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General time volume change equation for soils

Equation générale de variation de temps volume pour sols

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SYNOPSIS Traditionally the time volume change behaviour of soils under isotropic and confined conditions has been divided into instantaneous and delay deformations. This time a general unifying equation for soils is presented. The time volume change behaviour of soils is described by 2 parameters: the "coefficient of volume viscosity δ " and the "characteristic time t^* ".

"παντα ρει" (Heraclitus)

INTRODUCTION

A general compressibility equation for soils has already been presented by the author (1981) using some general philosophic principles. These philosophic ideas have also been used to obtain a general permeability change equation for soils (1983a). This time the same philosophic ideas are used to obtain the evolution of such volume changes with time. It includes the instantaneous and delay volume changes in dry coarse soils (sands and gravels) and the delay or secondary compression of fine saturated soils (silts and clays). Primary compression or consolidation due to the retardation caused by the dissipation of pore pressure in saturated fine soils has already been the subject of previous papers (Juarez-Badillo, 1983b, 1985; Juarez-Badillo and Chen, 1983).

BASIC CONSIDERATIONS

Let us consider a sample of dry coarse soil subject to an isotropic stress σ_1 for a very long time. Let now the stress σ_1 be increased "instantaneously" to σ_2 . The problem is to find the infinitesimal changes in volume dV taking place in the infinitesimal times dt . Let V_i be the initial volume for $t = 0$ and V_f be the final volume for $t = \infty$. The relation between dV and dt should produce an equation satisfying the following philosophic principle:

"The equation relating V and t may exist only through a non dimensional parameter and should, independently of critical points, satisfy the extreme boundary conditions, namely: $V = V_i$ for $t = 0$ and $V = V_f$ for $t = \infty$ ".

The connection between dV and dt can be obtained through the following steps which are thought to be philosophically supported:

- 1) The real domain for t is complete, that is, from 0 to ∞ , while the real

domain for V is incomplete and inverse, that is, from V_i to V_f . We need to find a function $f(V)$ with real domain complete and straight, that is $f(V) = 0$ for $t = 0$ and $f(V) = \infty$ for $t = \infty$. Fig 1 illustrates the obtention of $f(V)$ that results to be

$$f(V) = \frac{1}{V - V_f} - \frac{1}{V_i - V_f} \quad (1)$$

- 2) Now $f(V)$ and t are ready to be connected. For philosophic reasons, which includes the philosophic principle enunciated above, the relationship should be

$$\frac{df(V)}{f(V)} = \delta \frac{dt}{t} \quad (2)$$

where δ is a non dimensional parameter of proportionality, called the non linear "coefficient of volume viscosity".

GENERAL EQUATION

Let V_1 be the known volume for $t = t_1$ ($t_1 \neq 0$). Integrating eq (2) between the limits (t_1, V_1) and (t, V) we get

$$\ln f(V) \Big|_{V_1}^V = \delta \ln t \Big|_{t_1}^t$$

$$\therefore \frac{f(V)}{f(V_1)} = \left(\frac{t}{t_1} \right)^\delta \quad (3)$$

Introducing eq (1) into eq (3) we obtain

$$\frac{\frac{1}{V - V_f} - \frac{1}{V_i - V_f}}{\frac{1}{V_1 - V_f} - \frac{1}{V_i - V_f}} = \left(\frac{t}{t_1} \right)^\delta \quad (4)$$

Multiplying numerator and denominator of the first term by $V_i - V_f$, this eq (4) may be written as

$$\frac{V_i - V_f}{V - V_f} = 1 + \left(\frac{V_i - V_f}{V_i - V_f} - 1 \right) \left(\frac{t}{t_1} \right)^\delta \quad (5)$$

In practice, a simpler and more convenient way of writing eq (5) is using alternatively the symbols

$$\begin{aligned} V_i - V &= \Delta V = x \\ V_i - V_f &= (\Delta V)_T = x_T \end{aligned} \quad (6)$$

where $\Delta V = x$ is the volume change at time t and $(\Delta V)_T = x_T$ is the volume change at $t = \infty$.

Introducing eqs (6) into eq (5) we obtain

$$\begin{aligned} \frac{x_T}{x_T - x} &= 1 + \left(\frac{x_T}{x_T - x_1} - 1 \right) \left(\frac{t}{t_1} \right)^\delta \\ \therefore \frac{x}{x_T - x} &= \frac{x_1}{x_T - x_1} \left(\frac{t}{t_1} \right)^\delta \\ \therefore \frac{x_T - x}{x} &= \frac{x_T - x_1}{x_1} \left(\frac{t_1}{t} \right)^\delta \end{aligned} \quad (7)$$

and therefore we arrive at the useful equation

$$\frac{x_T}{x} = 1 + \left(\frac{x_T}{x_1} - 1 \right) \left(\frac{t_1}{t} \right)^\delta \quad (8)$$

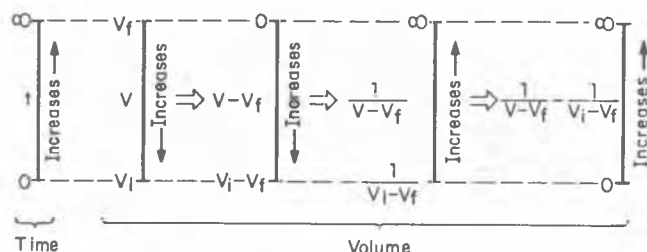


Fig.1 Scheme for the obtention of

$$f(V) = \frac{1}{V - V_f} - \frac{1}{V_i - V_f}$$

If the degree of compression U is defined as $U = \frac{x}{x_T}$, then eq (8) may be written as

$$\frac{1}{U} = 1 + \left(\frac{1}{U_1} - 1 \right) \left(\frac{t_1}{t} \right)^\delta \quad (9)$$

A further simpler form for eq (9) is defining the "characteristic time t^* " as the time for which $U_1 = 0.5$. In this way eq (9) may be written in the very simple form

$$\frac{1}{U} = 1 + \left(\frac{t^*}{t} \right)^\delta \quad (10)$$

In order to feel the progress of compression with time Fig. 2 shows the graphs of eq (9) for the special case that $t_1 = t_{0.9}$ for $U_1 = 0.9$. Different values of δ are shown. The Terzaghi's linear solution for onedimensional consolidation is shown for comparison. Fig. 3 shows the same curves for the case that $t_1 = t_{0.1}$ for $U_1 = 0.1$ and Fig. 4 shows also the same curves for $t_1 = t_{0.5}$ for $U_1 = 0.5$. From Figs. 2, 3 and 4 it appears obvious that they are not very convenient plots to deal with in practice.

