Dilatancy modeling for granular sand in simple shear condition

Modelage de dilatant pour sable granulaire dans une condition de cisaillement simple

CHING S. CHANG, Associate Professor of Civil Engineering, University of Massachusetts, Amherst, Mass., USA

SYNOPSIS A particle chain model is proposed for the derivation of soil dilatancy equation in a simple shear condition. By considering the deformation of a particle chain, the inclination of the mean sliding plane can be related to the mean inter-particle force vector of the assemblage, and the dilatancy equation is derived as a function of external stresses. This proposed particle chain model can be extended to more general stress conditions other than simple shear.

INTRODUCTION

Deformation of a soil sample, considered as an assemblage of particles, can be regarded as a rearrangement of particles caused by particle sliding. The directions of sliding vary randomly from particle to particle, and the average direction of the overall slidings represents the inclination of the sliding plane. This sliding plane is related to both the stress conditions and the dilatancy behaviour of the assembly.

The relationships between the inclination of sliding plane and the stress conditions have been developed and successfully applied to describe the dilatancy behaviour of sand by various investigators. Among these approaches, notably, are: (1) Rowe hypothesized a minimum energy principle (1962) stating that particles tend to slide along the direction of minimum energy; (2) Tokue (1979) and Nemat-Nassar (1980) assumed a probability distribution function for the planes of sliding. Through an integration of this distribution, the average plane of sliding can be obtained.

In this paper, the average plane of sliding is derived as a function of overall stress based on the deformation mechanism of particle chains. The dilatancy equation is then derived and compared with the existing dilatancy equations.

DEFORMATION MECHANISM

The deformation of an assemblage of particles may be caused by: (1) the sliding and rolling between particles, (2) the compression of solid particles, and (3) the crushing of particles.

According to Barden, Khayatt, and Nhightman (1969), the deformation of a specimen caused by the compression of sand particles is generally negligible compared to that caused by the sliding of particles. The crushing of sand particles also seems negligible if an ordinary sand is subjected to ordinary stress levels (Rowe, 1962; Lee and Farhoomand, 1967; Vesic and Clough, 1968.) Thus, neglecting the factors of compression and crushing of particles, deformation of an assembly is caused only by the relative movement between contact particles, which could be sliding, rolling, or a combination of both.

Horne (1965) studied the mechanism of particle rolling and sliding. He concluded that the angular rotation involved in rolling may be ignored in relation to the relative velocity due to sliding of two contact particles. Therefore, it is reasonable to assume that, at an ordinary stress level, the deformation of an assembly of particles occurs primarily as a result of sliding between particles, which is governed by the law of Coulomb friction.

The inter-particle forces between two particles are shown in Fig. 1a. The angle between the inter-particle force $\underline{F}$ and the contact normal $\underline{n}$ is α. The contact normal is defined as the vector perpendicular to the contact area of the two particles. The inter-particle force $\underline{F}$ can be decomposed into two components: th normal force perpendicular to the contact plane and the shear force parallel to the contact plane. Due to a small increment of loading applied to the soil sample, the inter-particle force changes its direction. As the increment of loading is infinitesimal, the incremental inter-particle force dF is perpendicular to $\underline{F}$. Let $\underline{f}$, $\underline{n}$, $\underline{df}$, and $\underline{s}$ be the unit vectors representing the directions of inter-particle force, contact normal, incremental particle force, and sliding, respectively. The following conditions must be satisfied for two particles at the threshold of sliding: (1) The angle between $\underline{f}$ and $\underline{n}$ is equal to the frictional angle, ϕ_μ, between two particles; (2) The direction of inter-particle force, due to the external loading, tends to change such that the angle between $\underline{f}$ and $\underline{n}$ has the tendency to increase.

The sliding deformation can be classified into two types: 1) elastic slip - When the contact forces have not yet reached the limit of sliding, two particles slip but are still remained in contact. 2) plastic slide - When the contact forces have reached the limit of sliding, the two particles slide relative to one another, and they may not remain in contact. The elastic slip is relatively small compared to the plastic sliding deformation; therefore, only the latter is considered.

