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On limit criteria for plastic failure in strain rate softening soils

Sur les critères de limite pour la rupture plastique aux sols de ramollissement de taux de dilatation

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SYNOPSIS

Strain softening and strain-rate softening in sensitive soils presents a major problem to the geotechnician because it invalidates his basic analytical tool - the plastic failure model. Marked brittleness in soil must be analyzed by some progressive failure model such as that treated in ref |3|, |6|. The present article deals with aspects on stability criteria for extensive slopes if the limiting large strain strength of the soil is strain rate dependent. The issue is exemplified by means of mechanical model - in the form of a train set - which offers striking similarities to the model used by the author for progressive failure analysis. The conclusions from the example are used to illustrate problems with the conventional safety factor according to the plastic failure model and how the assessment of safety will have to be modified in strain rate softening soils.

GENERAL

In analysis of geotechnical stability problems - based on the concept of plastic failure - the degree of safety is attributed a numerical value defined as the ratio between an assumed shear strength of the soil and the mean shear stress along a slip surface. Yet, in geotechnical literature there is abundant evidence - especially in soft clayey and silty soils - of considerable strain softening. The ultimate strength at large strains is often referred to as the limit state strength. Bernander et al have in |7|, |10| shown that the limit state stress-strain properties are also dependent on strain rate and overconsolidation ratio (OCR). The dependence of shear strength properties of strain rate and of OCR in the critical state has also been demonstrated by other workers e.g. Lefebvre & La Rochelle. |16| and Torstensson |22|.

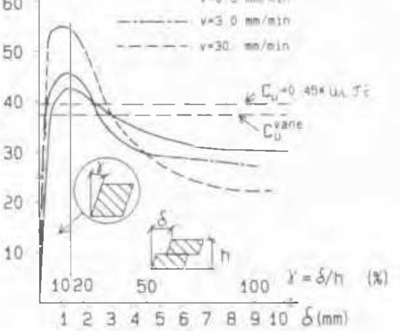

Fig. 1

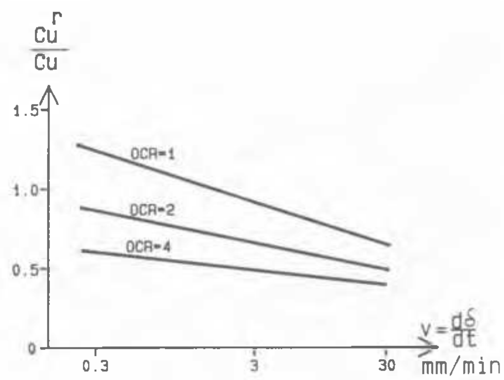

Fig. 2

Fig. 1 & 2 Typical dependence on strain rate and of OCR of stress-deformation properties in soft Swedish Clays acc. to |7| and |10|.

Many geotechnicians e.g. |15|, |18| suggest that the difficulties involved in strain-softening may readily be solved by merely substituting the shear strengths according to standard tests (cone, vane and unconfined compression tests etc) by the limit state shear strength values derived from standard (slow) shear tests at large deformations. This, of course, is an appealing approach as there would then still be no need to analyze the strains and deformations in the soil in e.g. slope stability analysis - i.e. the plastic failure concept would still be valid.
However, back-analysis of ocurred slides in Sweden indicate that this easy approach is not always valid - especially if the analysis is expected to explain the events and final outcome of a slope failure. If the limit-state strength ($C_u r$) is strain-rate dependent, then obviously time must be a factor to consider. For instance, the initiation, propagation and final extent of a slide cannot possibly be predicted correctly unless the response of the soil for the particular strain rates which are relevant in the different stages of a slide development are introduced into the analysis. Or - to put it bluntley - it is ludicrous to conceive that shear strengths from tests at strain rates of e.g. 0.001 mm/sec can have any bearing on the development or on the final extent and configuration

of a slide which evolves at a pace of e.g. 1 m/sec. As is seen in fig 1 the response of the soil already at a deformation rate of 30 mm/min (=0,5 mm/sec) is quite different from the response at 0.005 mm/sec. In the opinion of the author few soil stability problems can be solved - in principle - without due consideration to the timing of a potential slide and its effect on strain rates - although the practical importance of this is probably limited to situations where, rapid triggering agents, rapid creep and marked strain-softening (e.g. as in Scandinavian quick clays) is involved. In order to evaluate the applicability of the plastic failure approach it is thus necessary somehow to consider relevant strain-rates. This is related to the simple fact - that before the desired limit state for a portion of a slope is attained - considerable stress redistributions and deformations may have to take place and these deformations and associated strain-rates will under certain circumstances be decisive for the stability of the entire slope.

EXAMPLE

In order to illustrate this line of thought let us consider a train set on a sloping rail track (Fig. 3). - a mechanical model with striking similarities to the model for progressive failure in large planar landslides used by the author in references |3|, |4| and |6|.