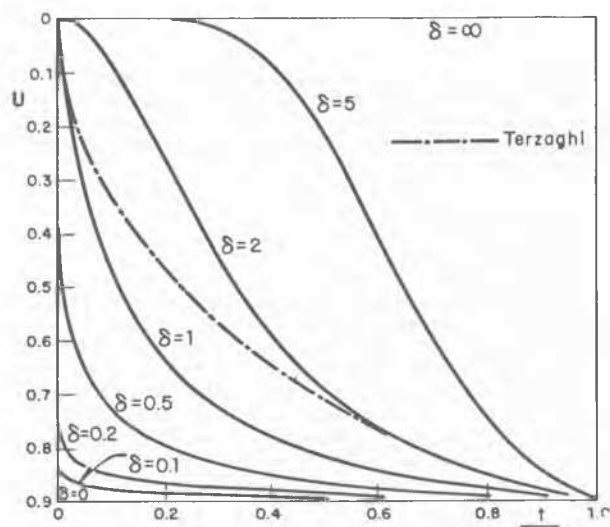


Fig.2 Graphs of $\frac{1}{U} = 1 + \left(\frac{1}{U_1} - 1 \right) \left(\frac{t_1}{t} \right)^\delta$ for $U_1 = 0.9 (t_1 = t_{0.9})$

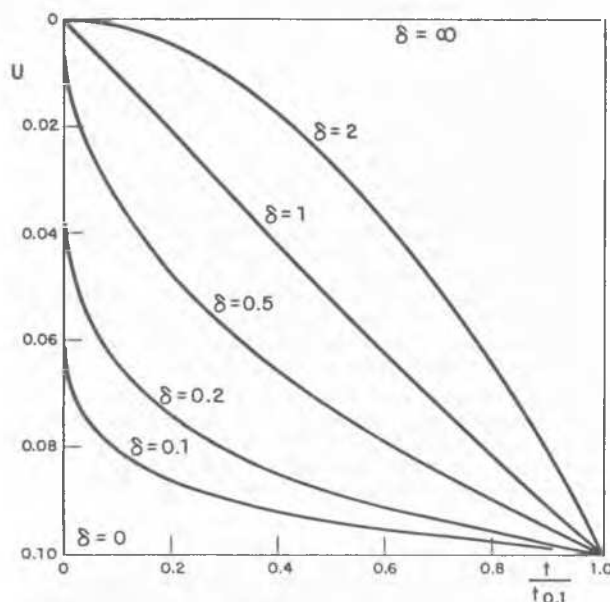


Fig.3 Graphs of $\frac{1}{U} = 1 + \left(\frac{1}{U_1} - 1 \right) \left(\frac{t_1}{t} \right)^\delta$ for $U_1 = 0.1 (t_1 = t_{0.1})$

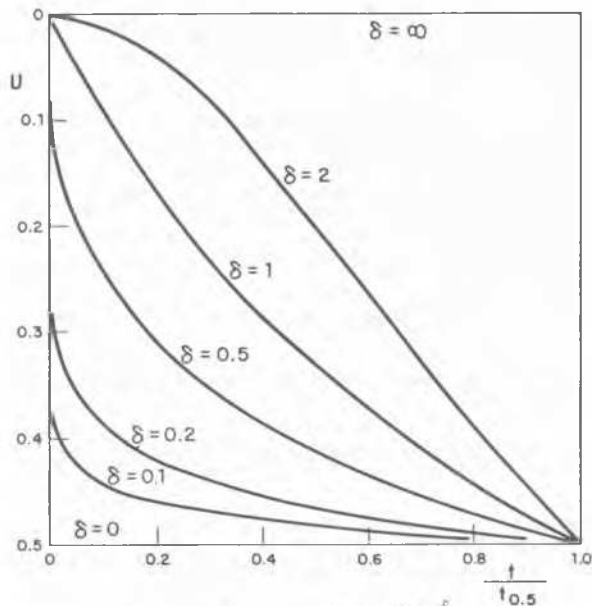


Fig. 4 Graphs of $\frac{1}{U} = 1 + \left(\frac{1}{U_1} - 1\right) \left(\frac{t}{t_1}\right)^\delta$ for $U_1 = 0.5 (t_1 = t_{0.5})$

Fig. 5 (a and b) shows the same curves than Fig. 2 but in a semi-log plot, that is, the time in log scale.

If the "time factor τ " is defined by

$$\tau = \left(\frac{t}{t^*}\right)^\delta \quad (11)$$

then eq (10) may be written as

$$\frac{1}{U} = 1 + \frac{1}{\tau} \quad (12a)$$

or in the form

$$\frac{U}{1-U} = \tau \quad (12b)$$

Fig. 6 presents the graph of eq (12) as well as the values of τ for different values of U .

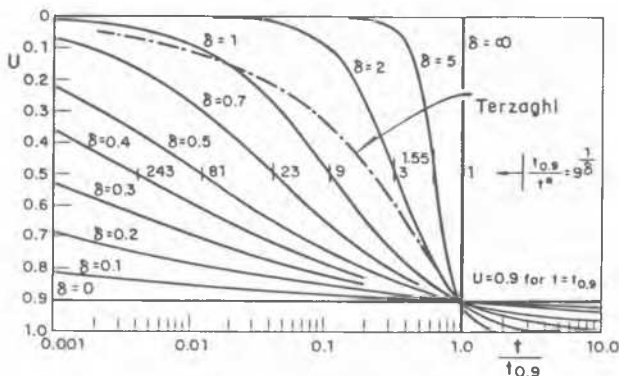


Fig. 5a Graphs of $\frac{1}{U} = 1 + \left(\frac{1}{U_1} - 1\right) \left(\frac{t}{t_1}\right)^\delta$

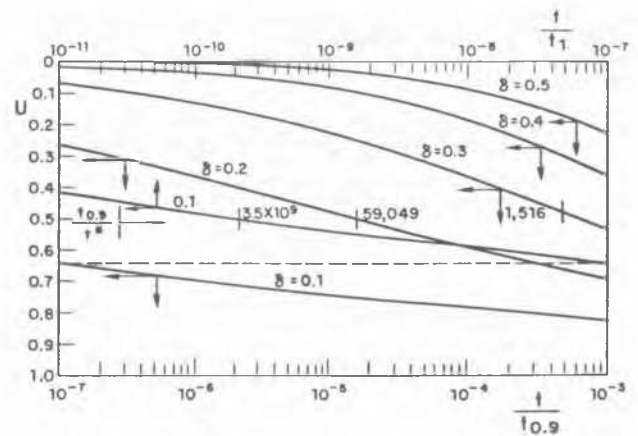


Fig. 5b Graphs of $\frac{1}{U} = 1 + \left(\frac{1}{U_1} - 1\right) \left(\frac{t}{t_1}\right)^\delta$

