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State surfaces for partially saturated soils

Surfaces d'état pour des sols nonsaturés

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SYNOPSIS A number of analytical simple models to describe state surfaces for volume and degree of saturation change of partially saturated soils subjected to confined or isotropic compression are proposed on the basis of observed qualitative behaviour. The results of a number of tests performed and others taken from published results are then used to find, through the appropiate optimization techniques, the optimum analytical expressions.

INTRODUCTION

Constitutive behaviour of partially saturated soils has been investigated, mainly through experimentation, in the past few decades. Initially Bishop (1959) and Aitchison and Bishop (1960) suggested the existence of an effective stress, σ', defined as

$$\sigma' = \sigma + \chi(p_a - p_\omega) \tag{1}$$

where σ is the total stress, p_a and p_ω the air and water pressure and χ a parameter highly dependent on degree of saturation, stress history and soil structure.

The concept of effective stress was soon questioned by Jennings and Burland (1962), Bishop and Blight (1963), Aitchison (1965), Blight (1965) and Burland (1965). In fact, a reduction in σ' through a decrease in (p_a-p_ω) may led to either collapse or swelling of the soil as a function if the value of σ. In the reduction in σ' comes from a decrease in total stress, swelling takes place. These phenomena are not compatible with an effective stress definition given by equation (1).

Coleman (1962) suggested that, under isotropic loading condition, the total (V) and water (V_ω) volume changes could be expressed in terms of changes in $(\sigma-p_a)$ and (p_a-p_ω):

$$-\frac{d\,V_\omega}{V} = C_{11}\, d(p_a-p_\omega) + C_{12}\, d(\sigma-p_a) \tag{2}$$

$$-\frac{d\,V_\omega}{V} = C_{21}\, d(p_a-p_\omega) + C_{22}\, d(\sigma-p_a) \tag{3}$$

Coefficients C_{ij} vary and may even change their signs as a function of the current value and the history of stresses. Bishop and Blight (1963), Blight (1965) and Burland (1965) suggested the convenience of formulating functional relationships between void ratio and $(\sigma-p_a)$ and (p_a-p_ω). By means of null tests Fredlund and Morgenstern (1977) showed that the mentioned pair of stress states are in fact a significant stress system for a partially saturated soil.

Matyas and Radhakrishna (1968) put forward the concept of state surfaces to relate void ratio (e) and degree of saturation (S_r) with $(\sigma-p_a)$ and (p_a-p_ω). They sowed that these surfaces are unique for monotonic loading sequences and changes in degree of saturation. This uniqueness was confirmed in the work reported by Barden et al. (1969).

Fredlund and Morgenstern (1976) performed a number of tests with different stress and suction paths and emphasized the notorious influence of hytheresis loops. In the same work constitutive relations of Coleman's type are proposed.

For significant portions of the state surfaces, Fredlund (1979) proposed the following expressions

$$e = e_o - C_t \log\frac{(\sigma-p_a)}{(\sigma-p_a)} - C_m \log\frac{(p_a-p_\omega)}{(p_a-p_\omega)} \tag{4}$$

$$\omega = \omega_o - D_t \log\frac{(\sigma-p_a)}{(\sigma-p_a)} - D_m \log\frac{(p_a-p_\omega)}{(p_a-p_\omega)} \tag{5}$$

where ω is the water content and C_t, C_m, D_t and D_m are constants.

Some attempts have also been made to derive more complex functions to relate degree of saturation and soil suction in the absence of external load variations. For instance, Casteleiro (1975) proposed the following expression for the volumetric water content of the soil

$$\theta = \theta_s \exp\left[-\frac{\mu(p_a-p_\omega)}{(p_a-p_\omega)_{cr}}\right] + \theta_{cr} \tanh\left[\frac{\nu(p_a-p_\omega)}{(p_a-p_\omega)_{cr}}\right] \tag{6}$$

where θ_s, θ_{cr}, μ, ν and $(p_a-p_\omega)_{cr}$ are constants to be found for every particular soil.

State surfaces allow a unified modelling of swelling and collapse effects in partially saturated soils. Lloret and Alonso (1980) developed a onedimensional consolidation model which includes state surface description for void ratio and degree of saturation. They were analytically described by means of twodimensional spline interpolation through experimental data. This approach has two shortcomings:

(a) The number of testing points necessary to cover a moderate range of significant stress changes is large. The necessary tests are, on the other hand, rather involved and time consuming.

