Yield criterion for anisotropic soils

Critère de fluage pour des sols anisotropes

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SYNOPSIS For the purpose of investigating the three-dimensional internal friction law which controls not only the shear strength but also the stress-strain behavior of anisotropic soils, a series of true triaxial tests was performed on cubical specimens of K_o-consolidated clay and sand with cross-anisotropic fabric. It has been made clear that the stress conditions at all levels of equishear strain could be prescribed with the same manner as the classic Mohr-Coulomb's criterion. In this paper, it is clarified that three envelopes for the three-dimensional Mohr's stress circles of anisotropic soils, too, can be drawn as straight lines and the influence of anisotropy does not appear in the point of their intersection d_γ. Based on the newly extended three-dimensional internal friction law, a new yield criterion for anisotropic soils is proposed and proved the usefulness showing the good agreement with the experimental results.

INTRODUCTION

The stress-strain and strength behavior of soils is considered to be governed by the friction law which has been represented by the Mohr-Coulomb's criterion. However, the constants of shear strength based on the classic conception of internal friction have been decided in defiance not only of the strain but also of the intermediate principal stress. Additionally, they can not describe the influence of anisotropy in soils. One of the authors has elucidated that stress conditions at all levels of equishear strain γ_{13} can be prescribed by the same equation as the Mohr-Coulomb's criterion, and three envelopes for the three-dimensional Mohr's stress circles at any level of equishear strain can be recognized. Then, in consideration of the three-dimensional friction angles obtained from these envelopes, the yield criterion for isotropic soils has been presented (Hayashi & Yamanouchi, 1983).
In the present paper, for the purpose of extending the above conception, the three-dimensional internal friction law of anisotropic soils is investigated by a series of true triaxial tests on anisotropic clay and sand. A yield equation for anisotropic soils is then derived on the basis of the newly extended three-dimensional internal friction law, and it is proved the proposed yield curves are well adaptable to anisotropic soils in comparisons with the experimental results.

ANISOTROPIC SPECIMENS AND TRUE TRIAXIAL TESTS

The materials used here are white clay having the properties : $G_S = 2.705$, clay 60%, silt 40%, $w_L = 50.0\%$, $w_p = 25.0\%$, and uniformly graded Cambria sand with particle sizes from 0.84 to 2.00 mm, having the properties : $G_S = 2.708$, $e_{max} = 0.80$, $e_{min} = 0.51$. Anisotropic clay specimens were prepared in a shape of rectangle (60 x 59 x 44 mm) by cutting after the clay sample was remolded at the moisture content of about 100% and was consolidated in a special cylindric mold up to the maximum pressure of 589 kPa. Cross-anisotropy of the clay specimen was confirmed by the result of isotropic compression test on the cubical specimen (70 x 70 x 70 mm). Cubical anisotropic sand specimens with side lengths of 76 mm were prepared in a specially designed mold by pouring and shaking sand grains in several layers, which had void ratios of 0.53 - 0.54 corresponding to relative densities of 93%-90%. By means of photographs taken in horizontal and vertical sections through central regions of the specimen, it was confirmed that the particles in the specimens had strong preferred orientations in the vertical section, but almost completely random orientations in the horizontal section (Ochiai & Lade, 1983).

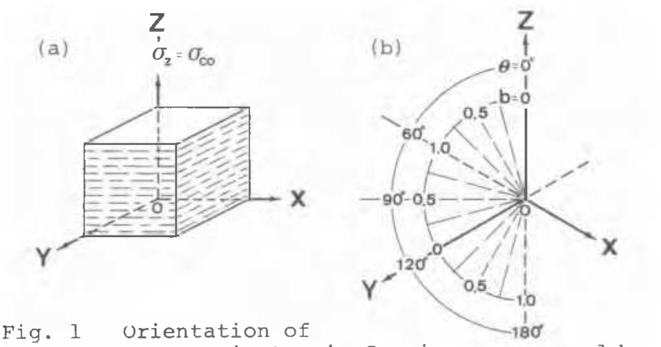

Fig. 1 Orientation of Cross-Anisotropic Specimen expressed by: (a) Cartesian Coordinate System, (b) Octahedral Plane

