On the modelling of the consolidation response of end bearing piles
La modélisation de la réponse en consolidation des pieux portant en pointe

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SYNOPSIS: This paper applies the finite element technique to examine the consolidation response of purely end bearing piles. The modelling focusses on the utilization of composite finite and infinite elements to represent respectively the near field and far field consolidation response. The consolidation behaviour of the end bearing zone is assessed by examining two limiting cases where the end bearing zone is located either in a deep borehole or embedded in a medium of infinite extent. Numerical results developed illustrate the manner in which drainage conditions at the end-bearing zone influences the degree of consolidation.

1. INTRODUCTION

The end bearing response of axially loaded piles have been examined in great detail due to their importance to foundation design. Accounts of these developments are given by Meyerhof (1976); Vesic (1977); Pells (1980); and Poulos (1987). The assessment of the consolidation response of an end bearing pile should take into consideration a range of interface traction, displacement and pore pressure boundary conditions that are applicable to the load transmitting interface. This paper applies the finite element technique to study the time-dependent settlement behaviour of an end bearing pile. The depth of application of the end bearing loads is assumed to be such that the medium is represented as an infinite space. The finite element scheme uses the theory of primary consolidation developed by Biot (1941) to examine the end bearing pile problem. The finite element scheme also employs a coupled infinite element to correctly model the region of infinite extent. The numerical results presented in the paper illustrates the influence of drainage and displacement and/or traction boundary conditions on the magnitude and rate of consolidation settlement. The results can also form the basis for the evaluation of certain aspects of the consolidation response of deep borehole plate load tests and screw plate tests (Selvadurai and Nicholas, 1979; Selvadurai et al. 1980; Selvadurai and Gopal, 1986).

2. FINITE ELEMENT MODELLING OF SOIL CONSOLIDATION

The basic equation for one-dimensional consolidation first proposed by Terzaghi (1925) was further extended by Biot (1941) to include three dimensional effects. This generalized theory was used by many investigators to develop analytical solutions for a variety of problems associated with consolidating media (see e.g., McNamee and Gibson, 1960a,b; Gibson et al. 1970; Chiarella and Booker, 1975; Booker and Small, 1986). Sandhu and Wilson (1969) were the first to apply finite element methods to the study of consolidation problems. Since these initial studies, a number of authors (Ghabbousi and Wilson, 1972, 1973; Booker and Small, 1975; Sandhu et al., 1975; Selvadurai and Gopal, 1986; Simoni and Schrefler, 1987) have applied the finite element technique to the solution of a variety of soil consolidation problems of interest to geotechnical engineering. The review of both the analytical and numerical approaches to the study of soil consolidation presented here is not meant to be exhaustive. A comprehensive account of past and recent developments are given by Lewis and Schrefler (1987).

Following the standard variational principles discussed in detail in the previous articles, the finite element equations of consolidation can be derived from the governing equations in the following form:

$$\begin{bmatrix} \mathbf{K} & \mathbf{C} \\ \mathbf{C}^T & -(\mathbf{E}+\delta\Delta t\mathbf{H}) \end{bmatrix} \begin{Bmatrix} \mathbf{u}_{t+\Delta t} \\ \mathbf{p}_{t+\Delta t} \end{Bmatrix} = \begin{Bmatrix} \mathbf{F}_{t+\Delta t} \\ \mathbf{G}_{t+\Delta t} \end{Bmatrix} \qquad (1)$$

where $\mathbf{u}$ are the nodal displacements; $\mathbf{p}$ is the pore fluid pressures; δ = convolution integration factor ($\delta > 0.5$ for stable integration); Δt = time increment; $\mathbf{K}$ = stiffness matrix of soil; $\mathbf{C}$ = interaction stiffness between soil skeleton and pore fluid; $\mathbf{H}$ = permeability matrix governing the dissipation of pore fluid; $\mathbf{E}$ = compressibility matrix of fluid; $\mathbf{F}$ = external traction and body force vector and $\mathbf{G}$ = force vector due to externally specified pore pressures and dissipation forces.

The solution of (1) is usually achieved by employing composite type finite elements. These elements have both displacement and pore pressures as nodal variables. In a typical quadrilateral composite element the displacements are defined at all 8 nodes and the pore pressures defined at the four corner nodes. This special finite element technique, first introduced by Sandhu and Wilson (1969) ensures the same order of linear stress variation for both the effective stresses in the soil skeleton and the pore pressures in the fluid.

