Sand properties governing foundation settlement

Propriétés de sable qui provoquent le tassement des fondations

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SYNOPSIS The aim of this paper is to find which aspect of sand behaviour governs the settlement of footings on natural sand deposits. Settlement observations are compared with the predictions of different soil models. The most suitable model for the behaviour of larger footings on sand proved to be a half-space whose modulus increases with the square root of depth. With this model some results of practical interest are calculated: effect of footing breadth and side ratio, settlement outside loaded footings, and thickness of the soil layer contributing to the settlement of the footing. For actual settlement estimation, penetration test data are used.

INTRODUCTION

One of the fundamental problems in geotechnics always has been to predict the displacements of loaded footings. For this purpose, methods for soil exploration and sample investigation have been developed, constitutive equations established and calculation methods improved. Constitutive equations for soil material are very complex if they are meant to describe soil behaviour under all possible loading conditions. This, however, is not necessary: According to Truesdell and Noll (1965) constitutive equations mostly describe isolated aspects of material behaviour. Thus no general constitutive model will be established in this paper but the question is: Which aspects of sand behaviour governs foundation settlement?

The earliest mechanically consistent model for the ground is the elastic homogeneous half-space by Boussinesq (1885). This model still is widely used for settlement prediction, though it is well known that sand does not behave elastically during monotonic first loading. Boussinesq's solution was modified by Fröhlich (1934) such that predicted and observed stress distributions agree better. Several authors have investigated an elastic half-space whose modulus increases with depth, Ohde (1939), Borowicka (1943), Gibson (1967). On the other hand various constitutive models have been established, which, as far as I know, have not much improved settlement prediction in practice.

PREDICTIONS AND OBSERVATIONS

Many authors have reported on observations of settlements of footings. Burland et al. (1977) have summarized the results of many investigations in a diagram where settlement w per average pressure q in the contact area has been plotted against breadth B of the footing. Though their data show considerable scattering, a certain trend can be preceived: The settlements w/q increase with B. For small breadths the increase approximately is linear, for B >1 m it is weaker, roughly $w/q \sim \sqrt{B}$.

This scale effect shall be used now to test the applicability of different models. The system considered is the footing with breadth B and length L as shown in Fig.1. The footing is placed on a natural homogeneously deposited sand half-space. The unknown is the settlement w due to monotonic loading by the average foundation pressure q. The investigation of this system by Holzlöhner (1985) will be expanded here.

First consider a half-space that is characterized by a modulus E with the dimension of a stress. This modulus may increase by

$$E = H \cdot z^{\alpha} \quad (1)$$

with depth. H is the "modulus of increase" with the dimension $kN/m^{2+\alpha}$. The settlement w of the footing depends by

$$w = f_1(B,q,H) \quad (2)$$

on the governing quantities. Here and in the following the symbol f_i denote some unknown function. The settlement depends on further quantities that are dimensionless and assumed to be constant as exponent α , side ratio L/B, Poisson's ratio ν and other ratios of elastic or plastic moduli if anisotropic material is considered. Applying dimensional analysis yields

$$\frac{w}{B} = f_2\left(\frac{q}{HB^{\alpha}}\right) \quad (3)$$

Linearity between measured settlement and load have often been found, Burland et al.(1977)

$$w \sim q \quad (4)$$

Thus

$$\frac{w}{B} = \frac{q}{HB^{\alpha}} \cdot f_3 \quad (5)$$

or

$$\frac{w}{q} \sim B^{1-\alpha} \quad (6)$$

Eq. (6) includes the result $w/q \sim B$ of the elastic homogeneous half-space, α = 0, and, for α = 1, the result of the Gibson (1967) soil where w/q is constant. In the latter case the soil is characterized by the modulus of subgrade reaction. α = 1/2 means

$$E = H\sqrt{z}, \quad (7)$$

which yields

$$\frac{w}{q} \sim \sqrt{B} \quad (8)$$

Eq (8) just describes the dominating trend for larger footings, see the beginning of this section.