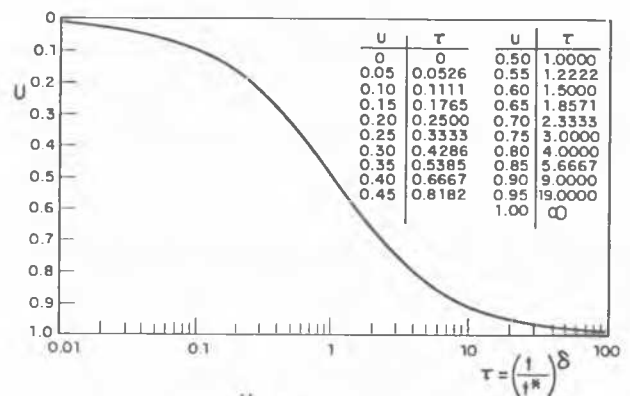


Fig. 6 Graph of $\frac{U}{1-U} = \tau$

In order to ascertain in practice the values of δ and t^* , some characteristics of the semi-log plots follows.

- 1) The semi-log plots are anti-symmetric, that is if $U_2 = 1 - U_1$, then $\frac{t_2}{t^*} = \frac{t_1}{t^*}$. From eq (12b) for $U = U_1$ we have

$$\tau_1 = \frac{U_1}{1 - U_1} \quad (13)$$

For $U_2 = 1 - U_1$ we similarly have

$$\tau_2 = \frac{U_2}{1 - U_2} = \frac{1 - U_1}{U_1} = \frac{1}{\tau_1} \quad (14)$$

that is, from eq (11)

$$\left(\frac{t_2}{t^*}\right)^\delta = \left(\frac{t_1}{t^*}\right)^\delta \quad (15)$$

therefore

$$\frac{t_2}{t^*} = \frac{t_1}{t^*} \quad (16)$$

In the semi-log plots, therefore, symmetrical values of U with respect to $U = 0.5$, have symmetrical values of t with respect to t^* . See Figs. 5 and 6.

- 2) The middle third of the semi-log plots resembles very close a straight line.

The slope of the curve in Fig. 6 may be found using eqs (12)

$$\frac{dU}{d \log \tau} = 2.3 \tau \frac{dU}{d\tau} = 2.3 \tau \frac{U^2}{\tau^2} = 2.3 \frac{U^2}{\tau}$$

$$\therefore \frac{dU}{d \log \tau} = 2.3 U(1-U) \quad (17)$$

The maximum slope occurs at $U = 0.5$ ($t=t^*$) and its value is

$$\left(\frac{dU}{d \log \tau} \right)_{\max} = \frac{2.3}{4} \quad (18)$$

Similarly, the slopes of the curves in Fig 5 are as follows. From eq (11) we have

$$\frac{d\tau}{\tau} = \delta \frac{dt}{t} \quad (19)$$

and therefore, we may write from eqs (19) and (17)

$$\frac{dU}{d \log t} = 2.3 t \frac{dU}{dt} = 2.3 \delta \tau \frac{dU}{d\tau} = 2.3 \delta \frac{U^2}{\tau}$$

$$\therefore \frac{dU}{d \log t} = 2.3 \delta U(1-U) \quad (20)$$

The maximum slope occurs at $U = 0.5$ ($t=t^*$) and its value is

$$\left(\frac{dU}{d \log t} \right)_{\max} = \frac{2.3}{4} \delta \quad (21)$$

The ratio of the slope at any point to the maximum slope is therefore given by

$$\frac{\frac{dU}{d \log t}}{\left(\frac{dU}{d \log t} \right)_{\max}} = 4U(1-U) \quad (22)$$

The values of this ratio for $U = 0, 0.1, 0.2, 0.3, 0.4, 0.5$ are: ratio = 0, 0.36, 0.64, 0.84, 0.96, 1.0 respectively. For $U = 1/3$, ratio = $8/9 = 0.89$.

Finally, the ratio of the slope of the straight line passing by the extreme points of the middle third of the graph (secant slope = $\frac{\Delta U}{\Delta \log \tau}$) to the maximum slope is, using eq (12b)

$$\frac{\Delta U}{\Delta \log \tau} = \frac{2/3 - 1/3}{\log \frac{2/3}{1/3}} = \frac{1/3}{\log \frac{2}{1}} = \frac{1}{6 \log 2} \quad (23)$$

The ratio is, therefore, from eqs (23) and (18)

$$\frac{\frac{\Delta U}{\Delta \log \tau}}{\left(\frac{dU}{d \log \tau} \right)_{\max}} = \frac{2}{3 \times 2.3 \log 2} = 0.96 \quad (24)$$

The above shows that, in practice, the middle third of the time curve may be considered, without serious error, as a straight line. See Figs. 5 and 6.

The straight line (middle third) of the time curve extends over certain number of cycles in the log scale of time. The number of cycles may be found as follows. For Fig. 6 and from eq (12b) we have that for $U = 1/3$, $\tau = 0.5$ and for $U = 2/3$, $\tau = 2$, therefore, the number of cycles given by $\Delta \log \tau$ is

$$\Delta \log \tau = \log \frac{\tau^{2/3}}{\tau^{1/3}} = \log 4 = 0.6 \quad (25)$$

Similarly, for Fig. 5 and from eq (11) we may write

$$\Delta \log t = \log \frac{t^{2/3}}{t^{1/3}} = \frac{1}{\delta} \log \frac{\tau^{2/3}}{\tau^{1/3}} = \frac{\log 4}{\delta} = \frac{0.6}{\delta} \quad (26)$$

For example, for $\delta = 0, 0.1, 0.3, 0.6, 1.0, \infty$, the straight lines extends over $\infty, 6, 2, 1, 0.6, 0$ cycles, respectively. See Fig. 5.

All the above characteristics of the time curves are very useful to determine, in practice, the parameters δ and t^* from the experimental data.

Once the values of δ and t^* are known the time for a give degree of compression may be found. From eqs (11) and (12b)

$$\left(\frac{t}{t^*} \right)^\delta = \frac{U}{1-U} \quad (27)$$

For example, for $U = 0.9$

$$\frac{t_{0.9}}{t^*} = 9^{1/\delta} \quad (28a)$$

Observe that for $(1-U)$ the time is just reciprocal. For the above example, for $U = 0.1$

$$\frac{t_{0.1}}{t^*} = \left(\frac{1}{9} \right)^{1/\delta} \quad (28b)$$

Fig 5 shows the values of eq (28a) for the different values of δ .