(b) If the interpolating surfaces are forced to pass through the experimental data, large errors may be associated to the directional derivatives (modulus) of the state surfaces. These errors affect, obviously, any consolidation model.

Relatively simple mathematical expressions of general applicability, defined by means of a reduced number of data points, easy to find in the laboratory, would reduce the experimental effort and would facilitate the implementation of numerical modelling.

In this work several types of analytical functions are proposed and their predictions compared with the results of tests on different soil types. An analysis of the applicability of the different expressions is then made and some optimum functions are selected on the basis of minimum fitting errors.

EXPERIMENTAL WORK

Two types of soils were tested. One of them is a commercial kaolin, moderately plastic (properties are given in Table I) and it was subjected to confined and isotropic loading.

TABLE I Characteristics of soils analyzed.

Type of soil (Reference)	w_L (%)	w_p (%)	IP (%)	%< #200	Unified Classification	γ_s	Composition	Sample Preparation
1. Silty Clay (Jennings and Burland, 1962)	56.4	21.2	35.2	23	CH	2.75	80% ball milling of quartzite 20% Wyoming bentonite	Formed in moulds and brought to equilibrium
2. Kaolin (Matyas and Radhakrishna, 1968)	29	25	4	8	ML	2.63	80% flint powder 20% Peerless clay	Statically compacted
3. West Water Clay (Barden et al. 1969)	20	10	10	10	CL	2.66	Illite	Half Proctor compaction
4. Kaolin (Lloret, 1982)	41	29	12	4	ML	2.65	Kaolin	Statically compacted
5. "Piñolén" Clayed sand	32.2	18.5	13.7	3	SC	2.72	--	Kneading compaction

Six different edometric tests were carried out with the first soil. Five samples were subjected to loading processes under different pore water suctions. The sixth sample was subjected to alternate steps of loading increase and suction decrease. In addition, the same soil was tested under isotropic conditions. Three loading sequences for three different water suctions were carried out. More details concerning these tests are given in Table II (Test series numbers 1 and 2).

TABLE II Characteristics of test series analyzed

Test series	Type of test (+)	Type of soil (Table I)	Initial void ratio	Initial Degree of Saturation	Suction range (10^5 N/m^2)	Range of Applied loading (10^5 N/m^2)	No. of points to define st. surface e/S_r	Type of stress path (*)
1	A	4	0.934	0.517	0 - 1	0.05 - 8	41/36	C
2	B	4	0.934	0.517	0.2 - 1	0.1 - 2	8/8	D
3	A	5	0.474	0.433	0 - 1	0.05 - 8	25/25	D
4	B	2	0.825	0.676	0 - 1.03	0.14 - 8.27	31/31	C
5	A	2	0.963	0.491	0 - 0.68	0.14 - 8.27	29/29	C
6	B	1	0.838	0.504	0 - 31.02	0.1 - 30.6	21/12	D
7	A	3	0.475	0.400	0.02 - 0.82	0.41 - 4.5	15/ -	C

(+): A: One dimensional deformation
B: Isotropic compression

(*): C: Loading under constant suction alternating with suction decrease
D: Loading under constant suction

The second soil tested, the so-called "Piñolén" clayed sand is a natural soil used to build the impervious core of the Limonero earth dam in Málaga, Spain. It is a detritic soil with some amount of fine material which is the responsible for its plastic behaviour. Five one-dimensional compression tests at different suction values were performed in this soil (see Tables I and II).

A special edometric cell was built according to the scheme shown in Fig. 1. Air and water pressures as well as water content variation are controlled during the tests. Cells somewhat similar to the apparatus built to perform these