The "large" footing is the limiting case in which soil

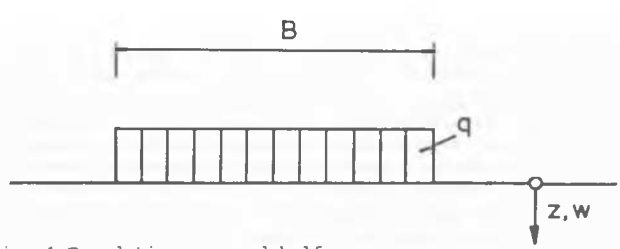

Fig. 1 Foundation on sand half-space

deformation due to the load of the footing is small compared to the deformation due to overburden pressure. Then the sand behaves as in the oedometer, Holzlöhner (1985). Because of the small additional stress the deformation is described by the tangent modulus and the observed linearity, Eq.(4) has a theoretical basis.

Non-linear material behaviour can also be incorporated in the solution procedure. Dietrich (1982) has shown that, if the material has the parabolic stress-strain relation $\sigma \sim \varepsilon^{\beta}$, the number of dimensionless quantities reduces as in the linear case. The same effect has the later finding of Winter and Hettler (1983) that, if $\sigma \sim \varepsilon^{\beta}$, the corresponding relation

$$w^{\beta} \sim q \qquad (9)$$

holds. Gl.(9) instead of Gl.(4) yields

$$\left(\frac{w}{B}\right)^{\beta} = \frac{q}{HB^{\alpha}} \cdot f_4 \qquad (10)$$

and, for the homogeneous half-space, $\alpha = 0$,

$$\frac{w}{q^{1/\beta}} \sim B \qquad (11)$$

Eq.(11) can be used to investigate the other limiting case, the "small" footing. Here, the soil deformation due to the load of the footing outweighs the deformation due to overburden pressure. Eq.(11) shows that, for linear or non-linear material, $\beta = 1$ or $\beta \neq 1$ in Eq.(9), the settlement is proportional to the breadth. The settlement observations compiled by Burland et al. (1977) show this trend indeed with smaller footings.

All the hitherto considered materials are governed by some stress-like modulus E. In Dietrich's (1977) psammic material, however, unit weight γ is the only dimensional quantity. Psammic material is the material model for rigid unbreakable grains. Replacing H by γ in Eq.(2) yields

$$w = f_5\ (B, q, \gamma) \qquad (12)$$

As Dietrich's psammic material behaves parabolically, the application of dimensional analysis yields with Eq.(9)

$$\left(\frac{w}{B}\right)^{\beta} = \frac{q}{\gamma B} \cdot f_6 \qquad (13)$$

or

$$\frac{w}{q^{1/\beta}} \sim B^{1-1/\beta} \qquad (14)$$

Hettler (1983) interprets the settlements of his small-scale footings on quartz sand on the basis of Dietrich's theory. He found $1/\beta = 1.63$. Thus

$$\frac{w}{q^{1.63}} \sim B^{-0.63} \qquad (15)$$

Eq.(15) disagrees with observed settlements. Eqs(15) and (8) represent different aspects of sand behaviour. Kögler (1933) spoke of "displacement of the soil sidewise and upwards" on the one hand and of "compression" on the other hand. The present investigation has shown that the different aspects of sand behaviour can better be characterized by their respective governing quantities: the unit weight for small footings highly loaded with respect to failure and a stress-like modulus for larger footings under moderate loads. This is not the same as Kögler's interpretation because an incompressible elastic half-space also reacts to surface loads by lateral displacement. On the other hand psammic compression is possible under certain conditions.

Summarizing we can say that the most suitable model for settlement prediction is a half-space whose modulus increases with the square root of depth. Holzlöhner (1985) has called this body "SQR half-space". The material is linear, elastic or plastic, isotropic or anisotropic. Constitutive equations need not be established. Therefore the SQR half-space is more general than the hitherto considered half-spaces. Though this study is restricted to monotonic first loading, it should be noted that, in the case of dynamically loaded footings, the scale effect is essentially the same, Holzlöhner (1969).

RESULTS OF THE SQR HALF-SPACE

Dimensional analysis can also be used to calculate stresses and displacements in the half-space, Holzlöhner (1985). Fig. 2 shows a vertical concentrated load Q acting on the half-space. The produced vertical surface displacement w_o can be written

$$w_o = C\,\frac{Q}{r^{1.5}} \qquad (16)$$

Though C remains indeterminate, results by superposition can be obtained concering the effect of breadth of the footing, side ratio, pressure distribution below rigid footings, mutual dependence of adjacent footings, and influence numbers for the calculation of flexible base slabs, Holzlöhner (1985). Some of these results will be discussed and expanded here.