The rate at which the degree of compression progresses may be found as follows. From eqs (12a) and (12b)

$$\frac{dU}{d\tau} = \frac{U^2}{\tau^2} = (1-U)^2 \quad (29)$$

From eq (11)

$$d\tau = \delta \left(\frac{t}{t^*} \right)^\delta \frac{dt}{t} \quad (30)$$

Introducing eq (30) into eq (29)

$$\frac{dU}{d\tau} = (1-U)^2 \frac{\delta}{t} \left(\frac{t}{t^*} \right)^\delta \quad (31)$$

For $t = 0$ ($U = 0$) we get

$$\begin{aligned}
 \text{If } \delta < 1 \quad \left[\frac{dU}{dt} \right]_{t=0} &= \infty \\
 \text{If } \delta = 1 \quad \left[\frac{dU}{dt} \right]_{t=0} &= \frac{1}{t^*} \\
 \text{If } \delta > 1 \quad \left[\frac{dU}{dt} \right]_{t=0} &= 0
 \end{aligned} \quad (32)$$

From expressions (32) we see that only for the case $\delta < 1$ we may expect to have "instantaneous" or "simultaneous" deformation with the "instantaneous" increase in stress. However the amount of this instantaneous deformation depends on the value of δ and on the time we take to register it. See Figs. 2, 3 and 4. For $\delta = 1$ the rate of deformation is finite and therefore any deformation needs some time to take place. For $\delta > 1$ the rate of deformation is zero.

The above theory, developed for soils under isotropic stresses is thought to be also true for triaxial conditions if the principal directions of stress and of strain do not change and if the ratio of the principal stresses are kept constant, as for example, in confined onedimensional compression.

The determination, in practice, of the parameters δ and t^* depends on the type of the experimental data. Using the semi-log plots we will refer to the three thirds by (c), (s), (c), that is, curve, straight line, curve. Data with points in the three thirds: (csc). Data with only in two thirds: (cs) and (sc). Data with only in one third: (c₁), (s) and (c₂). The best experimental data is (csc). The worse experimental data is (s) if it is a "short" straight line.

For obtaining δ and t^* from an experimental curve, the author has found convenient to proceed as follows. The procedure is general but, for simplicity, imagine we have an (cs) type data.

Let t_3 , x_3 be an initial point (See eq (8)). Let t_1 , x_1 be an intermediate point. Let t_2 , x_2 be a final point. The last two points located in the initial and final zones of the straight line. Observe that the values of the x may be any quantities proportional to the volume changes.

- 1) Guess a value for x_T making use of the characteristics of the semi-log plot. Find the value of $x = a$ for the point where the straight line starts. Then, $x_T = 3a$.
- 2) Compute δ using points 1 and 2 (they are usually better points than point 3). Points 1 and 2 satisfy eq (10)

$$\frac{1 - U_1}{U_1} = \left(\frac{t^*}{t_1} \right)^\delta \quad (33)$$

$$\frac{1 - U_2}{U_2} = \left(\frac{t^*}{t_2} \right)^\delta \quad (34)$$

Dividing eq (33) by eq (34)

$$\frac{1 - U_1}{U_1} \frac{U_2}{1 - U_2} = \left(\frac{t_2}{t_1} \right)^\delta \quad (35)$$

Solving eq (35) for δ and writing the U s in terms of the x s we get

$$\delta = \frac{\log \frac{x_2}{x_1} \frac{x_T - x_1}{x_T - x_2}}{\log \frac{t_2}{t_1}} \quad (36)$$

- 3) Check the value of x_T using the initial point 3. This point should satisfy eq (8)

$$\frac{x_T}{x_3} = 1 + \left(\frac{x_T}{x_1} - 1 \right) \left(\frac{t_1}{t_3} \right)^\delta \quad (37)$$

- 4) Repeat steps 1 to 3 in case eq (37) is not satisfied, changing the value of x_T in the correct direction (a greater x_T results in greater calculated x_3).
- 5) Compute t^* using preferable point 2. From eq (10)

$$\left(\frac{t^*}{t_2} \right)^\delta = \frac{x_T - x_2}{x_2} \quad (38)$$

- 6) Introduce x_T , δ and t^* in eq (10). The time equation is ready for use

$$\frac{x_T}{x} = 1 + \left(\frac{t^*}{t} \right)^\delta \quad (39)$$

Equation (39) written in terms of volumes and in terms of heights (for onedimensional consolidation) are as follows

$$V = V_i - \Delta V = V_i - \frac{(\Delta V)_T}{1 + \left(\frac{t^*}{t} \right)^\delta} \quad (40a)$$

$$H = H_i - \Delta H = H_i - \frac{(\Delta H)_T}{1 + \left(\frac{t^*}{t} \right)^\delta} \quad (40b)$$

For the case of (sc) type data use the antysymmetrical points. For (csc) type data use the extreme points of the straight line for points 1 and 2 and check two points 3 located at the extremes of the intire experimental curve.

For the case of (s) type data, the parameters may be determined if it is a "complete straight line". Intervals for x_T , δ and t^* may be found otherwise.

For this case let x_1 and x_2 the values of x for the initial and final points of the straight line. The straight line is complete if $x_2 = 2x_1$. If $x_2 < 2x_1$ it is incomplete. In this last case x_T should satisfy the inequality

$$1.5x_2 \leq x_T \leq 3x_1 \quad (41)$$

Assuming for x_T the extreme values given by eq (41), the corresponding values for δ and t^* may be found using eqs (36) and (38).

If the value of δ is known, x_T may be found solving eq (35) for x_T . It results

$$\frac{x_T}{x_2} = \frac{\left(\frac{t_2}{t_1}\right)^\delta - 1}{\left(\frac{t_2}{t_1}\right)^\delta - \frac{x_2}{x_1}} \quad (42)$$

The value of x_T found should satisfy inequality (41).

PRACTICAL APPLICATION

The above theory is now applied to some experimental data.

One of the earliest "Time curve for a typical load increment on sand" is Fig 10.4 in Taylor's book (1948) (Not included here). Comparison with Fig. 2, indicates approximate values of $\delta = 0.1$ and $t^* = 1$ sec.

Fig. 7 presents the experimental (Vesic and Clough, 1968) and theoretical curves for "medium, uniform, slightly micaceous sand, composed of sub-angular quartz grains" between 0.1 to 1.0 mm sizes, Chattahoochee River sand, during isotropic compression. "The volumetric strains are plotted as percentages of total volumetric strains under the particular load increments (degrees of consolidation) versus time". The deformation in question "predominantly breakdown of particles" situates these curves on the second phase (virgin) of the compression curves (Juarez-Badillo, 1981). An inspection of the curves obliged the author to make an allowance of 10 sec for primary consolidation for the first 3 increments of load, and to make an allowance of 3% for further compression for the last increment of load. With these "corrections" eq (10) was applied using $\delta = 1$ and the characteristic times t^* indicated for each load increment. It is to be observed that t^* increases with the level of stress.

