A simplified yet general model for constitutive behavior of soils

Loi simplifiée et générale pour le comportement des sols

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SYNOPSIS A general yet simplified yield function is proposed for describing elastic-plastic behavior of soils. It allows for effect of isotropic plastic hardening, state of stress, coupling of shear and volumetric responses and stress paths. Comprehensive test data obtained by using a truly triaxial device for five soils are used to illustrate determination of parameters and verification of the model.

INTRODUCTION

Development of constitutive models that can unify observed behavior of a material or a class of materials is the subject of significant current research activity. A constitutive model should satisfy the governing physical principles and possess a theoretical framework, but at the same time, it should be sufficiently simple for practical applications. In other words, a constitutive model should be established such that it can account for as many as possible significant factors affecting the behavior, and then simplified for practical use.

The objective of this paper is to present a general model for describing behavior of soils that is based on appropriate mathematical formulation, identification of required material constants and their determination from laboratory tests and verification (Desai, 1980), (Desai and Faruque, 1983, 1984).

Behavior of soils is affected by a number of factors such as initial stress, physical state defined by density, void ratio, and volume, stress path, coupling of volumetric and shear responses and type of loading. A rational constitutive model should incorporate these (or a part of) factors in a unified formulation. This is possible only through comprehensive and careful observations of the behavior of a soil; here it is important to identify responses or parameters that are invariant with respect to the factor(s). For instance, plastic volumetric strain or void ratio is often used to define hardening behavior of soil. In this paper, special parameters are delineated in order to allow for plastic hardening, stress path dependency and coupling of volumetric and shear responses.

LABORATORY TESTING

Cubical specimens of five different geological materials were tested by using a truly triaxial or multiaxial testing device. The tests involved a wide range of initial hydrostatic or confining stresses, and stress paths depicted in Fig. 1. About 20 to 40 such tests were performed for each material. Details of the five materials tested are given below:

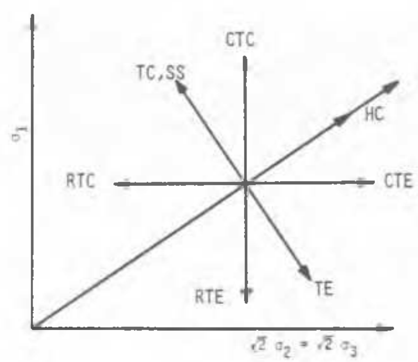

Fig. 1 Schematic of the Commonly Used Stress Paths in Triaxial Plane

Silty Sand

This soil was obtained from the site of Urban Mass Transportation Administration Test Section at Transportation Test Center, Pueblo, Colorado (Desai et al, 1983). It is a well graded material with uniformity coefficient of 3.2, and specific gravity of 2.59. Its maximum wet density is 2.23 g/m^3, with the optimum moisture content of about 9.0 percent. The initial density of the specimens tested is about 2.0 g/cm^3.

Artificial Soil

This material is a mixture of 50 percent Florida Zircon sand and 50 percent fire clay, with 10 percent No. 5 SAE mineral soil, which can be classified as skip-graded material (Desai, 1980). Its maximum and minimum densities are 2.65 and 1.00 gm/cm^3, respectively. The initial density of specimens is 2.00 gm/cm^3.

Ottawa Sand

This cohesionless soil is highly uniform medium sand with subrounded grains with specific gravity of 2.65 (Mould, 1979). Its maximum and minimum densities are 1.76 and 1.55 g/cm^3, respectively. The initial density for tests reported herein is about 1.75 gm/cm^3.

"Munich" Sand

This material is found near Munich, Germany, and is coarse to medium, well graded sand with round grains and about 3.0 percent silt particles and specific gravity of 2.758 (Scheele and Desai, 1983). Its maximum and minimum densities are 1.86 and 1.52 gm/cm^3, respectively. The test results reported herein are for initial density of 1.80 gm/cm^3 with moisture content of 4.0 percent.

Agricultural Soil

This soil exists near Blacksburg, Virginia, USA, and represents typical soil in agricultural farms (Samford, 1981). It has 48, 36 and 16 percent clay, silt and sand contents, respectively, with specific gravity of 2.65. Its liquid and plastic limits are 41 and 22 percent, respectively.

Testing and Interpretation

All the five materials were tested by following a wide range of stress paths depicted in Fig. 1. The test results were obtained in terms of measured values of (applied) principal stresses, σ_1, σ_2, σ_3 and corresponding strains, ε_1, ε_2, ε_3. In order to define the ultimate envelope, the stresses at ultimate for curves under various stress paths were noted and the results were plotted in $J_1 - \sqrt{J_{2D}}$ space. Typical envelope for the silty sand is shown in Fig. 2; here J_1 and J_{2D} = first and second invariants of the stress and deviatoric stress tensors, respectively. The ultimate state is adopted as the envelope corresponding to the asymptotic values of stresses to a given stress-strain curve.

