Bearing capacity of square footings on sand

La capacité portante de semelles carrées sur du sable

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SYNOPSIS: Model tests have been performed to investigate the bearing capacity of square footings on a surface of dry sand. The footings were loaded centrally and eccentrically with vertical as well as inclined loads. The test results have been compared to bearing capacity theory involving a combined bearing capacity and inclination factor calculated on the basis of a rupture figure which is statically determined. The test results have also been related to design practice as reflected by the Codes of Practice for Denmark, Canada and the Federal Republic of Germany.

1. BEARING CAPACITY THEORY

1.1 Vertical central load

The vertical bearing capacity V per unit length of a centrally loaded strip footing with the breadth B is normally calculated from the well-known Terzaghi bearing capacity formula, which in the case of sand (c = o) gives:

$$V = B\left(\tfrac{1}{2}\gamma B N_\gamma + q N_q\right) \qquad (1)$$

where

- γ is the unit weight of the sand,
- q is the unit load on the surface outside the footing, and
- N_γ and N_q are dimensionless bearing capacity factors, which depend on the angle of internal friction.

For a rectangular footing on a sand surface (q = o) Brinch Hansen (1970) modified (1) to:

$$V_o = B\,L\left(\tfrac{1}{2}\gamma B N_\gamma s_\gamma\right) \qquad (2)$$

and proposed

$$s_\gamma = 1 - 0{,}4\,\frac{B}{L} \qquad (3)$$

for the shape factor, where L is the length of the footing.

On the basis of a figure of rupture proposed by Lundgren and Mortensen (1953) and illustrated in figure 1 (at the top), Brinch Hansen suggested the values for N_γ indicated in figure 2 by a dashed line, and these values have been adopted by the Danish Code of Practice (1985).

1.2 Vertical, eccentric load

For a square footing, loaded eccentrically by a vertical load, the "effective foundation area" concept illustrated in figure 3 is introduced, which for a square footing yields:

$$V_e = B_e\,L_e\left(\tfrac{1}{2}\gamma B_e N_{\gamma e}\,s_\gamma\right) \qquad (4)$$

where:

$$B_e\,L_e = B^2\left(1 - \frac{2e_1}{B}\right)\left(1 - \frac{2e_2}{B}\right) \qquad (5)$$

e_1 and e_2 are the eccentricities in the two principal directions parallel to the side lines of the footing. e_1 is always the larger of the two eccentricities. For s_γ equation (3) yields:

$$s_\gamma = 1 - 0.4\,\frac{B_e}{L_e} = 1 - 0{,}4\,\frac{B - 2e_1}{B - 2e_2} \qquad (6)$$

Introducing (5) and (6) into (4) V_e may for a square footing be expressed as:

$$V_e = B^2\left(\tfrac{1}{2}\gamma B N_{\gamma e}\,k\right) \qquad (7)$$

where

$$k = \left(1 - 0{,}4\,\frac{B - 2e_1}{B - 2e_2}\right)\left(1 - \frac{2e_1}{B}\right)\left(1 - \frac{2e_2}{B}\right)^2 \qquad (8)$$

is a dimensionless factor, which depends solely on the two eccentricity ratios e_1/B and e_2/B.

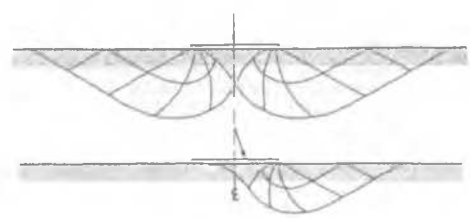

Figure 1. Figure of rupture for footings with vertical, central load (top) and inclined, eccentric load (bottom).