For the purpose of clearly indicating the directions of stress and strain relative to principal axes of the materials, a Cartesian coordinate system is employed as in Fig. 1 (a). The Z-axis coincides with the axis of rotational symmetry for the cross-anisotropic specimens. The angle θ, indicated on the octahedral plane in Fig. 1 (b), is measured counter-clockwise from the Z-axis and is calculated by Eq. (1). The relative magnitude of the intermediate principal stress

TABLE I Performed Cases in True Triaxial Tests

White Clay	$\theta°$	0	15	30	45	60	75	90	105	120	135	150	165	180
	b	0.00	0.268	0.50	0.732	1.00	0.732	0.50	0.268	0.00	0.268	0.50	0.732	1.00
Cambria Sand	$\theta°$	0	8.9	32.0	48.5	61.0	71.5	88.0	110.9	120	129.1	152.0	168.5	181.0
	b	0.00	0.17	0.53	0.79	0.98	0.79	0.53	0.17	0.00	0.17	0.53	0.79	0.98

$$\theta = \tan^{-1}\left[\sqrt{3}\,\frac{(\sigma_Y - \sigma_X)}{(\sigma_Z - \sigma_Y) + (\sigma_Z - \sigma_X)}\right] \quad \text{-----(1)}, \qquad b = \frac{(\sigma_2 - \sigma_3)}{(\sigma_1 - \sigma_3)} \quad \text{-----(2)}$$

has often been indicated by the b-value, which is defined by Eq. (2). Each series of the true triaxial tests on clay and sand was performed on thirteen cases of the angle θ from 0° to 180° as shown in Table I.

INTERNAL FRICTION LAW OF ANISOTROPIC SOILS UNDER THREE-DIMENSIONAL STRESS CONDITION

Envelopes for Three-Dimensional Mohr's Stress Circles at Equishear Strain

By way of example, Fig. 2 shows the three-dimensional Mohr's stress circles and their envelopes at the equishear strain of $\gamma_{13} = 5\%$ in diverse angles of θ on clay. It is clearly shown that the envelopes not only for the maximum circles but also for the second and third ones of three-dimensional Mohr's stress circles at all levels of equishear strain can be recognized and each of them gives an essentially straight line in the same manner as the classic Mohr-Coulomb's failure envelope. Moreover, not only these three envelopes have the same point of intersection on the σ coordinate axis but also the point of intersection d_γ at the equishear strain state can be recognized to be constant independent of the θ value, as shown in Fig. 2. From these experimental results, it is considered that the essential cohesion of soils can be detected with the d_γ, regardless of anisotropy.

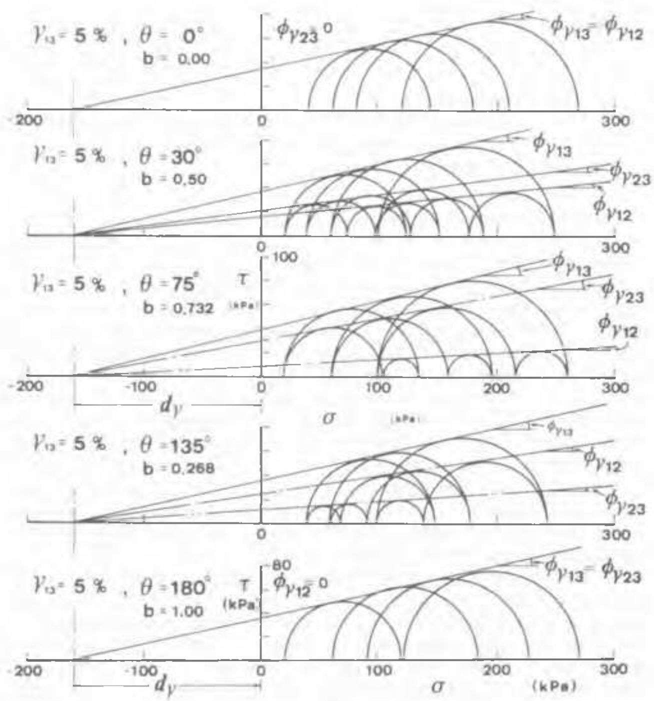

Fig. 2 Three-Dimensional Envelopes for Mohr's Stress Circles at Equishear Strain on Anisotropic Clay