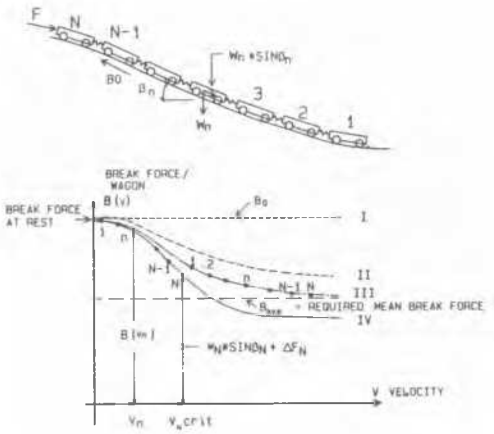

Fig. 3 Example of a mechanical analogy of progressive slope failure.

Let a force F act on the last wagon. Each wagon is assumed to be able to mobilize a breaking force B_o at rest. In motion the friction in the breaks is reduced with increasing velocity so that the mobilized friction force per wagon varies according to either of the curves I-IV in fig. 3.

Notations:

B_o = Breaking force/wagon at rest

$B(v)$ = Breaking force/wagon at velocity (v)

$B(v_n)$ = Breaking force/wagon at velocity (v_n)

B_{ave} = Average breaking force/wagon to keep the train in set place - $B_{ave} = \sum^N W_n \sin\beta_n / N$

W_n = Weight of wagon no n

N = total number of wagons f = safety factor

Other notations according to fig. 3.

If the train is at a stand still, then

$$f = B_o/(B_{ave}+F/N) = N\, B_o / \left(\sum^N W_n \sin\beta_n + F\right) \qquad (1)$$

The safety factor acc. to equ. 1 is however only apparent since conditions will change the very moment any part of the train set is activated. Thus if the velocity of a wagon is (v_n) the following expression for f is valid:

$$f_v = \sum^N B(v_n) \,/\, \left(\sum^N W_n \cdot \sin\beta_n + F\right) \qquad (2)$$

Equ. 2 implies that the safety factor f_v is defined only if the velocity of each individual wagon is defined i.e. if no part of the system may be subject to accelleration. However, some important conclusions may be drawn:

a) If the mean mobilizable breaking force for all conceivable values of (v_n) exceeds the required mean breaking force B_{ave} (which is the case for e.g. curves I, II and III) then the safety factor f_v will always be greater than unity.

Thus: if $\sum^N B(v_n) > \sum^N W_n \sin\beta_n + F$ (2a)

for all conceivable values of v_n

then the train set is always stable. - because although the safetyfactor f_v - under the circumstances - will render velocity dependent values - the train set as an integral structure cannot accellerate.

b) If on the other hand the mobilizable breaking force $B(v_n)$ for any conceivable values of $(v_n) = (v_n)_{max}$ can fall below the total driving force (the right member of equ 2a) then - if the safety factor f_v is to have any physical meaning - also other conditions have to be fulfilled. Thus a guaranteed stability can be evaluated with equ. 2 only if the application of the additional force F does not cause significant accelleration in parts of the train set. This requires that the springs in the connections are infinately stiff or - if there is resilence in the system - that the load F is applied gradually at a slow rate.
If, however, there is considerable resilence in the system and the rate of load application is not restricted, the displacements related to stress redistribution may involve accelleration of part of the train. This leads to reduced breaking force in some wagons which again leads to additional accelleration whereby $B(v_n)$ for one wagon after the other may fall below the required average breaking force B(ave) until the whole train is set in motion.
A stable situation can only be guaranteed as long as the driving force for any one wagon $W_{(n)} \cdot \sin\beta_n + \Delta F_n$ does not exceed the mobilizable breaking force $B(v_n)$ for that same wagon.
Thus, for a train not to accellerate the following conditions must be satisfied:

If for conceivable values of v_n

$$\sum^N B(v_n) < \sum^N W_n \sin\beta_n + F \qquad (2b)$$

then for every wagon in the train equation 3 must also apply:

$$B\,(v_n) \geqslant W_n \sin\beta_n + \Delta F_n \qquad (3)$$

CONCLUSIONS FROM EXAMPLE

The example with the train model demonstrates that:

a) if - in a resilent (elastic) structure of strain rate softening material - the limit strength C_u does not fall below the average prevailing shear stress - the structure will always be stable

$$f = \frac{C_u}{\tau} \text{ always } > 1 \qquad (1a)$$

b) if the integral effect of the limit material strength in the different elements of the structure for conceivable strain-rates may fall below the average shear stress, then - the stability of the structure may be guaranteed only when - the maximum stress does not exceed the mobilizable strength $C_u\,(d\gamma/dt)$ in any part of the structure.

If this last criterion is not fulfilled the resilent structure may - in a strain rate softening material - collapse prematurely and progressively due to the loss of strength associated with the deformations that may result from local overstressing and stress redistribution.

Thus if $f_v = Cu(d\gamma/dt)/\tau_{ave} < 1$ (2c)

for conceivable strain rates $(d\gamma/dt)$ then stability can still guaranteed

$$\text{if } \tau^n_{max} < C_u\,(d\gamma/dt)_n \qquad (3a)$$

or if the load is applied gradually and so slowly that no appreciable accelleration of strain rate occurs in the system.

