A model for analysis of viscoplastic behaviour of soils

Un modèle pour les analyses de comportement viscoplastique des sols

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SYNOPSIS The paper deals with the author's proposition of an elasto-viscoplastic model for soils. The considerations are limited to the axisymmetric homogeneous compression occurring during conventional drained triaxial tests of soils. The appriopriate constitutive equations are given and then applied to the mathematical description of the triaxial creep tests and shear ones executed with various constant strain rates. These theoretical studies are made to show a qualitative effectiveness of model.

INTRODUCTION

An incremental elasto-viscoplastic model is, in general, required, when rheological phenomena in soil, such as the non-vanishing creep or sensitivity of the shear strength to the strain rate, are to be considered in the geomechanical analyses. There are only few available papers presenting various constitutive relationships of that kind for soils (e. g. Perzyna, 1966, Runesson, 1978, Nova, 1982, Gryczmański, 1983).

In this paper the model being a little simplified version of the author's proposition (Gryczmański, 1983), neglecting the kinematic hardening, is applied to the theoretical study of rheological effects in the case of the homogeneous axisymmetric compression of soil. The stress and strain states are then represented usually by the following invariants:

$$p = \frac{1}{3}(\sigma_1+2\sigma_3) \qquad q = \sigma_1-\sigma_3 \tag{1}$$

$$\varepsilon_v = \varepsilon_1+2\varepsilon_3 \qquad \varepsilon_q = \frac{2}{3}(\varepsilon_1-\varepsilon_3) \tag{2}$$

where p is called the hydrostatic pressure, q may be interpreted as the shear stress, and ε_v, ε_q are assumed to be the volumetric and distortion strain respectively. The symbols σ_1, $\sigma_2 = \sigma_3$ and ε_1, $\varepsilon_2 = \varepsilon_3$ denote the principal stresses and strains. It is worth to notice that the case considered occurs in conventional triaxial soil tests. In the paper the general rate constitutive relations for any drained ($\sigma_1 = \sigma_1'$, $\sigma_3 = \sigma_3'$, $p = p'$, $q = q'$) test, and their particular cases describing the p-constant shear and creep processes, are specified. To illustrate model abilities some numerical results for those cases are shown.

CONSTITUTIVE EQUATIONS

The stress-strain, rate constitutive relations for any drained triaxial test can be derived summing the strain rates given by the Hooke's law and the viscoplastic flow rule, as in the author's paper (1983), and then passing into the axisymmetric state by help of the formulas (1), (2). The equations obtained can be expressed as follows:

$$\dot{\varepsilon}_v = A_{11}\dot{p}+A_{12}\dot{q}+C_p \qquad \dot{\varepsilon}_q = A_{21}\dot{p}+A_{22}\dot{q}+C_q \tag{3}$$

where

$$A_{11} = \frac{1}{K}+\frac{1}{H}\left(\frac{\partial F}{\partial p}\right)^2 \qquad A_{12} = \frac{1}{H}\frac{\partial F}{\partial p}\frac{\partial F}{\partial q}$$

$$A_{22} = \frac{1}{3G}+\frac{1}{H}\left(\frac{\partial F}{\partial q}\right)^2 \qquad A_{21} = A_{12} \tag{4}$$

$$C_p = \frac{1}{H}\frac{\partial F}{\partial p}\frac{\partial F}{\partial t} \qquad C_q = \frac{1}{H}\frac{\partial F}{\partial q}\frac{\partial F}{\partial t}$$

Here K and G are the elastic bulk and shear moduli respectively. F is the loading function forming the left side of the yield condition

$$F = p^2+\frac{1}{f^2}q^2-2clp-c_1l^2 = 0 \qquad c_1 = 1-c^2 \tag{5}$$

while H denotes the hardening modulus given in the form

$$H = (1+e)\frac{\partial F}{\partial p}\frac{\partial F}{\partial e^{vp}} \tag{6}$$

obtained from the consistency condition. Here e is the void ratio and e^{vp} is its inelastic part. In the stress space the yield condition (5) is represented by the vertical axial section of the ellipsoidal yield surface (Fig. 1) like that defining the Modified Cam-Clay Model. (Schofield and Wroth, 1968). However, the surface considered here undergoes not only expansion or contraction, governed by irreversible void ratio changes, but also an additional time-dependent evolution composed of contraction and flattening perprndicular to the hydrostatic axis.