The slopes of three envelopes in diverse stress states are designated as the three-dimensional internal friction angles ($\phi_{\gamma12}$, $\phi_{\gamma23}$, $\phi_{\gamma13}$). In consideration of the experimental results as shown in Fig. 2, the envelopes for the maximum, intermediate and minimum circles of three-dimensional Mohr's stress ones can be expressed respectively as follows:

$$\left.\begin{aligned} (\sigma_1 - \sigma_2) &= [2d_\gamma + (\sigma_1 + \sigma_2)]\cdot \sin\phi_{\gamma12} \\ (\sigma_2 - \sigma_3) &= [2d_\gamma + (\sigma_2 + \sigma_3)]\cdot \sin\phi_{\gamma23} \\ (\sigma_1 - \sigma_3) &= [2d_\gamma + (\sigma_1 + \sigma_3)]\cdot \sin\phi_{\gamma13} \end{aligned}\right\} \text{--------(3)}$$

Furthermore, through the geometric properties of three-dimensional Mohr's stress circles and their envelopes, the relationships between three-dimensional friction angles $\phi_{\gamma12}$, $\phi_{\gamma23}$ and $\phi_{\gamma13}$ can be obtained by introducing the b-value, as follows:

$$\sin\phi_{\gamma12} = \frac{(1-b)\cdot\sin\phi_{\gamma13}}{1 + b\cdot\sin\phi_{\gamma13}} \quad \text{-----(4)}, \qquad \sin\phi_{\gamma23} = \frac{b\cdot\sin\phi_{\gamma13}}{1-(1-b)\cdot\sin\phi_{\gamma13}} \quad \text{-----(5)}$$

Characteristics of Three-dimensional Internal Friction Angles

For an example at the case of $\gamma_{13} = 5\%$ on sand, the three-dimensional internal friction angles ($\phi_{\gamma ZX}$, $\phi_{\gamma YX}$, $\phi_{\gamma YZ}$) converted from the measured friction angles ($\phi_{\gamma12}$, $\phi_{\gamma23}$, $\phi_{\gamma13}$), are described on the octahedral plane using the cylindrical coordinates (θ , r), with the magnitude of friction angle ϕ_γ(°) as the r-coordinate axis, as in Fig. 3. Regardless of having anisotropy, the complete shape of three-dimensional internal friction angles of soils on the octahedral plane is drawn

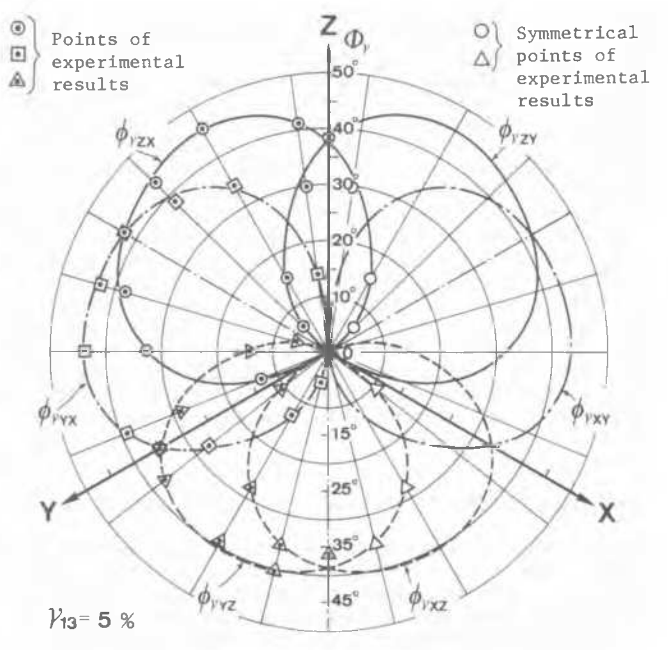

Fig. 3 Complete Shape of Three-Dimensional Internal Friction Angles on Octahedral Plane on Anisotropic Sand

