power spectral density functions were obtained when the convolution expressed by Eq. 13 was done for the acceleration response and for the displacement response, and when the numerical integrations were carried out by different schemes.

Figure 3 shows the same comparison between the standard U.S. Nuclear Regulatory Commission design response spectrum for 5 percent damping and the spectrum calculated from the iterated power spectral density function. The match is excellent for frequencies below about 20 Hz, but there is significant divergence at higher frequencies. This phenomenon is observed often when applying random vibration methods to standard, simplified response spectra. A similar effect is found when generating artificial accelerograms to match such spectra. This is not surprising, for artificial accelerograms are usually generated by a process that involves finding the power spectral density of the underlying process and then randomizing the phase of the individual components.

As noted by Vanmarcke (1976), the integral in Eq. 13 involves two terms, one of which is the amplification function for a single-degree-of-freedom system. This is equal to unity at low frequencies, has a sharp peak at the natural frequency, and goes rapidly toward zero above that frequency. Therefore,

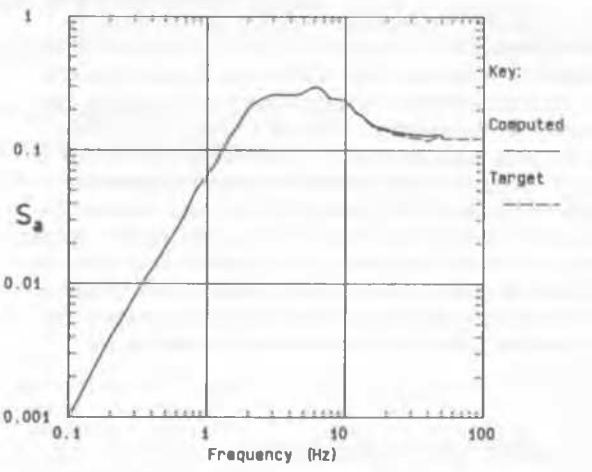

Figure 2. Comparison of 84th percentile response spectrum with iteration results

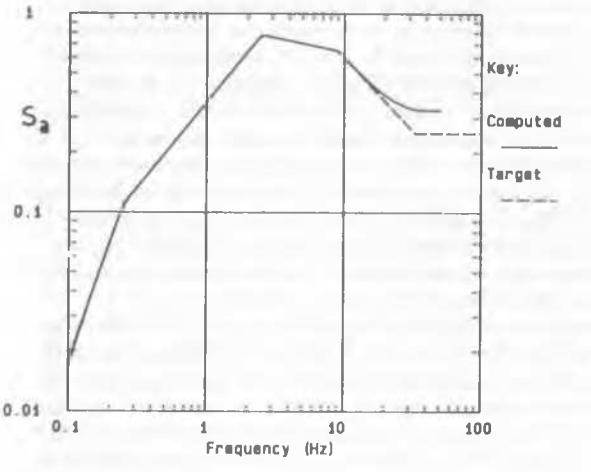

Figure 3. Comparison of the standard response spectrum with iteration results

the integral can be approximated by three portions. In the first, the amplification function is unity, and the integral is simply the integral of the power spectral density. The second is essentially the integral of the peak of the amplification function multiplied by the value of the power spectral density function at that frequency. The third vanishes because the amplification function goes to zero. One implication of this analysis is that the response at high frequencies can be dominated by the power at the lower frequencies; the low-frequency power causes response even at higher frequencies where there is no power at all. Physically this means that a relatively rigid body subjected to low-frequency oscillations will experience the accelerations of the input even though there is no energy input at the fundamental frequency of the responding body. It also im-

plies that the shapes of response spectra cannot be specified arbitrarily and still be consistent with the constraints of dynamic response.

It should be noted that minor differences in power spectral density functions also can have significant effects at very low frequencies. In the extreme, a non-zero value of the power spectral density function at zero frequency implies a steady source of energy and leads to infinite computed response. Clough and Penzien (1975) pointed this out and proposed corrections to the well-known Tajimi-Kanai power spectral density function to eliminate the low-frequency power. Even when the power spectral density function does vanish at zero frequency, minor variations in the power content at low frequencies can have significant effects on the computed response at low frequencies.

4 DISCUSSION AND CONCLUSIONS

There are many arguments in favor of the use of site-dependent descriptions of design motion instead of general spectra that apply to a broad range of conditions. Because the site can be described by a number of parameters and the choice of recorded motions will be affected by different interpretations of the site conditions and the governing seismic regime, several different site-dependent motions may be obtained. Rather than trying to reconcile the differences into a unique consensus, it may be better to combine the results of different approaches statistically, as described in Section 2.

The methods of random vibration theory and, in particular, the power spectral density function provide a powerful tool for describing ground motions in a way that is consistent with statistically derived ground motion spectra and with modern methods of dynamic analysis. Techniques have been described for computing probabilistically based response spectra from the power spectral density function of the underlying process. The inverse calculation can be done by iteration.

It should be noted that the power spectral density functions and the response spectra are treated as continuous functions. Their values at particular frequencies are just that: local values of a function. In contrast, artificial accelerograms are composed of many discrete components at different frequencies, and the energy in one component represents the contributions of a cluster of frequencies near the discrete value. The distinction is the same as that between a continuous and a discrete Fourier transform. Using discrete descriptions of the motion, such as artificial accelerograms, can lead to very jagged response spectra and considerable sensitivity to the selection of frequencies at which calculations are done. On the other hand, a continuous representation tends to give much smoother results and eliminates the extreme conservatism induced by efforts to force all results to lie above a smoothed curve.

Power spectral density functions are more easily generated to match response spectra generated by statistical processes from real records than artificially smoothed idealized response spectra. Mismatches appear especially at the high-frequency end of the spectra. However, difficulties also can arise at the low-frequency end because of the strong effects of power at very low frequencies.

One potential advantage of random vibration methods is that they permit a more rational treatment of questions involving the probability of exceeding a given level of motion. They also free the analyst from dependence on the quirks of a particular accelerogram. A major disadvantage is the lack of practical experience in their use, which reduces the intuitive confidence required for successful application of any technique.

There is a need for further examination of the use of random vibration techniques. To date there is relatively little practical experience with these methods, so engineers have little intuitive grasp of what can and cannot be done and of where the pitfalls lie. In particular, it would be worthwhile for studies to be made of the power spectral density functions for a variety of different site conditions and seismic regimes.

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