Reliability based partial coefficients - A simplified approach

Coefficients partiels basés sur la confiabilité - Une approche simplifiée

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SYNOPSIS: Under the new Swedish Building Code safety checking should preferably be done in a partial coefficient format. In geotechnical and foundation engineering the implementation of the new code is more complicated than in structural engineering. In this paper the problems are outlined and a method suggested by which the partial coefficient method can be applied in a simple yet stringent manner. In the method the partial coefficients are calculated for a required safety index ß using empirical values for the sensitivity factors α_i and the coefficient of variation of the soil resistance. The coefficient of variation is calculated using a simple Bayesian stochastic model which takes spatial correlation properties into account.

1 INTRODUCTION

In Sweden the current Building Code is being revised into a partial coefficient format. For structural engineering this has been implemented, but for foundations the problems have been greater. This comes from the fact that each site is individual, so that the "characteristic value" must be determined from the soil investigation data. Another problem has been the choice of magnitude of the partial coefficients.

The verification can be done in one of two ways according to the code:

Using the partial coefficient method or

Using a well documented statistical method.

The partial coefficient method is the standard method but the purpose of using it is of course to fulfill the requirements stated in terms of the safety index ß in order to get constructions with a consistent reliability.

The safety checking format of the partial coefficient method is the standard one:

For the limit state show that the dimensioning load effect S_d is less than the dimensioning resistance R_d.

S_d and R_d are calculated using a characteristic value F_k and f_k respectively which are multiplied (loads) or divided (resistances) by a partial coefficient γ.

For structures material properties and partial coefficients can be taken from tables, but in geotechnical work the problems are greater:

The characteristic value must be determined from test data from case to case.

For practical work the amount of statistical computations must be restricted, which implies that the characteristic value should be taken as the mean which most users would understand and feel comfortable with. Using a percentile would require the users to compute also the variance from a restricted set of data.

The penalty for using a simple characteristic value comes in the choice of partial coefficients. Using a small percentile as characteristic value, the theoretically correct partial coefficient does not differ very much and can for practical use be taken as a constant value.

With the mean as a characteristic value, the partial coefficients will vary with the problem and must thus be calculated from case to case.

One of the important parameters that determine the value of the partial coefficients is the variance of the soil properties. The problem is thus twofold, a simple method to calculate the coefficients is needed as well as a suitable stochastic soil model for calculating the variances.

2 CALCULATION OF PARTIAL COEFFICIENTS

It is possible to calculate "exact" partial coefficients which gives a design with the prescribed safety index ß using the relationship (see for instance Thoft-Christensen & Baker, 1982)

$$\gamma_i = x_{k,i} / (\mu_i + \alpha_i \beta \sigma_i) \qquad (1)$$

γ_i = the i:th partial coefficient of safety

$x_{k,i}$ = the i:th characteristic value

α_i = the i:th sensitivity factor

β = the required safety index

σ_i = the standard deviation of the i:th stochastic variable

The problem is however, that even given the statistical parameters, the sensitivity factors α_i must be calculated. These sensitivity factors contain partial derivatives of the limit function and the best method to calculate them is the ß-method.

This means that introducing the partial coefficient method poses a dilemma: either the partial coefficients have to be chosen on a subjective basis following today's praxis or the partial coefficient method should be replaced by the more complex ß-method.

The first method, to adjust the partial coefficients to conform

to today's total safety factor, just trades a known system for an unknown, something which can give rise to serious mistakes. The method does not take advantage of the statistical method e.g giving constructions with a known and balanced reliability.
To avoid the dilemma, the use of empirical sensitivity factors has been suggested. One method of determining such factors is described by Thoft-Christensen & Baker (1982). In this method, the variables are ranked and then the sensitivity factors are calculated according to that ranking:
$\alpha_i = 0.8\ (\sqrt{i} - \sqrt{i-1})$ for resisting variables and
$\alpha_i = 0.7\ (\sqrt{i} - \sqrt{i-1})$ for load variables
with i being the rank of the variable.