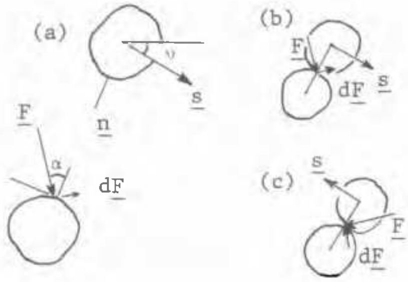

Fig. 1 Inter-particle Forces Between Two Particles

It is noted that the four vectors- s, df, f, and n - are on the same plane. The sliding vectors on this plane have two possible orientations as shown in Fig. 1b and 1c. The orientation of sliding depends on the relative positions of f and n. For either orientation: the angle between f and s is 90 - ϕ_μ; f is perpendicular to df; and s is perpendicular to n. The geometrical relationship among the unit vectors (f, df, and s), shown in Fig. 2, is expressed in the following form.

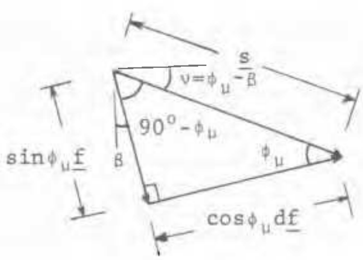

Fig. 2 Geometrical Relationship Between f, df, and s.

$$\underline{s} = \sin\phi_\mu \underline{f} + \cos\phi_\mu \underline{df}, \qquad (1)$$

and the inclination of sliding, ν, is

$$\nu = \phi_\mu - \beta, \qquad (2)$$

where β is the angle between f and vertical line.

PARTICLE CHAIN DEFORMATION

Let us consider a vertical particle chain within an assembly as shown in Fig. 3. This chain consists of particles continuously in contact. The number of particles in the chain is sufficiently large so that the deformation characteristics of this chain represent the deformation characteristics of the assemblage. Displacement of any particle in the chain depends significantly on the friction and interlocking resistance due to the physical restraint provided by the surrounding particles.

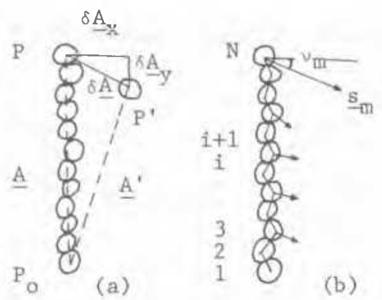

Fig. 3 Particle Chain Deformation

Consider two particles Po and P in the vertical chain. After deformation, grains slide relative to one another. The previous chain breaks; a new chain is formed. Po and P move to Po and P' respectively. The vector A deforms into the vector A'.

$$\underline{A}' = \underline{A} + \delta\underline{A} \qquad (3)$$

The components of $\delta\underline{A}$ are $\delta\underline{A}_x$ and $\delta\underline{A}_y$, and the strains for the simple shear condition can thus be defined as the relative displacement of these two end particles:

$$d\varepsilon_y = \frac{\delta\underline{A}_y}{\underline{A}} \quad ; \quad d\gamma_{xy} = \frac{\delta\underline{A}_x}{\underline{A}} \qquad (4)$$

Consider the same particle chain in which the particles are numbered from 1 to N for the purpose of illustration as shown in Fig. 3b. $\delta\underline{A}$ (i.e. the relative displacement of particle number N with respect to particle number 1) can be obtained by summing up the movement of all sliding contacts. Let $r_{j,i}$ be the relative displacement of particle j with respect to particle i. Then the relative displacement of particle N with respect to particle 1 is:

$$r_{n,1} = \delta\underline{A} = r_{2,1} + r_{3,2} + r_{4,3} + \ldots\ldots + r_{i+1,i} + \ldots + r_{N,N-1} \qquad (5)$$