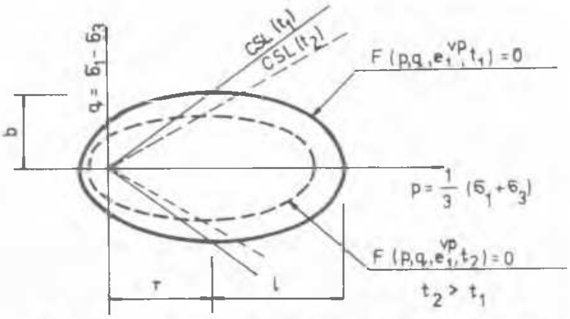

Fig. 1. The model yield surface section

The full evolution is defined by the functions

$$l(e^{vp},t) = l_o \exp[\varkappa(e_o^{vp}-e^{vp})-A_1\tau^{\mu_1}] \qquad (7)$$

$$f(t) = f_o \exp[\varkappa(A_1\tau^{\mu_1}-A_2\tau^{\mu_2})] \qquad (8)$$

where τ denotes the dimensionless time parameter $\tau = t : t_m'$, while $f = b : l$. The quantities b, l, (and r = cl) are identified in Fig. 1, t_m is any sufficiently large time value. It must be fixed before estimation of the material constants $l_o, f_o, c, \varkappa$ and A_1, A_2, μ_1, μ_2, as the latter subset is affected by that. One can estimate the whole set for a given soil making the regression analyses of results of rheological shear strength and hydrostatic compression tests (Gryczmański, 1983). Finally, it is necessary to notice that the loading function derivatives occuring in the constitutive laws can be determined when accounting for the equations (5), (7), (8).

CREEP ANALYSIS

In this case the stress rates are equal to zero ($\dot{p}=\dot{q}=0$), and so the constitutive relations (3) take on the fairly simple form

$$\dot{\varepsilon}_v = c_p \qquad \dot{\varepsilon}_q = c_q \qquad (9)$$

To point out the model effectiveness the equations (9) have been used in the analysis of drained triaxial creep tests, in which a kaolin sample was subjected to the constant hydrostatic pressure p=200 kPa, and to the various constant shear stress q values. The incremental nonlinear analysis has been done for the following material constants estimated: c=0.9, f_o=0.8, A_1=0.005, μ_1=0.1, A_2=0.02, μ_2 = 0.1, l_o=105.26 kPa, e_o=0.6. The results of analysis are shown in Fig. 2 in the form of the time-distortion strain plots.

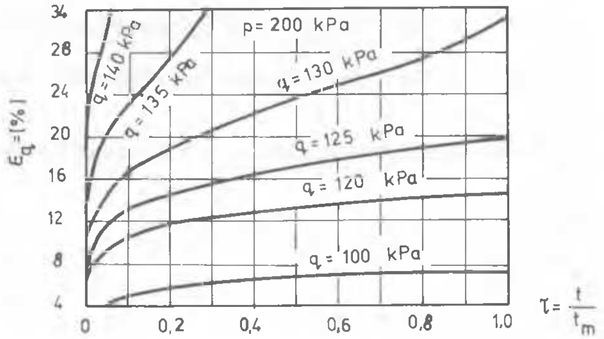

Fig. 2. Theoretical creep curves for kaolin

STRAIN RATE EFFECT ANALYSIS

When considering p=const shear and dilatancy characteristics ("ε_q - q" and "$\varepsilon_q - \varepsilon_v$" curves), the equations (3) should be transformed as follows

$$q = \frac{1}{A_{22}}(\dot{\varepsilon}_q - c_q) \qquad \dot{\varepsilon}_v = \frac{A_{12}}{A_{22}}\dot{\varepsilon}_q + \left(c_p - \frac{A_{12}}{A_{22}}c_q\right) \qquad (10)$$

The equations (10) are used in the analysis of drained triaxial tests in which the distortion strain rates are constant like as the hydrostatic pressure p = 200 kPa .

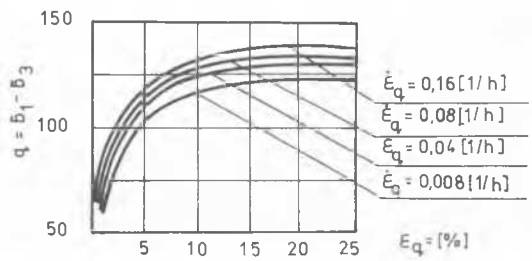

Fig. 3. Stress-strain curves (p=const)

The theoretical curves shown in Fig.2 and 3 are qualitatively consistent with the corresponding ones obtained from experiments.

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