Fig. 8 (a and b) present the experimental (Zepeda and Diaz, 1982) and theoretical time curves for Mexico City pumice sand, between 0.84 to 4.76 mm sizes, under the particular load increments indicated. The loads were sustained from 2 to 17 hrs. Tests were made on a very dense state of the sand and on a very loose state. The experimental data includes the types: (csc), (cs) and (s). The values of $x_2 = (\Delta H)_T$, δ and t^* were ascertained as indicated above, using eqs (36), (37) and (38) and eq (40b) was used to plot the theoretical curves. The check was almost entirely within 0.01 mm with few zones (at the beginning of the curves) where the check was within 0.02 mm. It was necessary, however, to make 2 small "corrections" to H_1 , as indicated in Figs. 9 and 10.

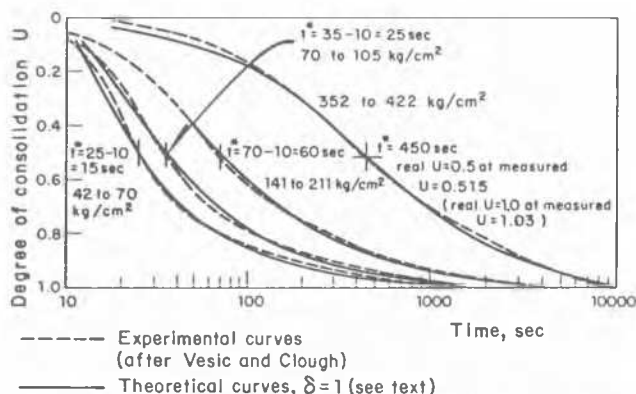


Fig. 7 Progress of volumetric strain with time during isotropic compression. Chattahoochee River Sand

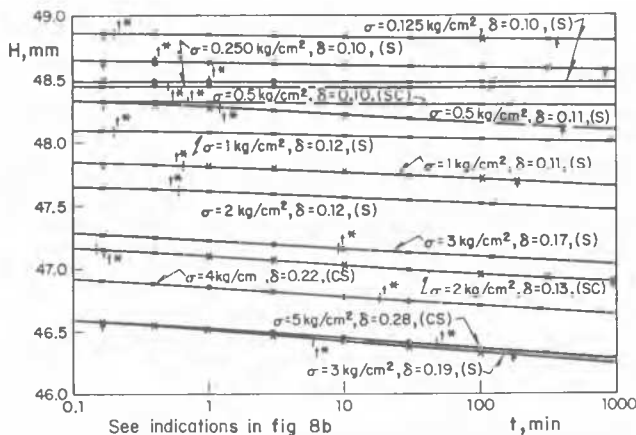


Fig. 8a Time Volume change of Mexico City pumice sand

Table 1 presents the time parameters of all these time curves. Figs. 9 and 10 present the compression curves for both, very dense and very loose states. Note that for stresses below 1 or 2 kg/cm² the experimental data was of the (s) type, and they were "short" straight lines with $x_2 < 2x_1$. However, by interpolation in Figs. 9 and 10 it was possible to choose the "correct" values for x_T , δ and t^* . Fig. 11 shows the variation of δ with the level of stress. Zepeda reported he believed there was breakdown of particles at all increments of load. The author believes there was not breakdown of particles for stresses below 2 kg/cm², due to the constant small value of $\delta = 0.1$. The feeling of the author, at present, is that, for granular soils, δ is small and constant ($\delta \approx 0.1$), before the breaking of particles and that δ is large and constant ($0.5 < \delta \leq 1$) when the level of stress is already in the virgin curve where a generalized breakdown of particles exists. In these experiments it

appears that the "critical zone" where there is partial breaking of particles extends from 2 to 8 kg/cm². Fig 11 also shows that for the very dense state the values of δ were somewhat smaller than for the very loose state. Observe also from Table 1 that t^* appears to be constant (~ 0.5 min) before the breakdown of particles.

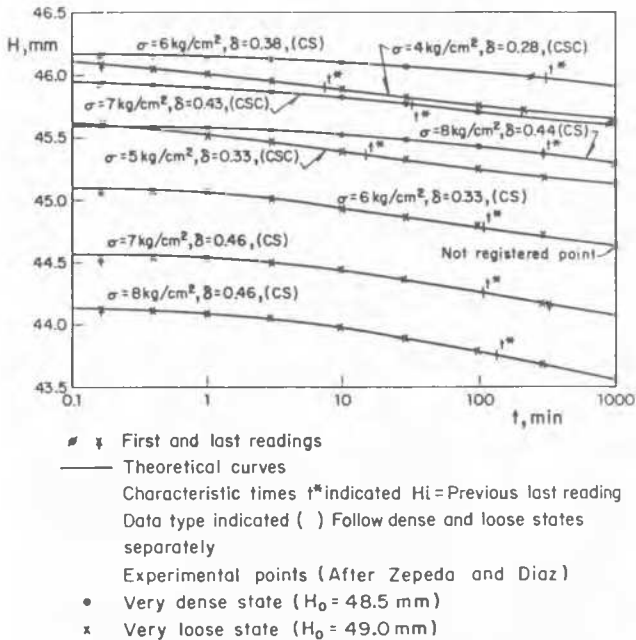
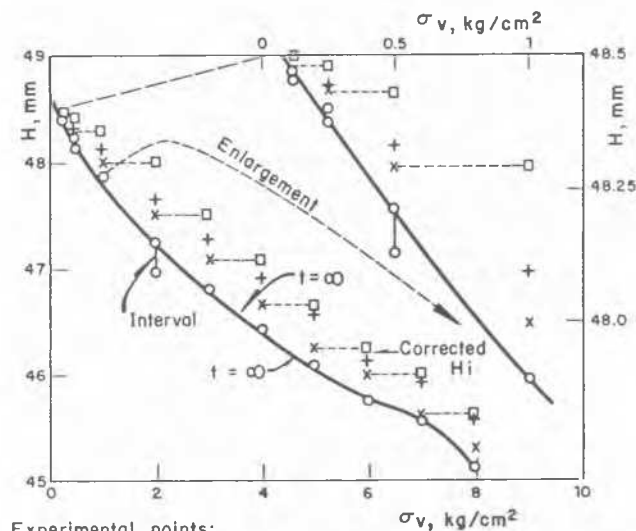