The concept of unbounded soil regions is employed quite extensively in the treatment of problems in geotechnical engineering. Analytical approaches are by far the most convenient methods for dealing with problems involving unbounded domains. Such analytical treatments are however restrictive, in the sense that three-dimensional conditions required for foundation geometries, soil conditions, etc. associated with practical problems cannot be conveniently handled through analytical schemes. Finite element methods are therefore of particular interest in dealing with complicated soil-foundation interaction problems involving soil consolidation. The conventional finite element techniques however have limitations in accurately modelling domains of infinite extent. The finite element based numerical solution procedure should be suitably modified to take into account the special requirements of zones of infinite extent. The use of infinite elements offers a computationally efficient and accurate technique for modelling unbounded domains such as infinite and halfspace regions. Examples of the application of infinite elements to the study of soil consolidation are given by Selvadurai and Gopal (1986), Schrefler and Simoni (1987) and Simoni and Schrefler (1987). In this paper an infinite element approach is employed in the study of the end bearing pile problem. The finite element technique employs eight noded composite finite elements and five noded composite infinite elements to model the unbounded consolidating medium. By using mesh configurations of different sizes it is observed that both computational efficiency and accuracy of the solution scheme can be improved by the incorporation of infinite elements. A reduced integration scheme is utilized for the infinite elements and the conventional Gauss-Legendre quadrature scheme is employed for both infinite and finite elements.

3. THE END BEARING PILE PROBLEM

The finite element procedure outlined previously is applied to the study of the end bearing pile problem. Prior to the numerical solution of the title problem, the accuracy of the numerical scheme which utilizes composite infinite elements is established by comparison with existing analytical results available in the literature on consolidating semi-infinite media. A detailed account of these comparisons are given by Selvadurai and Karpurapu (1989). The consolidating behaviour of an end-bearing pile is influenced by a number of factors including the pore pressure and displacement boundary conditions at the soil-pile interface, the depth of location of the end bearing zone and the relative stiffness of the soil-pile system at the load transmitting region. In this study it is explicitly assumed that the end bearing consolidation behaviour of the pile can be modelled by restricting attention primarily to the situation where the end bearing loads are applied to the soil medium via a rigid circular base. When this simplification is invoked the consolidation response of the end-bearing zone can be examined by imposing the pore pressure boundary conditions at the soil-pile interface. Finally, these boundary conditions cannot be specified with certainty. The exact nature of the boundary conditions will depend significantly upon the method of construction of the pile. For this reason it is prudent to consider extreme cases of the end bearing response which are characterized by the following problems. In the first case we assume that the end bearing region of the pile is located at the base of a bore hole (Figure 1). Pertaining to this model, two specific problems are examined; one referring to a fully drained soil-base interface (Case A1) and the second referring to an impermeable soil-base interface (Case B1). The boundary conditions appropriate for the Case A1 are as follows.

$$\begin{aligned}
&p = 0 \; ; \; z = 0 \; ; \; 0 < r < a \; ; \; t > 0 \\
&p = 0 \; ; \; r = a \; ; \; -\infty < z < 0 \; ; \; t > 0 \\
&\sigma_{rr} = 0 \; ; \; r = a \; ; \; -\infty < z < 0 \; ; \; t > 0 \\
&\sigma_{rz} = 0 \; ; \; r = a \; ; \; -\infty < z < 0 \; ; \; t > 0 \\
&u_z = const \; ; \; z = 0 \; ; \; 0 \le r \le a \; ; \; t > 0 \\
&\sigma_{rz} = 0 \; ; \; z = 0 \; ; \; 0 < r < a \; ; \; t > 0 \\
&P = 2\pi \int_0^a r\sigma_{zz} dr \; ; \; z = 0 \; ; \; t > 0
\end{aligned} \tag{2}$$

The boundary conditions appropriate for the case B1 are as follows:

$$\begin{aligned}
&\frac{\partial p}{\partial z} = 0 \; ; \; z = 0 \; ; \; 0 < r < a \; ; \; t > 0 \\
&p = 0 \; ; \; r = a \; ; \; -\infty < z < 0 \; ; \; t > 0 \\
&\sigma_{rr} = 0 \; ; \; r = a \; ; \; -\infty < z < 0 \; ; \; t > 0 \\
&\sigma_{rz} = 0 \; ; \; r = a \; ; \; -\infty < z < 0 \; ; \; t > 0 \\
&u_z = const \; ; \; z = 0 \; ; \; 0 \le r \le a \; ; \; t > 0 \\
&\sigma_{rz} = 0 \; ; \; z = 0 \; ; \; 0 < r < a \; ; \; t > 0 \\
&P = 2\pi \int_0^a r\sigma_{zz} dr \; ; \; z = 0 \; ; \; t > 0
\end{aligned} \tag{3}$$

In the second case we assume that the end bearing region is located within a consolidating medium of infinite extent (Figure 2). The end bearing region maintains contact only over one face of the circular plate region. The plate remains unadhered on one face.