such as the shape shown in Fig. 3.
The classic internal friction angle ϕ obtained by the Mohr-Coulomb's law on isotropic materials makes a circle on the octahedral plane, but the circle consists of six discontinuous ϕ_{ij}, such as ϕ_{ZX} in the region of θ from 0° to 60°, ϕ_{YX} in 60° to 120°, ϕ_{YZ} in 120° to 180° and so on. This fact means that the plane on which the internal friction is mobilized changes discontinuously one after another.
On the other hand, as shown in Fig. 3, the three-dimensional friction angles based on the newly proposed internal friction law for soils can vary smoothly from the maximum angle to the intermediate or the minimum one in turn. Therefore, it is considered that the classic ϕ which makes a circle on the octahedral plane is an approximation of the external circumference of presented three-dimensional friction angles as shown in Fig. 3. Then the presented friction law can be designated as the true three-dimensional internal friction law for soils in the rigorous sense.

DERIVATION OF YIELD CRITERION FOR ANISOTROPIC SOILS BASED ON THREE-DIMENSIONAL FRICTION LAW

Derivation of Yield Criterion

By means of Eqs. (4) and (5), if the maximum angle of three-dimensional friction angles in each region of the angle θ from 0° to 60°, 60° to 120°, 120° to 180° and so on, may be expressed mathematically, both of the intermediate and minimum angles can be calculated numerically. While, at any level of equishear strain, the distribution of three-dimensional friction angles on the octahedral plane obtained from the experimental results can be fairly approximated by three circles in the region of θ from -60° to 240°, as shown in Fig. 4 as an example.

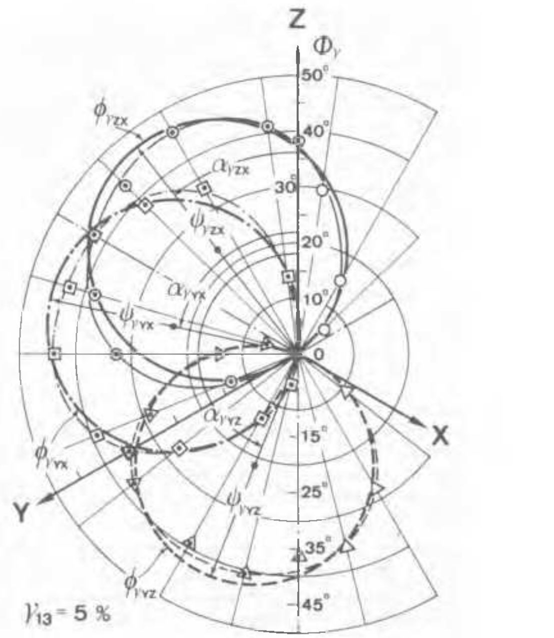

Fig. 4 Approximated Circles of Three-Dimensional Internal Friction Angles on Octahedral Plane on Anisotropic Sand

Therefore, by means of each diameter of the approximated circles ($\psi_{\gamma ZX}$, $\psi_{\gamma YX}$, $\psi_{\gamma YZ}$) and the rotation angles of these centers ($\alpha_{\gamma ZX}$, $\alpha_{\gamma YX}$, $\alpha_{\gamma YZ}$), the maximum friction angle $\phi_{\gamma 13}$ in each region of the angle θ from 0° to 60°, from 60° to 120° and from 120° to 180° are expressed as follows:

$0° \le \theta \le 60°$

$$\phi_{\gamma 13} = \psi_{\gamma ZX}\cdot(\cos\alpha'_{\gamma ZX}\cdot\cos\theta' + \sin\alpha'_{\gamma ZX}\cdot\sin\theta') \quad \text{---(6)}$$

$60° \le \theta \le 120°$

$$\phi_{\gamma 13} = \psi_{\gamma YX}\cdot(\cos\alpha'_{\gamma YX}\cdot\cos\theta' + \sin\alpha'_{\gamma YX}\cdot\sin\theta') \quad \text{---(7)}$$