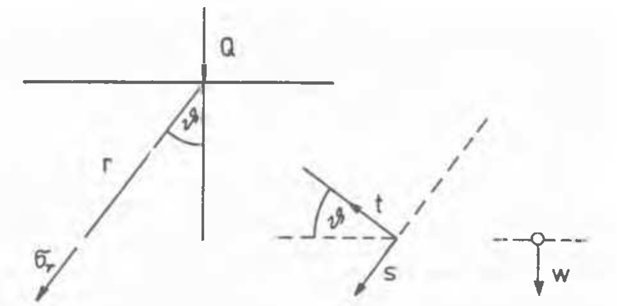

Fig. 2 Concentrated load on half-space

Foundation settlement depends not only on breadth B of the footing but on length L too. Table 1 gives the displacement of rectangular rigid footings divided by the displacement of a square footing loaded by the same pressure.

Table 1 Effect of the side ratio on foundation settlement. (a) SQR half-space, (b) Observations by Schultze and Sherif (1973), (c) Approximation $\sqrt[4]{L/B}$

L/B	(a)	(b)	(c)
1	1	1	1
2	1.185	1.25	1.189
3	1.280	-	1.316
4	1.341	-	1.414
5	1.384	1.45	1.495
10	1.497	-	1.778
∞	1.842	1.60	-

On the SQR half-space the settlement of a strip foundation, $L/B \to \infty$ is finite. The equivalent length $\sqrt{L \cdot B}$ often is used in problems of the homogeneous elastic half-space. Substituting $\sqrt{L \cdot B}$ for B in Eq.(8) yields

$$w_{L/B} \sim \sqrt[4]{L \cdot B} \qquad (17)$$

Dividing by $\sqrt{B}$ yields the approximation given in the last column of Table 1. The difference to the correct value is not more than 8% if the side ratio does not exceed 5. In case of non-rectangular footings the square root of the area may be used instead of B.

The SQR half-space can also be checked by comparing calculated and observed surface displacements outside loaded footings. Small-scale tests are not made use of, because, as the discussion of the preceding section has shown, the deformation may be of another kind than that typically occurring with foundation settlement in practice. Fig. 3 shows observed surface displacements in the neighbourhood of six nearly rigid footings at three sites in

Berlin, Muhs (1952). At each site, two towers have been constructed with foundation areas of 82m x 82m and 33m x 60m. The soil was fine to medium sand at site (a); at sites (b) and (c) sand was overlain by sandy silt and till layers of 10m to 20 m thickness. The displacements are normalized to the average foundation settlement; the distance is normalized to the breadth or, in case of rectangles, also to the length of the footings. Fig.3 includes the predictions of half-spaces with different modulus increase. Obviously, the homogeneous half-space, $\alpha = 0$, predicts much too large settlements. The SQR half-space, $\alpha = 0.5$, fits the data of sites (b) and (c) well, the displacement of site (a) decays more rapidly with distance. Here, α appears to have a value between 0.5 and 1; $\alpha = 1$ is the case of linear increase where the surface outside the footing does not settle at all, Gibson (1967). There can be no doubt that the reason for the observed surface displacement is the increase of the modulus with depth.

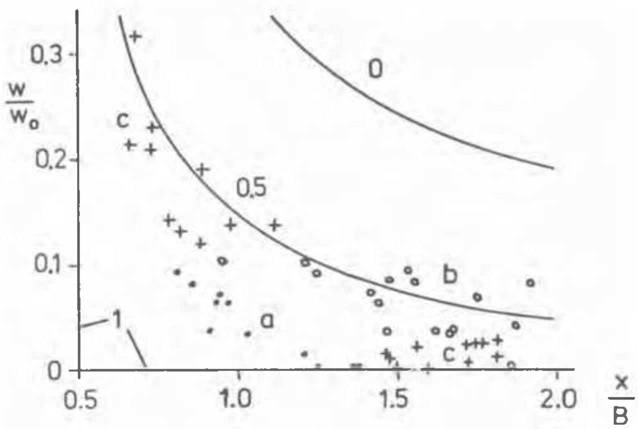

Fig. 3 Observed settlement outside loaded footings at sites a, b, c, Muhs (1952). Different predictions characterized by parameter α.