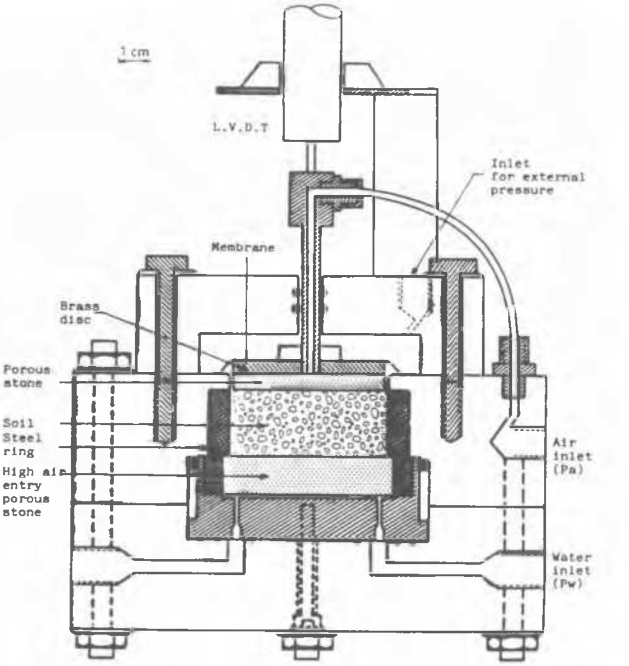

Fig. 1 Scheme of cell used to perform confined compression tests under controlled air and water pressures.

tests have been described by Escario (1967), Barden et al. (1969), Aitchison and Woodburn (1969), Moore and Millar (1971) and Fredlund and Morgenstern (1976). A triaxial cell was modified to carry out the isotropic compression tests following previous work done by Bishop and Donald (1961), Matyas and Radhakrishna (1968) and Fredlund and Morgenstern (1976).

Types of State Surfaces

Relationships such as equations (4) with constant C coefficients are not able to model simultaneously the collapse behaviour induced by wetting (when the applied load is relatively high) and the swelling which takes place if the applied total load is relatively small. Even though this mixed behaviour is not always observed for every type of soil, the deformability associated to changes in total load increases when the water suction decreases.

Accordingly, one is led to think that coefficients C and D in equation 4 cannot be constants. A first approximation is to accept a linear dependence of them with $(\sigma-p_a)$ and (p_a-p_ω). In other words surface gradients $de/d(\sigma-p_a)$ and $de/d(p_a-p_\omega)$ depend linearly on (p_a-p_ω) and $(\sigma-p_a)$ respectively. The simpler equation fulfilling these requirements has the following type

$$e = a + b(\sigma-p_a) + c\,(p_a-p_\omega) + d(\sigma-p_a)(p_a-p_\omega) \quad (7)$$

where a, b, c and d are constants. If Coleman's relationships (equation 3) are followed, the C_{2j} coefficients are given by

$$C_{21} = -\frac{1}{1+e_o}\left[c + d(\sigma-p_a)\right] \quad (8)$$

$$C_{22} = -\frac{1}{1+e_o}\left[b + d(p_a-p_\omega)\right] \quad (9)$$

A higher number of terms and coefficients would probably increase the degree of approximation but the physical interpretation of parameters is rapidly lost. If only linear terms are considered,

$$e = a + b(\sigma-p_a) + c(p_a-p_\omega) \quad (10)$$

the formulation becomes equivalent to Bishop's initial proposal of effective stress (equation 1). In fact,

$$\frac{e-a}{b} = (\sigma-p_a) + \frac{c}{b}(p_a-p_\omega) = (\sigma-p_a) + \chi(p_a-p_\omega) \quad (11)$$

A logarithmic scaling of significant stresses is another choice, perhaps better justified for extended ranges of stress variation (specially with regard to water suction). However they cannot be directly used when total stress (if $p_a = 0$) and suction become null.

A number of linear and nonlinear functions, given in Table III were finally selected. They cover different possibilities but maintain a desirable simple format. Similar types of functions are also proposed to predict the degree of saturation. In this case equations such as (6) suggest hyperbolic and exponentially decaying variations with suction. Accordingly, two more types of functions were added (Cases 9 and 10 in Table III).

TABLE III Functions used for state surface approximation.