These values have been compared by Olsson, Rehnman & Stille (1985) to exact values calculated using the ß-method with the characteristic value taken as the mean. This comparison shows that the partial coefficients for resistance variables sometimes are very much on the unsafe side. One reason for this might be the choice of characteristic value.

2.1 Suggested empirical sensitivity factors

The authors suggest that, with a characteristic value equal to the mean, the following approach might be used:
The variables are ranked in order of descending coefficients of variation (standard deviation divided by the mean). Then, observing that the squared sum of the sensitivity factors is always equal to one, the following sensitivity factors are assigned:
$(\alpha_1)^2 = 0.9$
$(\alpha_2)^2 = 0.9\ (1\text{-}0.9)$
$(\alpha_3)^2 = 0.9\ (1\text{-}0.9\text{-}0.09)$
and so on according to the principle that in each step 90% of the remainder of the total sum of one is assigned to the sensitivity factor.
If two variables are equally ranked the available part of $(\alpha_i)^2$ is divided equally between them.
For resistance variables α_i is negative and for load variables α_i is positive.
For some examples of typical foundation problems partial coefficients calculated using these sensitivity factors have been compared to exact values and a good agreement has been found. However, until more examples have been checked it is suggested that a conservative 10% be added to the sensitivity factors. Also, it should be observed that there might be cases where the simple system of ranking according to coefficient of variation is not applicable.
It seems, however, that the method of using empirical sensitivity factors in calculating partial coefficients of safety is a most promising way out of the dilemma of the non-constant partial coefficients caused by the choice of characteristic value.

2.2 Stochastic soil model

In both the ß-method and in the partial coefficient method the soil parameters should be stochastic variables. This means that a stochastic soil model is needed which is simple to use and understand, and at the same time capable of the following:

- representing the spatial variations of the soil
- being able of combining local experience with "hard" data
- taking the small sample uncertainty (statistical uncertainty) into account.

The emphasis should be made on simplicity as most engineers do not have a background in statistics. The model should if possible be so simple as to allow a "cook-book" use.
In choosing a model the following considerations should be made:
-The soil is elasto-plastic so that all soil element contribute at failure
-The soil has an inherent physical variation. The effect of this can be reduced by physical averaging
-The mean of the soil parameters is modelled as a stochastic variable. This is not a physical property but is caused by our lack of information and can be reduced by more sampling
-The soil is modelled as consisting of strata with no trend in each stratum.
One model that considers the above points is suggested by Rackwitz and Peintinger (1981).
In this model the soil properties are described as consisting of two superimposed parts, a regional mean ("Geologiestreuung") and a spatial variation ("Baustellestreuung") around this mean, see figure 1.

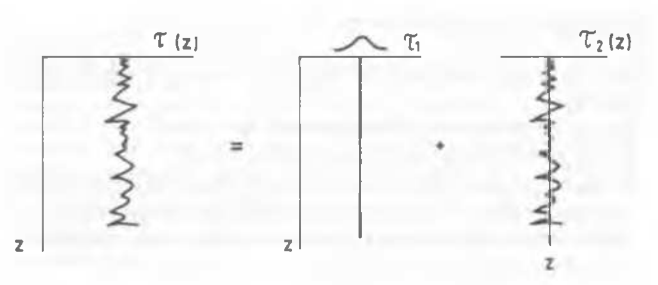


Figure 1. The parts of the stochastic soil model

As was mentioned earlier, the variance is of great importance. In the suggested model the variance consists of two parts, one caused by lack of complete knowledge and one caused by the physical structure of the soil.
A complete deterministic description of the physical variation can not be obtained but the variation can be described statistically.
In the model, the variation round the mean is considered to be known as regards its magnitude as well as its statistical "law of variation" i.e its correlation structure.
This is because a determination of this correlation structure from case to case would require a very great number of samples and rather complicated statistical calculations. The authors suggest that for practical work correlation data be given in the Code.
An important effect is that of physical averaging e.g in a slip surface. If the soil is elasto-plastic the spatial mean of the shear strength should be used in calculations. The variance of

a spatial mean is smaller than the point variance, see for instance Vanmarcke (1977).
Different methods exist for calculating the variance reduction. As a general method the authors suggest the use of numerical integration. In this the variance reduction is calculated as the mean of the correlation between all points on the surface. In practical use MonteCarlo techniques are applied with a random sampling of a number of points in the order of 100. Also nomograms can be designed which give the variance reduction as a function of some characteristic measure of the construction and the correlation distance b_z, see figure 2.