STRESSES AND DISPLACEMENTS WITHIN THE SQR HALF-SPACE

The stress and displacement fields need not be known for the calculation of foundation settlement on the SQR half-space. The fields affect, however, the thickness of the layer that contributes to the settlement of the footing. These fields remain indeterminate in the solution obtained by dimensional analysis. They can only be determined by an additional assumption.

Fröhlich (1934) heuristically established the formula

$$\sigma_r = -\frac{nQ}{2\pi r^2}\cos^{n-2}\theta \qquad (18)$$

for the radial stress σ_r due to the vertical load Q acting at the surface, see Fig. 2. The other stress components were assumed to be zero. The concentration factor n modifies the stress distribution: the greater n the more the stresses concentrate near the axis of the load. For n = 3, Eq.(18) becomes Boussinesq's solution for the homogeneous elastic half-space for $\nu = 0.5$. Originally, Eq. (18) satisfies only the equilibium conditions. Ohde (1939) has shown, however, that for $n > 3$ Eq.(18) is the exact solution for elastic half-spaces with parabolic increase of the modulus by Eq.(1), if α and Poisson's ratio ν satisfy the condition

$$\nu(2+\alpha) = 1 \qquad (19)$$

Then, the concentration factor n is

$$n = \frac{1}{\nu} + 1 \qquad (20)$$

For the SQR half-space with $\alpha = 0.5$ Eqs. (19) and (20) yield $\nu = 0.4$ and $n = 3.5$. Eq.(18) becomes

$$\sigma_r = -\frac{1.75\,Q}{\pi r^2}\cos^{1.5}\theta \qquad (21)$$

The pertaining displacements are, see Fig. 2,

$$s = 0.1485\,\frac{2.5\,Q}{H\cdot r^{1.5}}\cos\theta \qquad (22)$$

$$t = -0.1485\,\frac{Q}{H\cdot r^{1.5}}\sin\theta \qquad (23)$$

The latitudinal displacement component is zero because of symmetry. Eqs. (22) and (23) can easily be verified by calculating the strains from the displacements. Inserting the strains in the elastic material equations yields Eq. (21) as the only non-zero stress component.

For further discussions only the vertical displacement w is needed, see Fig.2, which is calculated by

$$w = s\cdot\cos\theta - t\cdot\sin\theta = 0.1485\,\frac{Q}{H\cdot r^{1.5}}\,(2.5\cos^2\theta + \sin^2\theta) \qquad (24)$$

For $\theta = \pi/2$ the surface displacement results

$$w_o = 0.1485\,\frac{Q}{H\cdot r^{1.5}} \qquad (25)$$

Comparing Eq.(25) with Eq.(16), which was obtained by dimensional analysis, yields

$$C = 0.1485\,\frac{1}{H} \qquad (26)$$

Assuming a special material - the elastic body with $\nu = 0.4$ - leads to a relation between the hitherto unknown C and modulus H. Such relation, however, is not a necessary condition for the calculation of settlements for actual footings as will be shown in the next section.

To answer the question as to the thickness of the contributing soil layer the more general formula

$$w = \frac{CQ}{r^{1.5}}\,(2.5\cos^2\theta + \sin^2\theta) \qquad (27)$$

can be used instead of Eq.(24). Thus the originally very general SQR half-space is specified only as much as necessary. The only addition is the assumed variation of w with θ.

The thickness of the soil layer contributing to foundation settlement cannot be obtained by the theory of the homogeneous elastic half-space. As this theory predicts infinite displacement for the strip footing, the contributing thickness also is infinite. Therefore the question usually is answerd by "experience". Often the thickness T = 2B is assumed, Schultze and Sherif (1973), Parry (1978). The German standard DIN 1054 uses T = 3B for individual footings and T = 1.5B for base slabs. If the thickness depends on the breadth, it must depend on the length of the footing, too. The degree of increase of the modulus with depth will also have an effect. Here, the thickness of the contributing layer will be calculated for the SQR half-space. For this, the displacements below the loaded area are needed.