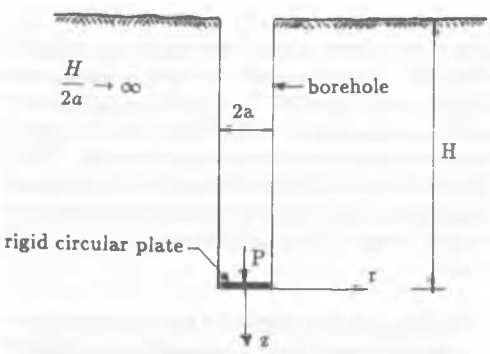

Figure 1: Base loading of an unlined semi-infinite borehole

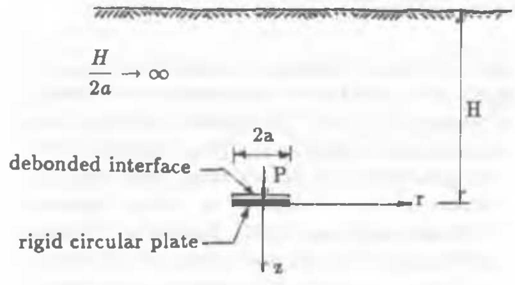

Figure 2: Interior loading of an infinite medium

Again, pertaining to this model, two specific problems can be examined; one which deals with a fully drained soil-base interface (case A2) and another dealing with an impermeable soil-base interface (B2). The unbonded surface of the soil medium is considered to be fully drained.

The boundary conditions appropriate for the case A2 are as follows:

$$
\begin{aligned}
&p = 0 \;;\; z = 0^- \;;\; 0 < r < a \;;\; t > 0 \\
&p = 0 \;;\; z = 0^+ \;;\; 0 < r < a \;;\; t > 0 \\
&u_z = const \;;\; z = 0^+ \;;\; 0 \leq r \leq a \;;\; t > 0 \\
&\sigma_{rz} = 0 \;;\; z = 0^- \;;\; 0 \leq r \leq a \;;\; t > 0 \\
&\sigma_{rz} = 0 \;;\; z = 0^+ \;;\; 0 < r < a \;;\; t > 0 \\
&\sigma_{zz} = 0 \;;\; z = 0^- \;;\; 0 < r < a \;;\; t > 0 \\
&P = 2\pi \int_0^a r\sigma_{zz} dr \;;\; z = 0^+ \;;\; t > 0
\end{aligned} \tag{4}
$$

where the superscripts $(\;)^-$ and $(\;)^+$ refer to faces of the rigid plate which relate to the unadhering and the adhering soil-plate interfaces respectively. The boundary conditions applicable for the case B2 are as follows.

$$
\begin{aligned}
&p = 0 \;;\; z = 0^- \;;\; 0 < r < a \;;\; t > 0 \\
&\frac{\partial p}{\partial z} = 0 \;;\; z = 0^+ \;;\; 0 < r < a \;;\; t > 0 \\
&u_z = const \;;\; z = 0^+ \;;\; 0 \leq r \leq a \;;\; t > 0 \\
&\sigma_{rz} = 0 \;;\; z = 0^- \;;\; 0 < r < a \;;\; t > 0 \\
&\sigma_{rz} = 0 \;;\; z = 0^+ \;;\; 0 < r < a \;;\; t > 0 \\
&\sigma_{zz} = 0 \;;\; z = 0^- \;;\; 0 < r < a \;;\; t > 0 \\
&P = 2\pi \int_0^a r\sigma_{zz} dr \;;\; z = 0^+ \;;\; t > 0
\end{aligned} \tag{5}
$$

The finite element discretization used to model the axisymmetric problem illustrated in Figure 2 is shown in Figure 3. Both composite finite elements and composite infinite elements are used to model the problems. The accuracy of the finite element technique was verified by comparison with the analytical solution developed by Chiarella and Booker (1975) for the indentation of a halfspace region by a rigid circular foundation. This analytical solution was chosen particularly in view of the fact that a rigid plate model is used to depict the effects of the end bearing load. In the numerical scheme the rigid behaviour of the foundation is simulated by constraining the vertical displacements under the loaded area to be equal. The multipoint constraint method proposed by Abel and Shepherd (1979) was used for implementing the constraint conditions. The meshes have been finely discretized near the foundation edge to account for the high stress gradients that are associated with the singular stress field at the boundary of the rigid plate.