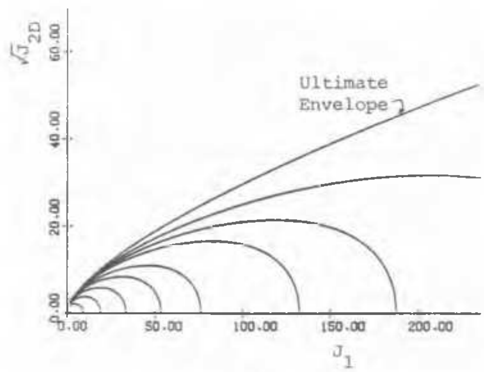

Fig. 2 Plots of Proposed Yield Function in J_1 vs $\sqrt{J_{2D}}$

Since the geological materials involve effect of coupling between volumetric and deviatoric responses and stress paths, a large number of parameters were identified and plotted to find the most suitable ones for incorporation in the model. The results of this study are shown in Fig. 3, in which typical plots of ratios r_v vs r_D for three soils are shown for tests with different stress paths; tests for the other two are not shown because of space limitations. Here

$$r_v = \xi_v/\xi, \; r_D = \xi_D/\xi \tag{1}$$

where $\xi_v = 1/\sqrt{3}\int(d\varepsilon^p_{ii})$, trajectory of volumetric plastic strain, $\xi_D = \int(de^p_{ij}\, de^p_{ij})^{1/2}$ trajectory of deviatoric plastic strain, $\xi = \int(d\varepsilon^p_{ij}\, d\varepsilon^p_{ij})^{1/2}$ trajectory of total plastic strain, ε_{ii} = volumetric strain, e_{ij} = deviatoric strain tensor and ε_{ij} = total strain tensor. Use of r_v, r_D and/or ξ can thus permit inclusion of the coupling effect. Figure 3 indicates that irrespective of the stress path followed, the plots of r_v vs r_D can be assumed to be essentially invariant, with their limiting values to be unity. Hence inclusion of r_v, r_D and/or ξ can also allow for the stress path effects. In the following model, the plastic hardening function is expressed in terms of ξ and r_D.

PROPOSED MODEL

The proposed model is derived from the consideration that (in the context of plasticity), the yield function can be expressed as

$$F = F\,(J_i,\, I^p_i,\, a_m) \tag{2}$$

where J_i (i = 1, 2, 3) = invariants of the stress tensor, I^p_i (i = 1, 2, 3) = invariants of the plastic strain tensor and a_m (m = 1, 2..N) = scalar parameters such as plastic strain. A special case of F was proposed by Desai (1980) as a complete polynomial in J_1, $J_2^{1/2}$ and $J_3^{1/3}$, in which the coefficients of the polynomial can be expressed in terms of r_v, r_D and/or ξ. It has been shown that various truncated forms of the polynomial can provide F for a given material with a given set of test data. One such function is given below (Desai and Faruque, 1983):

$$F = J_{2D} + \alpha J_1^{\,2} - \beta J_1 J_3^{\,1/3} + \gamma J_1 - k^2 = 0 \tag{3}$$

where α, β, γ, k = material response parameters. The function in Eq. 3 plots as continuous and convex in various stress spaces; a typical set of plots only in the $J_1 - \sqrt{J_{2D}}$ space is shown in Fig. 2. As a result, it possesses the certain advantages compared to two-surface models such as critical state (Schofield and Worth, 1968), Cap (DiMaggio and Sandler, 1971), and (Lade, 1980):
(1) Since only one function defines the plastic hardening process, there is no need to use two or more functions; (2) since it is continuous, its normal is uniquely defined avoiding computational difficulties at the intersection of the two surfaces; (3) its ultimate state contains failure or critical states as special cases; (4) for associative plasticity, the function automatically intersects the J_1-axis orthogonally; and (5) the function can be modified to incorporate softening, non-associative characteristics and anisotropic hardening, the latter by including joint or mixed invariants of stress and plastic strain tensors (Desai and Faruque, 1983, 1984; Baker and Desai, 1984).