Fig.8b Time Volume change of Mexico City pumice sand



Experimental points:
(After Zepeda and Diaz)
□ Initial points H_i
+ First lectures H_1 at $t=10$ sec
x Last lectures H_2 at $t=2$ to 17 hrs
Theoretical points:
○ Calculated final points H_f ($t=\infty$)

Fig.9 Compression of Mexico City pumice sand. Very dense state

Table 1. Time parameters for Mexico City pumice sand tests

State	σ_v , kg/cm ²	Time of loading, min	H_i , mm	x_m , mm	H_f , mm	δ	t^* , min
Very dense	0.125	120	48.50	0.03	48.47	0.10	1.0
	0.25	120	48.48	0.08	48.40	0.10	0.5
	0.5	160	48.43	0.22	48.21	0.10	0.7
	1	850	48.29	0.40	47.89	0.12	0.2
	2	120	48.00	0.75	47.25	0.12	0.6
	3	120	47.51	0.70	46.81	0.17	9
	4	270	47.08	0.65	46.43	0.22	18
	5	1000	46.66	0.55	46.11	0.28	47
Very loose	6	240	46.20	0.45	45.75	0.38	310
	7	940	45.99	0.45	45.54	0.43	33
	8	1000	45.62	0.50	45.12	0.44	300
	0.125	340	49.00	0.28	48.72	0.10	0.2
	0.25	790	48.81	0.35	48.46	0.10	0.4
	0.5	380	48.57	0.55	48.02	0.11	1.2
	1	180	48.21	0.80	47.41	0.11	0.65
	2	925	47.69	1.05	46.64	0.13	0.15
	3	170	46.89	0.90	45.99	0.19	6
	4	215	46.30	0.80	45.50	0.28	7.5
	5	1000	45.74	0.75	44.99	0.33	15
	6	1010	45.14	0.70	44.44	0.41	110
	7	340	44.61	0.70	43.91	0.46	110
	8	1000	44.17	0.85	43.32	0.46	140

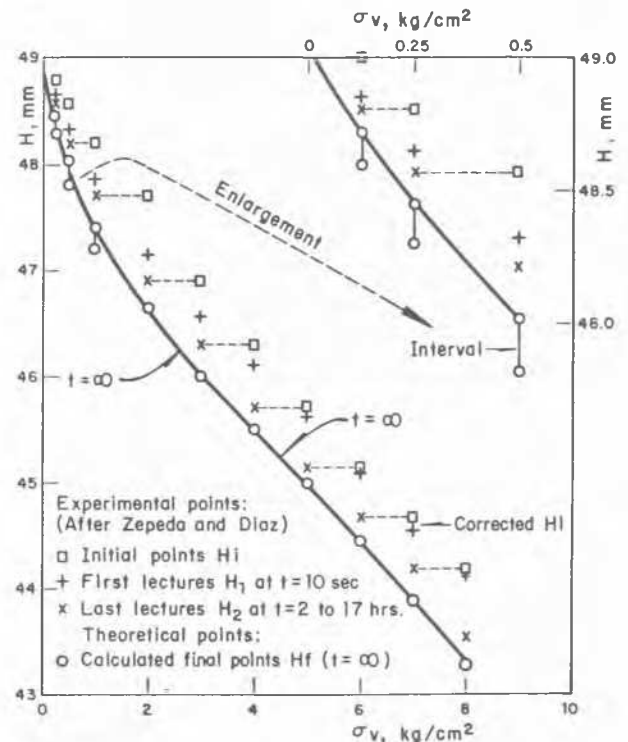


Fig.10 Compression of Mexico City pumice sand. Very loose state

A similar experimental work (Porras and Diaz, 1984) was made on Guadalajara pumice sand (Jal) in a medium dense dry and saturated

states. The load increments were sustained one day but the "short" straight lines were more abundant making somewhat difficult their analysis. However the author found that the values of δ for the saturated sand were, in the average, 1.5 times the corresponding values of δ for the dry sand.

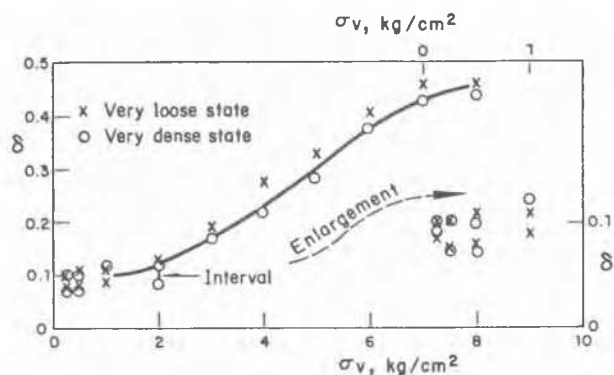


Fig. 11 Variation of δ for Mexico City pumice sand

Fig. 12 presents the time curve for a long time oedometer test made on a typical sample of Mexico City clay. The sample was 15 mm height. A vertical pressure of 0.96 kg/cm^2 was sustained 250 days. The previous load was 0.74 kg/cm^2 sustained 8 days. The increment load ratio was therefore 0.30. Eq (40b) was applied with $H_i = 12.12 \text{ mm}$, $(\Delta H)_T = 0.84 \text{ mm}$, $\delta = 0.73$ and $t^* = 7.2 \text{ days}$. After 90 days the experimental points presented some inconsistencies and bacterial growth was very noticeable. A second test was made in identical conditions and its behaviour was practically identical but with higher inconsistencies after 90 days. It should be observed that, for this test, primary consolidation was almost unnoticeable. Probably it was very small and took place in less than 0.1 day making the "secondary compression" data very consistent.

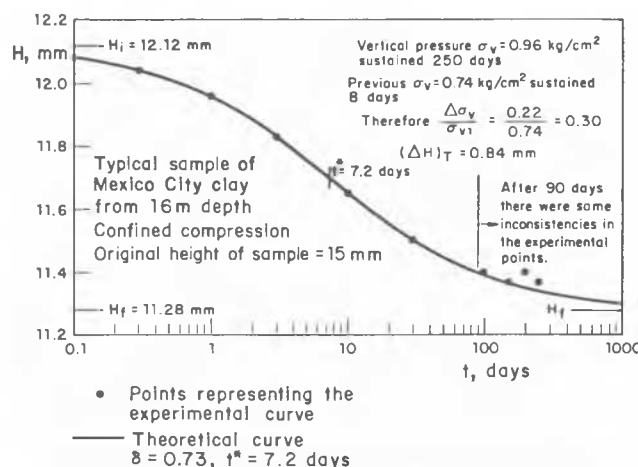


Fig 12. Time volume change of Mexico City clay

A "coefficient of secondary compression" that may prove to be useful in practice due to its similarity with the ones already in use (Mesri, 1973) is the following. From eq (21)

$$\epsilon_{\alpha}^* = \left(\frac{dH}{d \log t} \frac{1}{H} \right)_{t=t^*} = \frac{2.3}{4} \delta \frac{(\Delta H)_T}{H^*} \quad (43)$$

where H^* is the value of H for $t = t^*$

In terms of volumes, eq (43) would read

$$\epsilon_{\alpha}^* = \frac{2.3}{4} \delta \frac{(\Delta V)_T}{V^*} \quad (44)$$

For the test on Mexico City clay of Fig. 12 we have, therefore

$$\epsilon_{\alpha}^* = \frac{2.3}{4} 0.73 \frac{0.84}{11.70} = 0.030 \quad (45)$$

Note, however, that this parameter is not enough to completely specify secondary behaviour.