Consider some sub-region A of the surface loaded by load q. Let w_1 be the displacement by Eq.(27) due to the load $Q = q\cdot dA$. Then, the displacement at any place (r,θ) within the soil is

$$w(r,\theta) = \int_{(A)} w_1(r,\theta)\cdot q(r,\theta)\,dA \qquad (28)$$

Fig. 4 shows the vertical displacements below the center of uniformly loaded areas. A square, a rectangle and an infinite strip have been considered. The displacements all are divided by the surface displacement of the center of their respective areas. Therefore, all curves begin with 1 at the surface. In this logarithmic plot the curves for the rectangles have asymptotes with the inclination 1:1.5. This corresponds to the decrease $w \sim r^{-1.5}$

due to a concentrated load, see Eq.(27). The displacement below the strip eventually decreases by $w \sim r^{-0.5}$, which corresponds to the solution for a line load. In a homogeneous half-space the exponents of decrease would have been 1 and 0, respectively.

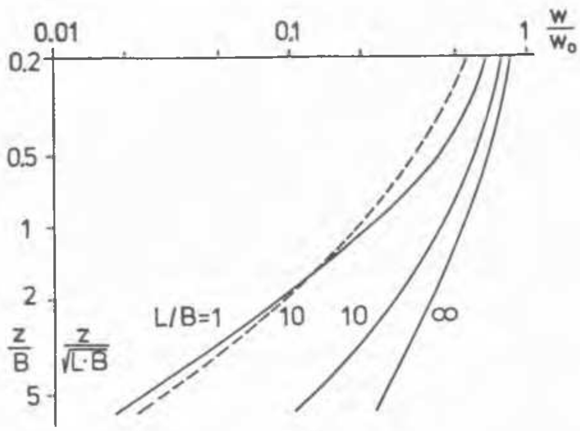

Fig.4: Vertical displacements below footings of different side ratios L/B.

In the depth z/B = 2 the curve for L/B = 1 shows 9% of the surface displacement, for L/B = 10 28%. If we define the thickness of the contributing layer such that a certain percentage, say 10%, of the displacement of the loaded area is due to deeper layers, the contributing thickness obviously depends not only on the breadth but also on the length of the footing. The dashed line in Fig.4 gives the displacement for L/B = 10 plotted against $z/\sqrt{L \cdot B}$. Thus, only small differences remain between the curves of different side ratio. The thickness should be related rather to $\sqrt{L \cdot B}$ than to B.

In the case of spread footings the contributing thickness depends not only on the lengths of the footings but on their mutual distance, too. Eq.(28) includes this case.

ASSESSMENT OF SETTLEMENTS

The indeterminate factor should be determined by calibrating the analytic results with settlement observations. Obviously, the SQR half-space cannot be the model for any soil. It represents average behaviour. Each actual sand soil will have some variations in density. Then, the soil is characterized by an average density. On the basis of Zolkov's (1974) tests, Holzlöhner (1985) relates the average density to factor C by

$$C = 0.704 \cdot 10^{-7} (105 - D_r) \frac{m^{2.5}}{kN} \qquad (29)$$

where $45\% \leq D_r \leq 85\%$.

If the density is determined from penetration tests, C should be determined directly from the data. Holzlöhner (1985) suggests to fit the penetration profile with the parabola

$$N = k\, z^{0.5} \qquad (30)$$

where k is the increase with depth of the standard penetration test N value. Holzlöhner (1985) relates this increase k to C using Parry's (1978) settlement evaluation procedure.

$$C = 0.48 \cdot 10^{-4} \frac{1}{k} \frac{m^{2.5}}{kN} \qquad (31)$$

Cone penetration and any other local or individual experience may be used in the calibration. The main difficulties are due to the widely varying relations between penetration resistance and modulus.

CONCLUSIONS

The analysis of settlement observations of footings on sand showed (a) the quantity that governs settlement is a modulus with the dimension of a stress and (b) the SQR half- space whose modulus increases with the square root of depth is the most suitable model for larger footings (B > 1m).

Without additional assumptions some results of practical interest can be obtained for the SQR half-space: effect of foundation breadth and side ratio and settlements outside loaded footings.

The thickness of the soil layer that contributes about 90% to the settlement of the footing was found to be $T = 2 \cdot \sqrt{L \cdot B}$. Formulas are given how to incorporate penetration test data in settlement assessment.

Further settlement observations are recommended. In the interpretation of the results both the breadth and the length of the footing shall be considered. Another important quantity is the degree of increase of the modulus with depth. If all these conditions are properly considered the scatter in the presentation of settlement observations can certainly be reduced.

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