Hardening Function

In F above, α, γ, k are assumed to be constants associated with the ultimate surface, Fig. 2. The function β is called hardening or growth function and is expressed in terms of hardening parameters ξ and r_D as

$$\beta = \beta_u \left[1 - \frac{\beta_a}{\xi^{\eta_1} \{1 - \beta_b\ (r_D)^{\eta_2}\}} \right] \tag{4}$$

Here β_u = value of β at the ultimate (= 3α), and β_a, η_1, β_b, η_2 = hardening constants.

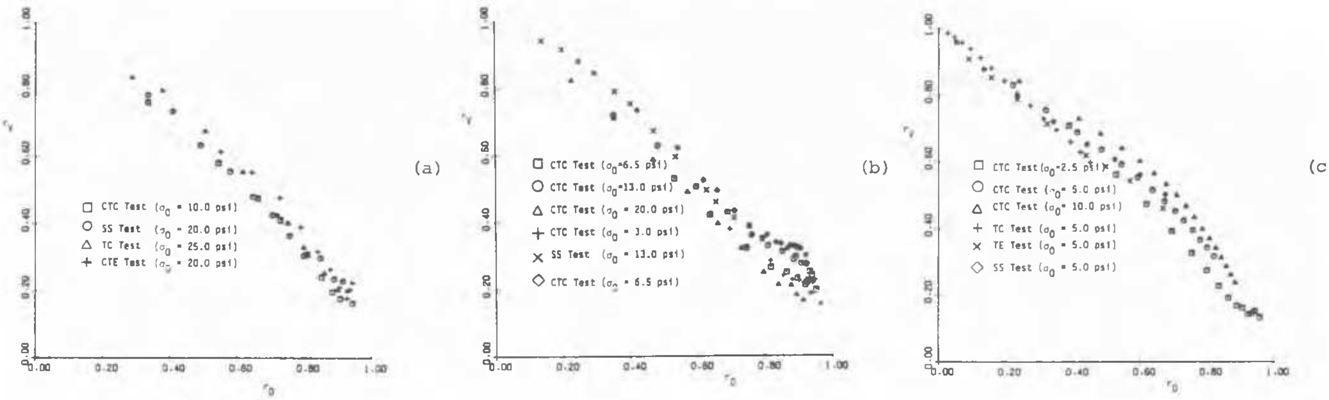

Fig. 3 r_v vs r_D For Different Stress Paths: (a) Silty Sand; (b) Munich Sand; (c) Agricultural Soil

Determination of Constants

It is quite straightforward to evaluate the constants. α, γ and k are obtained from the ultimate surface (Fig. 2) by using curve fitting; k is proportional to cohesive strength of the material, and, in general, is a function of stress path. The constants β_a and η_1 are obtained by plotting $\ell n\ (\xi_v)$ vs $\ell n\ (1 - \beta/\beta_u)$, Fig. 4, from hydrostatic test data (for the silty sand). Then the intercept along the ordinate gives β_a and the slope of the (average) line gives η_1.

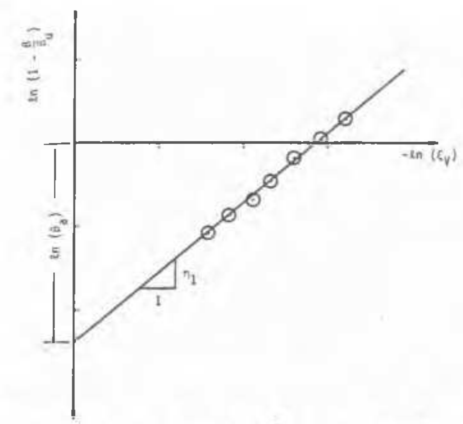

Fig. 4 Plot of $\ell n\ (1 - \frac{\beta}{\beta_u})$ vs $-\ \ell n\ (\xi_v)$ for Hydrostatic Compression Test

Results from the (shear) tests (for the silty sand) under various stress paths are plotted in Fig. 5. Then the intercept along the ordinate gives β_b and the slope of the average line gives η_2. It is interesting to note that for practical purposes, the line can be estimated even from standard (conventional) triaxial tests (CTC). For example, the weighted average values for β_b and η_2 from four stress paths (CTC, SS, TE and CTE) for the silty sand were 0.854 and 0.802, respectively, whereas those only from the CTC test were 0.839 and 1.03. Details of the parameters for the five soils are given in Table I.

Elastic Constants

The elastic constants, E and ν, are found as average initial moduli from unloading-reloading curves of different stress paths and for different confinements.