Several time curves were made available to the author by his colleague Leonardo Zeevart (1983) using 20 mm thick samples of typical Mexico City clay, with load increment ratios from 0.30 to 0.75 in the recompression branch and from 0.15 to 0.40 in the normally consolidation branch. The loads were sustained from 4 to 24 hours. An inspection of these curves clearly showed that in the recompression branch primary consolidation took place for $U \approx 0.3$. In the normally consolidation branch the author could not determine the termination of primary consolidation.

Mesri (1973) has published a very extensive experimental data on onedimensional tests using Organic (liquid limit $w_L = 70$) and Inorganic ($w_L = 54$) Paulling clays. Thixotropic hardening has not been considered in the above theory. Therefore, remoulded samples are not considered. Only sedimented samples behaviour on the normally consolidated branch will be considered (K_0 is not constant in the recompression and swelling branches). The data is mainly presented in terms of C_{α} and ϵ_{ap} defined by

$$\epsilon_{ap} = \frac{C_{\alpha}}{1 + e_p} = \frac{\Delta e}{\Delta \log t} \frac{1}{1 + e_p} \quad (46)$$

where e_p = void ratio at the beginning of the linear portion of the $e - \log t$ curve.

For our analysis let us substitute expression (46) for the equivalent expression

$$\epsilon_{ap} = \left(\frac{dH}{d \log t} \right)_{t=t^*} \frac{1}{H_p} \quad (47)$$

where H_p is the value of H for $e = e_p$.

Comparison of eqs (43) and (47) leads to

$$\epsilon_{\alpha}^* = \epsilon_{ap} \frac{H_p}{H^*} \quad (48)$$

The tests were made on 25.4 mm thick samples, using load increment ratios $\frac{\Delta\sigma}{\sigma} = 1$. The loads were from 0.25 to 32 kg/cm². The level of stress was reached "with only sufficient time allowed for excess pore pressure dissipation". Let us assume, in order to continue with the analysis, that this occurred for $U = 1/3$.

Fig. 13 presents a scheme illustrating primary and final compression curves in clays using the above assumption. The data was suggested by Organic Paulding clay. Approximate values of its coefficient of compressibility γ and of its void ratio e_0 for $\sigma = 8$ kg/cm² may be found using the expressions (Juarez-Badillo, 1975)

$$\gamma = 0.0016(w_L - 10) = 0.10 \quad (49)$$

and

$$e_0 = 7.5\gamma = 0.75 \quad (50)$$

The primary compression curve would, therefore, be (σ in kg/cm²)

$$\frac{V}{V_0} = \left(\frac{\sigma}{8}\right)^{-0.10} \quad (51)$$

or in terms of void ratios

$$\frac{1+e}{1.75} = \left(\frac{\sigma}{8}\right)^{-0.10} \quad (52)$$

The void ratio for $\sigma = 4$ kg/cm² is, from eq (52), $e = 0.875$, that is $(\Delta e)_p = 0.125$, and, therefore, $(\Delta e)_T = 0.375$, and for the total compression curve, we get, for $\sigma = 8$ kg/cm², $e_0 = 0.5$. The equation for this curve is, therefore,

$$\frac{1+e}{1.5} = \left(\frac{\sigma}{8}\right)^{-0.10} \quad (53)$$

The value of $\frac{H_p}{H^*}$, entering in eq (48), is given by (See Fig. 13)

$$\frac{H_p}{H^*} = \frac{H_i - (\Delta H)_p}{H_i - \frac{3}{2}(\Delta H)_p} = \frac{1 - \frac{(\Delta H)_p}{H_i}}{1 - \frac{3}{2} \frac{(\Delta H)_p}{H_i}} \quad (54)$$

where, for our case

$$\frac{(\Delta H)_p}{H_i} = 1 - \left(\frac{\sigma_2}{\sigma_1}\right)^{-\gamma} = 1 - 2^{-\gamma} \quad (55)$$

Introducing eqs (54) and (55) into eq (48) we obtain

$$\epsilon_{\alpha}^* = \epsilon_{ap} \frac{2}{3 - \left(\frac{\sigma_2}{\sigma_1}\right)^{\gamma}} \quad (56)$$

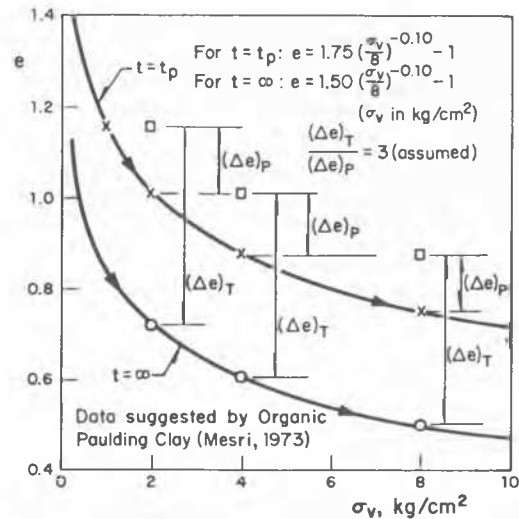


Fig.13 Scheme illustrating primary and final compression curves in clays

The value of $\frac{(\Delta H)_T}{H^*}$ entering in eq (43) is given by

$$\frac{(\Delta H)_T}{H^*} = \frac{3(\Delta H)_p}{H_i - \frac{3}{2}(\Delta H)_p} = \frac{3}{\frac{H_i}{(\Delta H)_p} - \frac{3}{2}} \quad (57)$$

Mesri has reported that for Organic Paulding clay, $\epsilon_{ap} = 0.013$, and that for Inorganic Paulding clay, $\epsilon_{ap} = 0.0035$ (Fig. 5 in Mesri paper).