Since the sliding limit may not be reached at every contact point within this particle chain, some of the terms in Eq(5) are zero. Let n' be the total number of sliding contacts. Thus Eq. (5) can be written as

$$\delta\underline{A} = \sum_{i=1}^{n'} r_i = \sum_{i=1}^{n'} L_i \underline{s}_i = n' L_m \underline{s}_m \qquad (6)$$

where r_i is the relative sliding of the upper particle to the lower particle at the i-th sliding contact with magnitude L_i and direction $\underline{s}_i$ (unit vector). L_m is the mean value of L_i, and $\underline{s}_m$ is a unit vector representing the mean sliding direction of all sliding contacts; and,

$$\underline{s}_m = \frac{1}{n'} \sum_{1}^{n'} \underline{s}_i . \quad (7)$$

It is noted that $\underline{s}_m$ has the same direction as the resultant displacement vector $\delta\underline{A}$. Substitute Eq (1) into Eq (7),

$$\underline{s}_m = \frac{1}{n'} (\sin \phi_\mu \sum_{1}^{n'} \underline{f}_i + \cos \phi_\mu \sum_{1}^{n'} d\underline{f}_i). \quad (8)$$

It is reasonable to assume that all contacts have the same sliding orientation due to shear stress change in a simple shear test. Eq(8) can thus be expressed as

$$s_m = (\sin \phi_\mu \underline{f}_m + \cos \phi_\mu d\underline{f}_m), \quad (9)$$

where $\underline{f}_m$ and $d\underline{f}_m$ are perpendicular, and the angle between $\underline{f}_m$ and $\underline{s}_m$ is $90 - \phi_\mu$, similar to the relationship shown in Fig.2. The inclination of the mean sliding plane can be obtained as

$$\nu_m = \phi_\mu - \beta \quad (10)$$

where β is the angle between the mean inter-particle force and the vertical line.

MEAN INTER-PARTICLE FORCE VECTOR

Fig. 4 shows a cubical free body with unit area on the top and bottom. In a simple shear condition, the forces on the top face, where the external shear stress is applied, should balance the inter-particle forces on the bottom face.

On the unit area of top surface, the total vertical force is σ and the total horizontal force is τ. On the bottom face the total force is Σf, the vertical component of which is Σf_y and the horizontal component Σf_x. Let β be the angle between the resultant force and the vertical force, then

$$\tan \beta = \frac{\tau}{\sigma} . \quad (11)$$

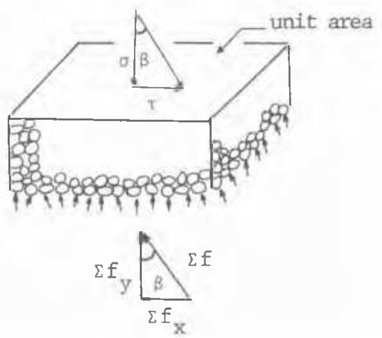

Fig. 4 Force Equilibrium of a Cubical Element

Since $\underline{f}_m$, by definition, has the same direction as the resultant force $\Sigma\underline{f}$, β in Eq(11) thus represents the angle between the mean inter-particle force and the vertical line.

DILATANCY EQUATION

Form Eq(4) and Fig. 3, the dilatancy ratio can be expressed as the displacement ratio of the particle chain; and subsequently, expressed as the inclination of the mean sliding plane.

$$\frac{d\varepsilon_y}{d\gamma} = \frac{\delta A_y}{\delta A_x} = \tan \nu_m \quad (12)$$

Substituting Eq (10) into Eq (12) yields

$$\frac{d\varepsilon_y}{d\gamma} = \tan (\phi_\mu - \beta). \quad (13)$$

Assuming that the $\underline{f}_m$ of a cubical element is the same as the $\underline{f}_m$ in a particle chain; and, substituting Eq (11) into Eq(13), the dilatancy equation becomes

$$\frac{d\varepsilon_y}{d\gamma} = \frac{\tan \phi_\mu - \tau/\sigma}{1 + \tan \phi_\mu \cdot \tau/\sigma} \quad (14)$$