1.3 Inclined, eccentric load

For a square footing loaded eccentrically by an inclined load an inclination factor $i_{\gamma e}$ is in-

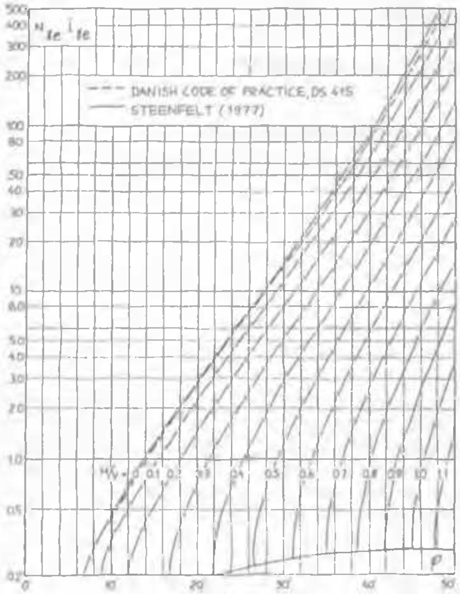

Figure 2. Bearing capacity factors for footings with inclined, eccentric load.

troduced; thus (7) gives:

$$V_e = B^2 \left(\frac{1}{2} \gamma B N_{\gamma e} i_{\gamma e} k\right) \tag{9}$$

The product $N_{\gamma e} i_{\gamma e}$ has been calculated by Steenfelt (1977) on the basis of a figure of rupture illustrated in figure 1 (at the bottom). The results of the calculations are given by full lines in figure 2 for various values of H/V, where H denotes the horizontal component of the force acting on the footing.

$N_{\gamma e} i_{\gamma e}$ is calculated by Steenfelt on the basis of a figure of rupture which is somewhat different from that used by Lundgren and Mortensen (1953) for their calculation of N_γ. For this reason the Steenfelt-values of $N_{\gamma e} i_{\gamma e}$ for H = o, which corresponds to $i_{\gamma e} = 1$, are not in total agreement with the N_γ-values from the Danish Code of Practice, DS 415. However, the difference between the two sets of values are so small that it may be disregarded from a practical point of view.

In conclusion it is thus proposed to calculate the bearing capacity for a square footing on a sand surface loaded eccentrically by an inclined load from formula (9) using the dimensionless bearing capacity factors in figure 2.

2. MODEL TESTS

A total of 67 model tests were performed with the aim of evaluating the bearing capacity theory described in section 1. A detailed description of the tests is given by P. Larsen (1982).

2.1 Model test equipment

The tests were performed in a container having a

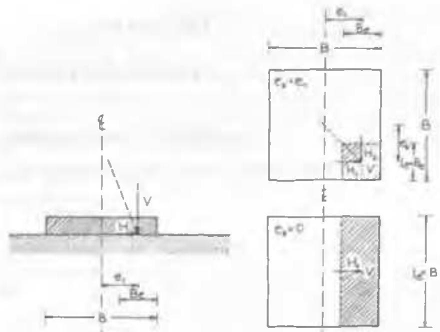

Figure 3. Basic definitions for the bearing capacity problem of square footings.

diametre of 1.5 m and a depth of 0.8 m. All tests were performed with square footings with the following area:

$B^2 = 0.15 \times 0.15\ m^2$ for $e/B \leqq 0.3$

$B^2 = 0.30 \times 0.30\ m^2$ for $e/B \geqq 0.3$

The base of the footings is made perfectly rough by means of sand grains glued on to the surface of the steel footings.

The load on the footing is produced by means of a screw spindle. The load is applied through a column which has a hinge at the top where it connects to the screw spindle and at the bottom at the level of the base of the footing. A force transducer is included in the column in order to measure the load on the footing.

2.2 Properties of the sand

All tests were made with sand from Lund's pit No 1 with a rather uniform grain size distribution and an average grain diametre of 0.5 mm. The sand has

$e_{max} = 0.89$

$e_{min} = 0.58$

$d_{60}/d_{10} = 1.3$ to 1.5

$\rho_s = 2650\ kg/m^3$

The sand was placed in the test container by means of pluviation. The sand was poured into the container from a hopper equipped with a system of sieves. The hopper and the sieves were moved upwards with such a speed that a constant height of fall for the sand grains was ensured. In this way sand samples with the same void ratio from test to test could be produced; also homogeneity within a specific sand sample could be ensured.