Applying eqs (49), (55), (56), (57) and (43) to Organic Paulding clay we get

$$\begin{aligned} \gamma &= 0.10 \\ \epsilon_{\alpha}^* &= 0.0135 \\ \delta &= 0.10 \end{aligned} \quad (58)$$

Similarly, applying the same equations to Inorganic Paulding clay we get

$$\begin{aligned} \gamma &= 0.07 \\ \epsilon_{\alpha}^* &= 0.0036 \\ \delta &= 0.04 \end{aligned} \quad (59)$$

Observe that there is no practical difference between ϵ_{α}^* and ϵ_{ap} . Observe also that these parameters can not be used in practice without the t^* parameter.

Mesri et al (1975) have published a very extensive experimental data on secondary compression of typical samples of Mexico City clay. The samples were 19 mm high. The applied pressures were up to 30 kg/cm². The samples were loaded in increments to a final

pressure using a load increment ratio of unity and a load increment duration just sufficient for completion of primary consolidation determined by Taylor's method (Taylor, 1948)". For undisturbed samples, first sustained loading, they report a constant value $\epsilon_{ap} = 0.033$ (their Fig. 25) for the virgin branch of the compressibility curve. From the compressibility curve (their Fig. 16) the author obtained a compressibility coefficient $\gamma = 0.43$. Making the assumption that primary consolidation occurred for $U \leq 1/3$, application of eqs (55), (56), (57) and (43) give

$$\begin{aligned}\gamma &= 0.43 \\ \epsilon_{\alpha}^* &= 0.040 \\ \delta &= 0.06\end{aligned}\quad (60)$$

The value of γ coincides with the value reported by the author (1975). Observe now that ϵ_{α}^* is 20% higher than ϵ_{ap} . Compare also this value of ϵ_{α}^* with the one given by eq (45) for Fig. 12. Compare also the small value of $\delta = 0.06$ with the high value $\delta = 0.73$ obtained in Fig. 12. This great difference, might be due to the fact that Fig. 12 is still in the recompression branch of the compressibility curve. Future research should clarify this point.

FINAL DISCUSSION

It has been shown that the time volume change behaviour of soils (when no retardation exists due to pore water pressure dissipation) under isotropic and confined compression (if $K_0 = \text{constant}$) may be described by a single equation containing two parameters, the coefficient of volume viscosity δ and the characteristic time t^* . The coefficient δ has to do mainly with the form of the time curve while the time t^* has to do mainly with the rapidity of the phenomenon. See Fig. 2.

This general time equation is identical in form to the general compressibility equation (Juarez-Badillo, 1981). Compare Fig. 3 of the present paper with Fig. 2 of the above mentioned paper. The coefficient δ in the volume time curve has the same significance than the coefficient γ in the volume pressure curve. Similarly, the characteristic time t^* has the same significance in the time curves than the characteristic pressure σ^* has in the pressure curves.

The compressibility equation reads

$$\frac{v_0}{v} = 1 + \left(\frac{\sigma}{\sigma^*} \right)^{\gamma} \quad (61)$$

while the time equation reads (eq (10))

$$\frac{1}{U} = 1 + \left(\frac{t}{t^*} \right)^{\delta} \quad (62)$$

If U' is defined by

$$U' = 1 - U \quad (63)$$

then eq (62) may be written as

$$\frac{1}{U'} = 1 + \left(\frac{t}{t^*} \right)^{\delta} \quad (64)$$

and the similarity of eqs (61) and (64) is evident. This is so because they satisfy the same philosophic requirements. Because of this similarity the time curves of Figs. 5 and 6 may also be interpreted, using eq (63), as the compressibility curves of granular materials before the braking of particles, that is, in the unvirgin curves.

It is the feeling of the author that for granular soils like sands:

1. In the first mechanical phase of the compressibility curve, unvirgin, δ is small and constant, say $\delta = 0.1$ and t^* is small and constant, say $t^* = 1$ to 30 sec (Taylor's figure, Fig. 11 and Table 1).

2. In the transition or critical zone, where partial breakdown of particles exists, δ and t^* increase when the stress increases (Figs. 8 and 11 and Table 1).

3. In the second mechanical phase, virgin curve, where a general breakdown of particles exists, δ is high and constant, say $\delta = 0.5$ to 1.0 and t^* increases when the compressive stress increases (Figs. 7, 8 and 11 and Table 1).

Similarly, it is the feeling of the author that for plastic soils like clays:

4. In the recompression branch δ is constant, apparently with high values (Fig. 12). Is t^* also constant?

5. In the normally consolidated branch δ is constant, apparently with low values (eqs (58), (59) and (60)). Is t^* also constant?

The above comments are mainly made to promote future experimental research. The published data does not allow the determination of t^* in clays. The author believes that a reconsideration of some of the experimental data of the past is highly desirable.

It should be noted that, in clays, primary consolidation will make necessary to introduce a modification in the time scale after the termination of the excess pore pressure dissipation, Fig. 7. Future research will indicate how this should be done.

It is important to realize that primary compression curves are functions of the loading programs while the final ($t = \infty$) compression curves are postulated to be unique for given soil samples under monotonic loading, Figs. 9, 10 and 13.

It should be noted that, according to this theory, the parameter ϵ_{α}^* , eq (43), very similar to the traditional ϵ_{op} , eqs (46), (47) and (48), is not a good parameter to study secondary compression. $\frac{(\Delta H)}{H^*T}$ is constant as far as the load increment H^* ratio is constant and no previous sustained loads have acted (eqs (57) and (55)).

It is suggested that eq (10) is also applicable to solids, liquids and gases. It should be very interesting to know the parameters δ and t^* for water. Very surely they both are very small.

CONCLUSIONS

The main conclusion are as follows:

1. A general time volume change equation for soils is presented, eq (10). It consist of two parameters: the coefficient of volume viscosity δ and the characteristic time t^* . The parameter δ has to do mainly with the form of the time curve while the parameter t^* has to do mainly with the rapidity of the phenomenon, Fig. 2.

2. We may expect to have "instantaneous" or "simultaneous" deformation with the "instantaneous" increase of stress only when $\delta < 1$. However its amount depends on the value of δ and on the time we take to register it, Figs. 2 to 4.

3. The semi-log plot seems most appropriate for the time curves, Figs. 5 and 6. In this plots, the time curves are antisymmetric and its middle third is very close to a straight line.

4. For sands, it appears that, for the unvirgin branch of the compressibility curves, the values of δ and t^* are small and constant, while for the virgin branch of the compressibility curves, the values of δ are high and constant and the values of t^* increases when the compressive stress increases.

5. For clays, it appears that the values of δ are constants, but different, in the recompression and in the normally consolidation branches of the compressibility curve.

6. In clays there is a high need of experimental data to study the t^* parameter.

7. There is a need of experimental data to study the factors that influence the values of the δ and t^* parameters in soils.

8. It is suggested that eq (10) is also applicable to solids, liquids and gases.

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