APPLICATION ON SLOPE STABILITY

The example given above and the implications thereof are directly applicable on the stability of extensive slopes as shown in fig. 4a and b. As may be seen in fig. 4b and equations 2d and 3b the studied slope is stable if the soil properties conform to curves I, II and III i.e. when for all values of strain rates

$$\tau_{ave} < C_u\,(d\delta/dt)ave \qquad (2d)$$

If the soil however displays deformation properties according to curve IV (Fig. 4b)

i.e. $\tau_{ave} > C_u\,(d\delta/dt)$ ave for conceivable high

strain rates - then with notations acc to fig. 4a and b - stability is only guaranteed as long as

$$\tau^n_{max} < C_u\,(d\delta/dt)_n \qquad (3b)$$

$$\text{or } \quad (d\delta/dt)_n < (d\delta/dt)crit \qquad (4)$$

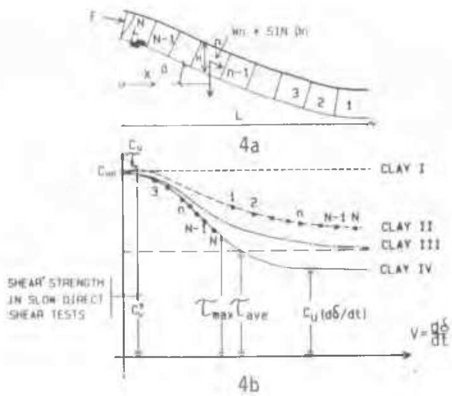

Fig. 4 Application on extensive slope

With symbols as in fig 4b the shear stress due to down slope forces is:

$$\tau_{ave} \approx \sum_1^N \frac{W_n \cdot \sin\beta_n \cdot \cos\beta_n \Delta x}{L} + \frac{F}{L} \cdot \sec\beta_{ave}$$

$$\approx \int_0^L \frac{\rho \cdot g \cdot h \cdot \sin\beta_n \cos\beta_n}{L}\,dx + \frac{F}{L} \cdot \sec\beta_{ave}$$

The criteria for slope stability are then:

$$f = \frac{\int_0^L C^r_u\,(d\delta/dt)\,\sec\beta_x \; dx}{\int_0^L \rho \cdot g \cdot h \cdot \sin\beta_x \cos\beta_x\,dx + F \cdot \sec\beta_{ave}} \geqslant 1 \qquad (2e)$$

If, however, for conceivable deformation rates $(d\delta/dt)$ $f < 1$ then stability is guaranteed only if:

$$\tau_x = \rho g\, h \sin\beta_x \cos\beta_x + dF_x \sec\beta/dx \leqslant C^r_u\,(d\delta/dt)_x \qquad (3d)$$

Equ. 3d indicates that for every part of the slope there is thus a critical rate of deformation $C_u(d\delta/dt)_{crit}$ corresponding to the prevailing shear stress defined by the equation:

$$C_u(d\delta/dt)_{crit} = \tau_{max} \qquad (4a)$$

However, as shear stress and rate of deformation (e.g. creep) are likely to be greatest in the steepest up hill parts of a slope it is there that equation 3d and 4a are of particular interest.
It should be observed that equ. 2e and 3d (4a) do not yield values of the safety factor, that reflect the true degree of safety. To achieve this the approach given in reference Bernander |6| may be used where the progressive failure analysis is based on a stress-deformation relationship relevant to the timing of the event subject to analysis. According to |6| the appropriate failure criteria are:

a) with respect to heave in the passive zone:

$$f = E_{Rankine}/E_{Current} \qquad (7)$$

or

b) with respect to progressive failure starting somewhere up slope:

$f = \Delta E_{cr}/\Delta E_{Current}$

where

E denotes earth pressures in the slope

ΔE increment of earth pressure due to some additional load effect.

CONCLUSIONS

Slope stability in strain rate softening materials cannot be solved without introducing the deformations and the time factor into the analysis. The examples and deductions above show that slope stability under these conditions is a question of strain or deformation rate rather than of stress and proven shear strength from standard laboratory tests, Cf |5|.

The application of large strain (limit state) strength - based on (slow) standard shear tests - in the analysis of stability in extensive slopes will as a rule yield results on the unsafe side. Only if the limit strength for tests at strain rates, relevant to all conceivable real events in nature, exceeds the mean shear stress will the safety factor have a guaranteed value in excess of 1,0 - a conclusion which may be of great value to the engineer. If however, this condition is not fulfilled there will be risk of progressive failure formation as soon as the rate of deformation $(d\delta/dt)$ exceeds $(d\delta/dt)_{crit}$.

A slope may therefore remain stable for thousands of years and yet fail progressively if the rate of deformation due to loading or creep exceeds past values, e.g. due to piling or other human activity.

The analytical treatment of progressive failure has previously been dealt with by the author e.g. in references |3|, |4| and |6|:

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