Using different heights of fall, sand samples with two different void ratios could be produced. 88 sand samples were produced for the testing. The density properties were the following:

Number of tests	Average void ratio e_m	Relative density I_D
35	0.68	0.7
53	0.58	1.0

The standard deviation on the average void ratio is 0.01.

In order to investigate the strength properties of the sand, 32 triaxial tests were performed at various densities on sand samples having a diametre of 200 mm. The tests were carried out with cell pressures equal to 10, 50 and 80 kN/m². This level of cell pressure amounts roughly to 10% of the actual pressure under the model footings at failure according to Danish practice. Based on the test results the tangential angle of internal friction φ_{tr} at the actual stress level can be expressed by the formula:

$$e \tan \varphi_{tr} = 0.566$$

Corresponding to this tangential angle of internal friction, the sand possesses an effective cohesion:

$$c' = 2.53 \text{ kN/m}^2$$

According to Danish practice this effective cohesion is disregarded in the interpretation of the load test results, and all bearing capacity calculations are based on the plane angle of internal friction for which it is assumed:

$$\varphi_{pl} = 1.1\ \varphi_{tr}$$

2.3 Test results, vertical load

31 bearing capacity tests were performed with a vertical load. 8 of these tests were conducted with a central load, 13 tests were conducted with the load acting eccentrically in one direction, while 10 tests were conducted with the load acting eccentrically in both directions. The ultimate bearing capacity from each test is expressed in terms of the factor k calculated from formula (7) as

$$k = \frac{2\ V_e}{B'\gamma N_{\gamma e}}$$

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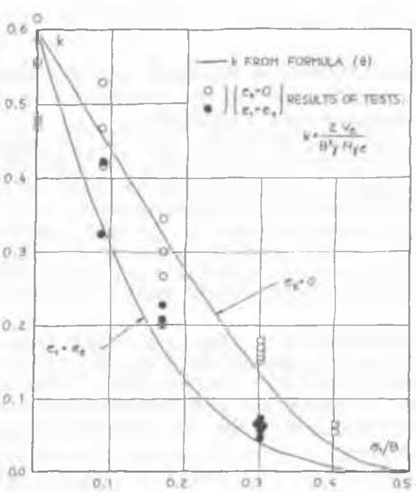

Figure 4. Results of model tests with footings with vertical, eccentric load.

and presented graphically in figure 4 as a function of the eccentricity ratio e_1/B.

The bearing capacity presented in figure 4 has been reduced for the q-term in the bearing capacity formula resulting from the settlement of the footing. The reduction amounts to between 7 and 20%.

Tests were conducted with sand having relative densities $I_D = 1.0$ and $I_D = 0.7$, respectively. In the dimensionless form presented in figure 4, the test results proved to be independent of the density of the sand within the test range.

For an eccentricity corresponding to e/B = 0.3, tests have been performed with footings having the breadth B = 0.15 m and B = 0.30 m as well. The test results in form of the dimensionless factor k showed no scale effect.

2.4 Test results, inclined load

36 tests were performed with two different ratios between the horizontal and the vertical load. 22 tests had H/V = 0.1, and 14 tests had H/V = 0.2. The tests were performed with sand having $I_D = 1.0$. The test results have been presented graphically in figure 5 in the same way as in figure 4.

In the interpretation of the test results the bearing capacity factor $N_{\gamma e} i_{\gamma e}$ has been calculated on the basis of H_1 which is the horizontal force perpendicular to the longer effective side length of the footing.

The reduction for the q-term from settlement amounts to between 5 and 10%. No scale effect appeared from the results of the tests with e/B = 0.3.