$120° \le \theta \le 180°$

$$\phi_{\gamma 13} = \psi_{\gamma YZ}\cdot(\cos\alpha'_{\gamma YZ}\cdot\cos\theta' + \sin\alpha'_{\gamma YZ}\cdot\sin\theta') \quad \text{---(8)}$$

where,

$$\left.\begin{aligned} &0° \le \theta \le 60°, && \alpha'_{\gamma ZX} = 90° - \alpha_{\gamma ZX}, && \theta' = 90° - \theta \\ &60° \le \theta \le 120°, && \alpha'_{\gamma YX} = \alpha_{\gamma YX} - 30°, && \theta' = \theta - 30° \\ &120° \le \theta \le 180°, && \alpha'_{\gamma YZ} = 210° - \alpha_{\gamma YZ}, && \theta' = 210° - \theta \end{aligned}\right\} \text{--(9)}$$

While, generally in the three-dimensional stress space, a yield equation is expressed as follows using the cylindrical coordinates:

$$\sigma_r = G\ (\sigma_0\ ,\ \theta\ ) \quad \text{------------------(10)}$$

where, σ_r : mean deviator stress, and radial distance on octahedral plane
σ_0 : mean principal stress
θ : rotation angle on octahedral plane

σ_r is also expressed using the principal stresses as follows:

$$\sigma_r = \frac{1}{\sqrt{3}}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{1/2} \quad \text{-------(11)}$$

By rearranging Eq. (3) and substituting it in Eq. (11), the following equation can be derived as a yield criterion for anisotropic soils in the form as Eq. (10), by means of $\phi_{\gamma 13}$ and d_γ as coefficients.

$$\sigma_r = 2\sqrt{3}\,\frac{\sin\phi_{\gamma 13}}{3 + (2b-1)\cdot\sin\phi_{\gamma 13}}[(1-b)^2 + b^2 + 1]^{1/2}\cdot(\sigma_0 + d_\gamma) \quad \text{---(12)}$$

where,

$$b = \frac{2}{1 + \sqrt{3}\tan\theta'}\ ,\quad 30° \le \theta' \le 90° \quad \text{-----(13)}$$

Then, $\phi_{\gamma 13}$ is given by Eqs. (6), (7) and (8).

Coefficients of Proposed Yield Criterion

The yield equation for anisotropic soils presented herein, finally includes three pairs of coefficients on the internal friction angles "($\psi_{\gamma ZX}$, $\alpha_{\gamma ZX}$), ($\psi_{\gamma YX}$, $\alpha_{\gamma YX}$) and ($\psi_{\gamma YZ}$, $\alpha_{\gamma YZ}$)" and one coefficient on the cohesion " d_γ ". As shown in Fig. 4, $\psi_{\gamma ZX}$, $\psi_{\gamma YX}$ and $\psi_{\gamma YZ}$ are the maximum values of three-dimensional internal friction angles $\phi_{\gamma ZX}$, $\phi_{\gamma YX}$ and $\phi_{\gamma YZ}$ respectively. It is contemplated that $\psi_{\gamma ij}$ give the substantial magnitude of internal friction angles of anisotropic soils at the state of equishear strain, and the measured $\phi_{\gamma ij}$ are the respective mobilized friction angles of them under the three-dimensional stress condition. It can be regarded that the other coefficient d_γ at equishear strain state is constant independent of the angle θ as above-mentioned, therefore d_γ is the intrinsic cohesive component independent of anisotropy at the state of equishear strain.

Thus, it is considered that the three-dimensional, anisotropic stress-strain and strength behavior of soils depend on the differences in magnitude of the three-dimensional internal friction angles $\phi_{\gamma ZX}$, $\phi_{\gamma YX}$ and $\phi_{\gamma YZ}$.

COMPARISONS OF PROPOSED YIELD CURVES WITH EXPERIMENTAL RESULTS

Fig. 5 gives a comparison between the anisotropic yield curves calculated with the proposed Eq. (12) and the experimental results of Cambria sand at the states of γ_{13} = 5% and 10%. Fig. 6 shows the comparison on the white clay as well. It is made clear that the proposed yield curves for anisotropic soils obtained by Eq. (12) agree well with the experimental results both on sand with anisotropic fabric and clay consolidated anisotropically.