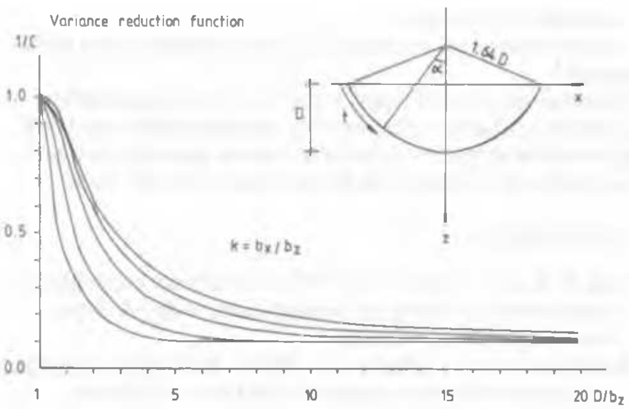


Figure 2. Variance reduction function for the slip surface shown

In order to be able to incorporate the experience of the engineer the authors advocate the use of Bayesian statistics in which probability is a subjective degree of belief and can be updated when more information is available. For details see e.g Ang &Tang (1975).
In soil mechanics the number of samples usually is small. To cope with this the so called predictive (or Bayesian) distribution of the soil property can be used, see Benjamin and Cornell (1970).

The following procedure is proposed:

1 Describe the prior knowledge of the mean μ as Normal distributed $N(m', \sigma')$

2 Take samples and calculate their mean $\bar{x}$

3 Update from σ' to σ'' and from m' to m'' using the solution given by Benjamin and Cornell (1970)

$$1/(\sigma'')^2 = 1/(\sigma')^2 + n/(\sigma_z)^2 \quad (2)$$

$$m'' = [(1/\sigma')^2 m' + (n/(\sigma_z)^2 \bar{x}] / [(1/\sigma')^2 + (n/(\sigma_z)^2] \quad (3)$$

where m' and s' are the prior mean and standard deviation of the mean μ

m'' and σ'' are the posterior (updated) mean and standard deviation of the mean μ

σ_z is the standard deviation of the data generating process (which is considered to be known in the model)

$\bar{x}$ is the sample mean

n is the number of samples.

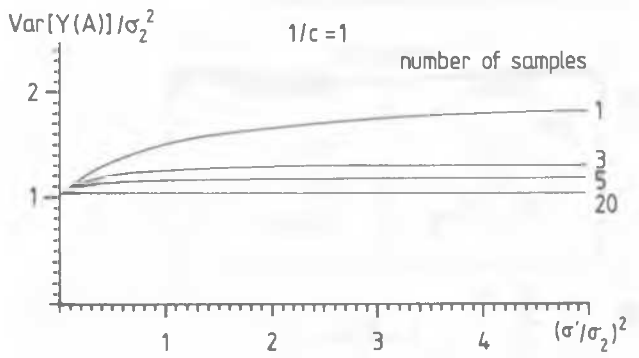
4 The prediction function for the spatial mean Y(A) will be Normal with moments

Expected value: $E[Y(A)] = m''$

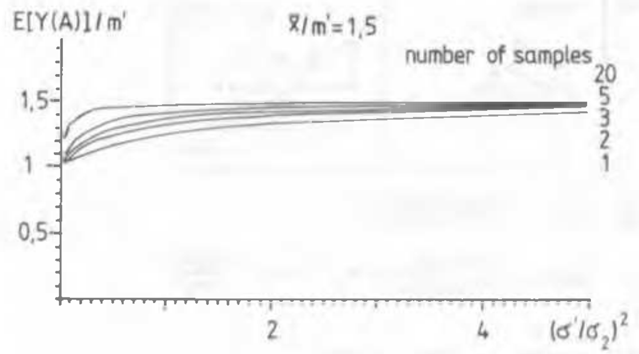
Variance: $Var[Y(A)] = (\sigma'')^2 + (1/c)\,\sigma_z$

where 1/c = the variance reduction factor, see Olsson (1986).