4. NUMERICAL RESULTS AND CONCLUSIONS

The results of the numerical investigations can be presented in diverse ways. In this paper, however, we focus on the evaluation of the degree of consolidation settlement of the end-bearing zone as

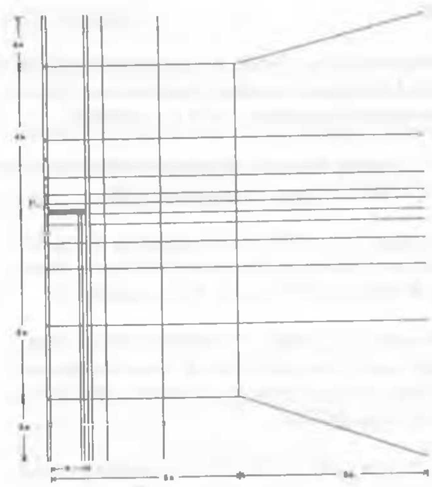

Figure 3: Finite element discretization of the interior loading problem

determined from the numerical modelling of the two basic problems and the associated drainage boundary conditions defined previously. The degree of consolidation U, at time t, is defined as

$$U = \frac{w_t - w_0}{w_\infty - w_0}$$

where w_0 and w_∞ are the instantaneous and ultimate settlements of the rigid end-bearing zone of the pile. The time factor T is defined as

$$T = \frac{ct}{a^2}$$

where $c(= 2Gk/\gamma_w)$ is the coefficient of consolidation, k is the permeability, G is the shear modulus, γ_w is the unit weight of water and a is the radius of the end bearing zone.

The Figures 4 and 5 illustrate the manner in which the time-consolidation response of the end bearing zone is influenced by the drainage conditions at the contact zone between the rigid end bearing region and the consolidating soil. In the case where the end bearing zone is located at the base of an unlined borehole, the consolidation response is influenced, quite significantly, by the nature of the drainage conditions at the soil-base interface. In the case of the end bearing region which is fully embedded in partial contact with a consolidation medium of infinite extent, the trends in the consolidation behaviour are very similar to those observed previously in connection with the deep borehole response. Owing to limitations of space the results presented are applicable for the case where $\nu = 0$. Similar results can be derived for other values of the soil skeleton Poisson's ratio. The initial and final settlements of the end bearing zone associated with the two models can be obtained from analytical and numerical results given by Selvadurai and Nicholas (1979). The paper shows that finite element techniques can be used quite successfully to investigate the consolidation response of foundations which are embedded within soil media. The use of infinite elements are an added computational advantage in treating consolidating media with unbounded domains or with dimensions greatly in excess of the embedded loaded zone.

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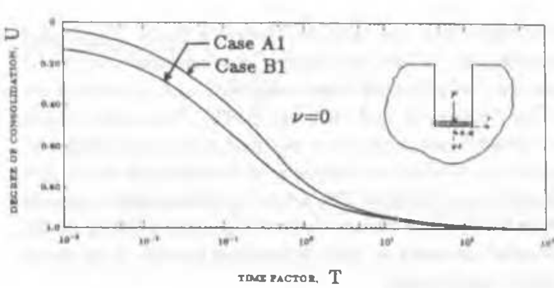

Figure 4: Time-consolidation behaviour of the end bearing region at the base of a borehole

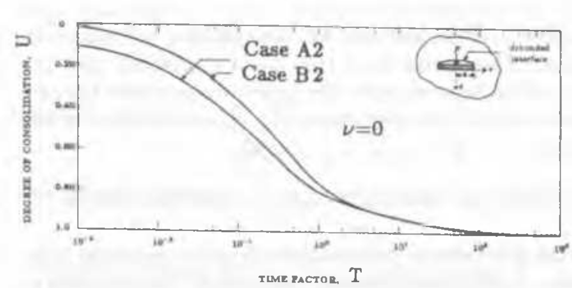

Figure 5: Time-consolidation behaviour of the end bearing region in an infinite medium