Fig. 5 shows the inclination of the sliding plane for four different stages. In all stages, the direction of sliding and the direction of mean inter-particle force form an angle of $90 - \phi_\mu$. As shown in Fig. 5, when β is less than ϕ_μ (i.e. τ/σ is less than $\tan \phi_\mu$), contraction occurs ($\nu_m > 0$); when β is equal to ϕ_μ (i.e. τ/σ equals to $\tan \phi_\mu$), the soil has zero volume change due to shear ($\nu_m = 0$); and, when β is greater than ϕ_μ (i.e. τ/σ is greater than $\tan \phi_\mu$), dilation occurs ($\nu_m < 0$).

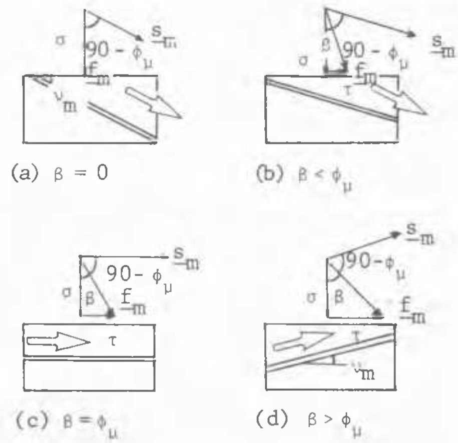

Fig. 5 Inclination of Sliding Plane for Four different Stages

Although the dilatancy equation of Eq. (14) derived above took an entirely different approach from that of Tokue (1979) and Nasser (1980), the form of the dilatancy equation turns out to be the same. This approach of the particle chain concept may provide an alternative way towards the objective of better understanding the granular behaviour.

Although the simple shear condition is used as a focus for the illustration of this approach, the methodology can be applied to more general stress conditions. (Chang, 1983)

CONCLUSION

By considering the deformation of a particle chain based on particle sliding mechanism, it can be derived that the angle between the mean sliding vector and the mean inter-particle force is $90 + \phi_\mu$. By force equilibrium the mean inter-particle force can be determined from the external stresses. Thus the relationship between the average sliding inclination and the external stresses can be established.

The dilatancy equation based on this methodology has been derived and yields reasonable results. The assumptions involved in this derivation are: 1) the particles are of convex, round shape, 2) particles are not breakable and rigid, 3) particle rolling is neglected, and 4) elastic deformation is neglected.

REFERENCES

Chang, C. S. (1983). Constitutive Modelling for Sand - A Particulate Approach. Proc. of Int. Conf. on Constitutive Laws for Engg. Materials - Theory and Application, Tucson, Arizona.

Horne, M. R. (1965). The Behaviour of an Assembly of Rotund, Rigid, Cohesionless Particles, Parts I and II. Proceedings, Royal Society, Series A, Vol. 286, pp. 62-92.

Vesic, A. S., and Clough, G. W. (1968). Behaviour of Granular Materials Under High Stresses. Journal of the Soil Mechanics and Foundation Engineering, Division, ASCE, Vol. 95, No. SM3, pp. 661-687.

Lee, K. L., and Farhoomand, I. (1967). Compressibility and Crushing of Granular Soil in Anisotropic Triaxial Compression. Canadian Geotechnical Journal, Vol. IV, pp. 68-86.

Barden, L., Khayatt, and Nhightman. (1969). Plane Strain Deformation of Granular Material at Low and High Pressures. Geotechnique, Vol. 19, No. 4, pp.441-452.

Nemat-Nasser, S. (1980). On Behaviour of Granular Materials in Simple Shear. Soils and Foundations, No. 20, pp.59-73.

Rowe, P. W. (1962). The Stress-Dilatancy Relation for the Static Equilibrium of an Assembly of Particles in Contact. Proceedings, Royal Society, Series A, Vol. 269, pp. 500-527

Tokue, T. (1979). Deformation Behaviours of Dry Sand under Cyclic Loading and a Stress Dilatancy Model. Soils and Foundations, No. 19, pp. 63-78.