A sensitivity analysis shows that this updating process is rather insensitive to the prior knowledge as soon as the number of samples is reasonably large. This is shown for expected value E[Y(A)] in figure 3a and for the variance Var[Y(A)] in figure 3b. The strength of the prior information is given by the ratio σ'/σ_z so that a strong prior gives a small value of this ratio. In the figures 3a and 3b one can also see that once a certain number of samples has been taken, further samples will not add much information. This number of samples is greater for the variance than for the mean. It must be observed, that this figures just illustrate the effect of the sample number on the statistical updating of a homogeneous soil layer. No regard has been taken to other purposes of the soil investigation program.



3a. Expected value



3b. Variance

Figure 3.Effect of prior information on updated statistical parameters of the spatial mean Y(A), after Olsson (1986).

The model used has several restrictions, for instance that the correlation structure (correlation distance and σ_z) must be known in beforehand for the type of soil. The mean must be constant with no trend and the samples must be uncorrelated. This means that the location of the samples does not influence the result,which is a real drawback. These restrictions, however, are a consequence of making the model simple to use. It can therefore be viewed as a first step into a statistical approach for the practicing soil engineers.

In a code the soil model and the method of calculating the partial coefficient might be used as follows:
a. Achieve correlation data from the Code and possibly also accepted prior parameters representing a certain information level.
b. Take samples and update
c. Calculate the variance reduction
d. Calculate variances and coefficients of variation
e. Rank the variables, calculate the α_i:s and the partial coefficients
f. Make the design

The procedure is illustrated in figure 4

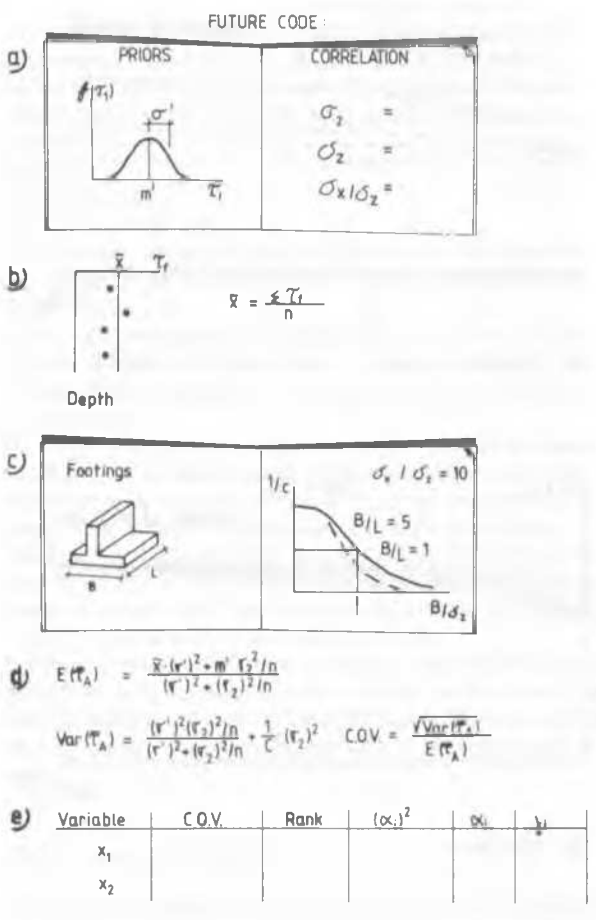


Figure 4. Design process using stochastic soil model and partial coefficients.

SUMMARY

A method is suggested that may be used in practical application of statistical methods to get a stringent system of partial coefficients of safety.
The method contains two main features:
A system for calculating the partial coefficients when the coefficients of variation are known
A simple stochastic soil model to calculate variances.

The aim when developing the method has been to get a method which
- gives designs with the prescribed safety index ß
- is simple but stringent
- can be used as an introduction to more sophisticated methods.
However there is still much work to be done, especially to determine suitable values for soil correlation structures and to check the empirical sensitivity factors against exact values, before the method can be introduced into the Code.

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