2.5 Conclusion

In figures 4 and 5 the theoretical values of k from formula (8) are presented by full lines. On the basis of the results of the 67 model tests with square footings on dry sand it can be concluded that satisfactory agreement exists between the test results and the commonly used bearing capacity formula (9) provided that the

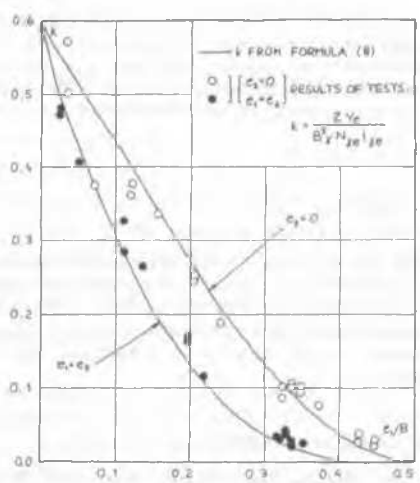

Figure 5. Results of model tests with footings with inclined, eccentric load.

dimensionless bearing capacity factors presented in figure 2 are used together with the effective foundation area concept.

3. COMPARISON TO DESIGN PRACTICE

3.1 Danish design practice

According to the Danish Code of Practice (1985) the bearing capacity of a footing is calculated from formula (9) using the effective foundation area concept and a shape factor defined by formula (3). The bearing capacity factor N_γ used in the Danish Code (dotted line in figure 2) deviates only slightly from the values proposed by Steenfelt and presented by full lines in figure 2. However, according to the Danish Code of Practice, the inclination factor is calculated according to the formula:

$$i_\gamma = (1 - H/V)^4 \qquad (10)$$

where H is the vectorial sum of H_1 and H_2.

The values of $N_\gamma i_\gamma$ calculated according to the Danish Code (formula (10) and the dashed line in figure 2) are in average between 10% (for vertical load) and 30% (for inclined load) smaller than the values proposed by Steenfelt (1977) and presented in figure 2. Taking into account that a relatively good agreement existed between the test results and the bearing capacity values presented in figure 2, it is reasonable to conclude that the Danish design practice is somewhat on the safe side. It should, however, be remembered that the present test results are valid only for the relatively small values of the ratio H/V and the relatively large values of the angle of internal friction in the tests.

3.2 Canadian design practice

According to the Canadian Foundation Engineering Manual (1985) the bearing capacity formula is used together with the effective foundation area concept and shape and bearing capacity factors similar to those used according to the Danish Code. For vertical eccentrically loaded footings Canadian and Danish design practice are in other words identical and about 10% on the safe side.

According to the Canadian Manual the inclination factor is calculated according to:

$$i_\gamma = (1 - \delta/\varphi)^2 \qquad (11)$$

where: δ = arc tan H/V

This formula yields values of i_γ which for actual values of H/V and the angle of internal friction are 15-35% higher than Danish design practice according to formula (10). This means that Canadian practice probably is in reasonably good agreement with the test results for inclined load.

3.3 German design practice

According to the German Code of Practice (DIN 4017, parts 1 and 2, 1979) a bearing capacity formula similar to formula (1) is used together with the effective foundation area concept and the shape factor:

$$s_\gamma = 1 - 0.3\ B/L \qquad (12)$$

The German version of the bearing capacity formula omits the factor $\frac{1}{2}$. Consequently, the German bearing capacity factor N_b is equivalent to the factor $\frac{1}{2} N_\gamma$ in formula (1).

The values of N_b given in DIN 4017 are about 35% higher than those given for $\frac{1}{2} N_\gamma$ in figure 2. Taking this into account and using the shape factor given by formula (12), it can be concluded that German design practice is somewhat on the unsafe side compared to the test results for vertical loads.

For inclined loads the inclination factor according to DIN 4017 is:

$$i_\gamma = (1 - H/V)^3 \qquad (13)$$

For the actual values of H/V formula (13) yields values of i_γ which are larger than those found from the Danish Code but smaller than those found from the Canadian one. Also for inclined loads German design practice thus seems to be somewhat on the unsafe side compared to the test results.

3.4 Conclusion

Based on an analysis of Danish, Canadian and German design practice, it may be concluded that Danish practice is somewhat on the safe side, Canadian practice is in reasonably good agreement, and German practice is somewhat on the unsafe side compared to the test results. It should be noted, however, that the comparison is based on shear strength properties for the sand interpreted according to Danish practice. No attempt has been made to determine the shear strength properties according to Canadian and German practice.

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