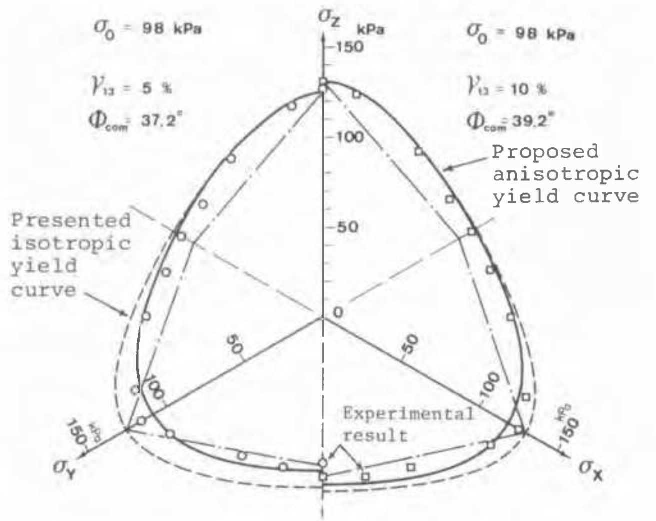

Fig. 5 Comparison of Proposed Yield Curves with Experimental Results on Anisotropic Sand

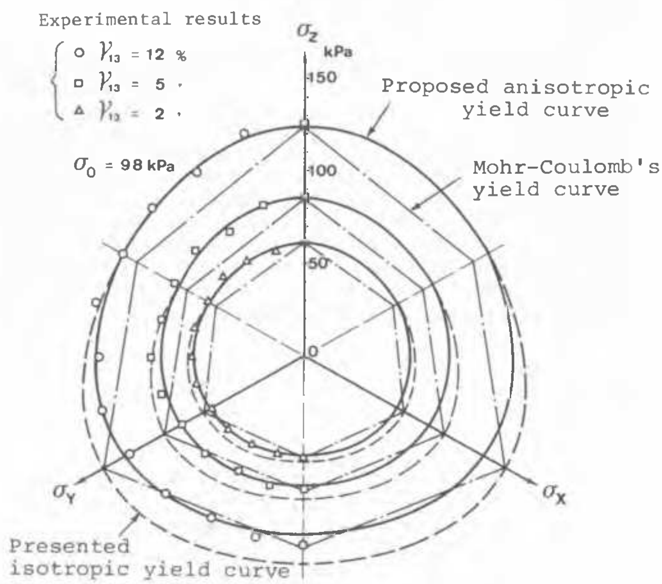

Fig. 6 Comparison of Proposed Yield Curves with Experimental Results on Anisotropic Clay

CONCLUSIONS

The main conclusions can be drawn out as follows:

(1) At any level of equishear strain, each envelope not only for the maximum circles but also for the second and third ones of three-dimensional Mohr's stress circles at the same b-value, gives an essentially straight line in the same manner as the classic Mohr-Coulomb's failure envelope.

(2) Moreover, regardless of anisotropy in soils, these envelopes have the same point of intersection on the σ coordinate axis, and the point of intersection d_γ can be recognized to be constant independently of the b-value. In other words, the influence of anisotropy in soils does not appear in the point of intersection d_γ.

(3) The three-dimensional internal friction angles ($\phi_{\gamma ZX}$, $\phi_{\gamma YX}$, $\phi_{\gamma YZ}$) converted from the measured friction angles depict the unique shape like a hexapetalous flower on the octahedral plane using the cylindrical coordinates.

(4) As the conception of presented three-dimensional internal friction angles includes the classic friction angle obtained by the Mohr-Coulomb's law to the first approximation, it can be designated as the three-dimensional internal friction law for soils in the rigorous sense.

(5) A yield equation for anisotropic soils is derived as Eq. (12) on the basis of the conception of three-dimensional internal friction law, having three pairs of coefficients on internal friction angles and one coefficient on cohesion which is independent of anisotropy.

(6) The proposed yield curves agree well with the experimental results at all levels of equishear strain not only on anisotropic sand but also on anisotropic clay.

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