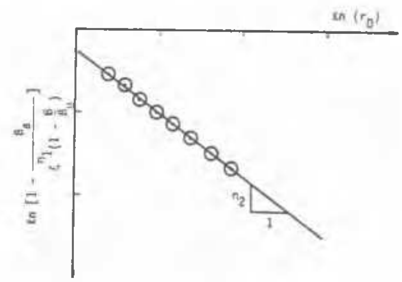

Fig. 5 Plot to Determine the Hardening Constants β_b and η_2

TABLE I

Values of Parameters

	Silty Sand	Artificial Soil	Ottawa Sand	Munich Sand	Agricultural Soil
E, psi (kPa)	9380 (65000)	4000 (27560)	38,600 (2.6×10^5)	18,000 (12×10^4)	4100 (28249)
ν	0.36	0.35	0.37	0.36	0.35
α	0.154	0.162	0.168	0.212	0.216
γ, psi (kPa)	3.007 (20.71)	1.542 (10.6)	3.10 (21.40)	3.67 (25.30)	1.425 (9.82)
k, psi (kPa)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)
β_a	0.00062	0.00217	0.000088	0.000278	0.00125
η_1	1.0554	1.376	1.0583	1.161	1.173
β_b	0.854	0.723	0.9730	0.805	0.808
η_2	0.802	0.660	0.567	0.704	1.00

VERIFICATION

The function in Eq. 3 can be treated as the yield function in the context of the theory of plasticity. Then, associative plasticity, and the consistency condition dF = 0, leads to the incremental elastic-plastic relation, in matrix notation, as

$$\{d\sigma\} = [C^{ep}]\ \{d\varepsilon\} \tag{5}$$

where $[C^{ep}]$ = elastic-plastic constitutive matrix and is expressed in terms of the foregoing constants: E, ν, α, γ, k, β_a, η_1, β_b and η_2.

This matrix differential equation (Eq. 5) was integrated starting from the initial (hydrostatic) stress state along various stress paths. The predictions have been compared with a large number of stress path tests for stress-strain and volumetric responses for the five materials. For want of space, only two typical predictions for the silty sand are shown in Fig. 6. The predictions are found to be satisfactory in almost all cases. One particular attribute of the model is that it can provide improved predictions of the volumetric behavior even for shear and extension paths because of the inclusion of r_D and total plastic strain ξ in the hardening model. Note that the previous two-surface models often include only the plastic volumetric strain as the hardening parameter. The proposed model is also incorporated in two- and three-dimensional finite element procedures and verified with respect to observed behavior of boundary value problems (Faruque, 1983).

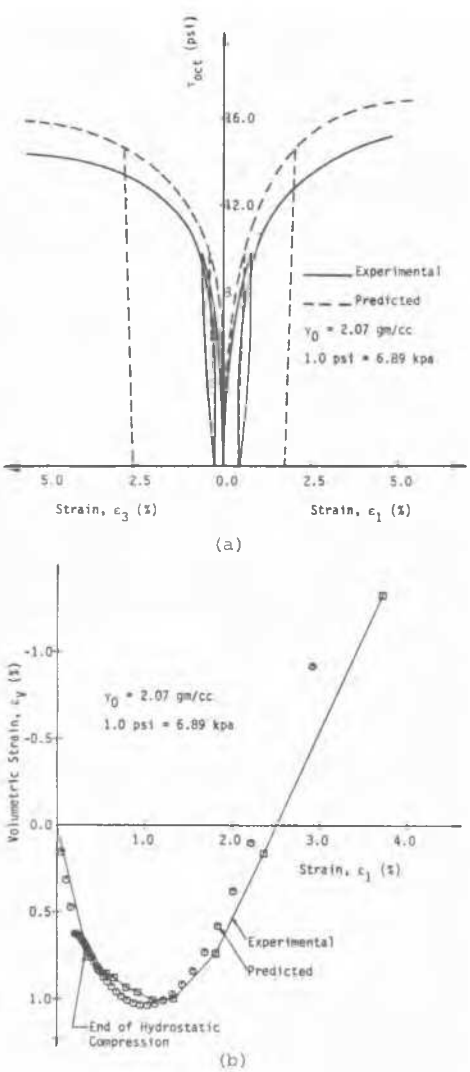

Fig. 6 Comparison of Predictions and Observations, Simple Shear Test, σ_o = 20 psi (Silty Sand): (a) Stress-strain Curves, (b) Volumetric Response

CONCLUSIONS

Based on comprehensive laboratory test data for five soils, a general yet simplified constitutive model is proposed. It is capable of capturing influence of a number of significant observed aspects such as state of stress, plastic hardening, coupling of volumetric and shear responses and stress path dependence of the behavior of soil. The determination of required parameters is straightforward, and hence the model can be implemented easily for solution of problems in soil mechanics.

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