CASE	FUNCTION	CODE
1	$\begin{matrix}e\\S_r\end{matrix} = a+b(\sigma-p_a)+c(p_a-p_\omega)$	L P, S
2	$\begin{matrix}e\\S_r\end{matrix} = a+b\log(\sigma-p_a)+c(p_a-p_\omega)$	L Log P, S
3	$\begin{matrix}e\\S_r\end{matrix} = a+b(\sigma-p_a)+c\log(p_a-p_\omega)$	L P, Log S
4	$\begin{matrix}e\\S_r\end{matrix} = a+b\log(\sigma-p_a)+c\log(p_a-p_\omega)$	L Log P, Log S
5	$\begin{matrix}e\\S_r\end{matrix} = a+b(\sigma-p_a)+c(p_a-p_\omega)+d(\sigma-p_a)(p_a-p_\omega)$	NL P, S
6	$\begin{matrix}e\\S_r\end{matrix} = a+b\log(\sigma-p_a)+c(p_a-p_\omega)+d\log(\sigma-p_a)(p_a-p_\omega)$	NL Log P, S
7	$\begin{matrix}e\\S_r\end{matrix} = a+b(\sigma-p_a)+c\log(p_a-p_\omega)+d(\sigma-p_a)\log(p_a-p_\omega)$	NL P, Log S
8	$\begin{matrix}e\\S_r\end{matrix} = a+b\log(\sigma-p_a)+c\log(p_a-p_\omega)+d\log(\sigma-p_a).\log(p_a-p_\omega)$	NL Log P, Log S
9	$S_r = a-\mathrm{Th}\,[b(p_a-p_\omega)]\,[c+d(\sigma-p_a)]$	NL P, Th S
10	$S_r = a-\left\|1-e^{-b(p_a-p_\omega)}\right\|\left\|c+d(\sigma-p_a)\right\|$	NL P, Ex s

RESULTS

Minimum squared error fitting techniques have been followed to find the coefficients of the different approximations shown in Table III better adjusted to the set of test series indicated in Table II. Predicted and measured values were then compared through conventional statistical regression techniques. Tables IV and V show the regression parameters for the best state surface equations approximated to the data obtained by Lloret (1982) (Test series 1 and 2 in Table II). Figs. 2 and 3 show straight line regressions between predicted and actual measurements for two particular state function models and test series No. 1 (Table II). The coefficient of correlation for the particular case plotted in Fig. 2 is 0.988. Dispersion of results is also minimum. A less satisfactory case is shown in Fig. 3. Here the regression coefficient is 0.952 and the dispersion of results probably reflects the testing difficulties associated to the measurement of degree of saturation. The dispersion of the measured values around the predicted ones has been characterized by the coefficient of variation in a point located at a distance from the origin equal to half the average value of observations. (Last column in Tables IV and V). This coefficient is 0.025 in Fig. 2 and 0.10 in Fig. 3. A similar analysis was done in the "Piñolén" clayed sand. In this case the computed dispersion was low and almost identical for both types of soil parameters (e and S_r).

TABLE IV Statistical Results for Different State Functions Proposed for Void Ratio. Confined Compression Tests on Kaolin (Lloret, 1982).

FUNCTION CODE (Table 3)	b	a	Em	Ea	σ_b	Vx
L P,S (S=0)	0.860	0.125	8.49	2.316	0.0556	0.0741
L P,S (S#0) (*)	0.972	0.025	3.33	0.939	0.0303	0.0379
L LogP,S(S=0)	0.651	0.312	17.12	4.688	0.0763	0.0859
L LogP,S(S#0) (*)	0.654	0.305	11.27	3.646	0.0869	0.0831
L P,LogS	0.972	0.025	3.32	0.945	0.0301	0.0377
L LogP,LogS	0.655	0.304	11.00	3.655	0.0868	0.0831
NL P,S (S=0)	0.905	0.085	7.20	1.988	0.0470	0.0652
NL P,S (S#0) (*)	0.986	0.012	3.16	0.564	0.0212	0.0270
NL LgP,S(S=0)	0.730	0.241	12.87	3.975	0.0711	0.0850
NL LgP,S(S#0) (*)	0.694	0.270	9.47	3.399	0.0841	0.0830
NL P,Log S	0.988	0.011	3.34	0.480	0.0200	0.0255
NL LogP,LogS	0.697	0.268	8.52	3.401	0.0839	0.0829

(*) Points lying in the plane Pa-Pw=0 are not considered

LEGEND :

b = Regression coefficient
a = Intercep at the origin
Em = Maximum error
Ea = Average error
σ_b = Standard deviation of regression coefficient
Vx = Coefficient of variation of a new estimation at point 1/2 average of measured parameters

TABLE V Statistical Results for Different State Functions Proposed for Void Ratio. Isotropic Compression Tests on Kaolin (Lloret, 1982). See legend in Table IV.

FUNCTION CODE (Table 3)	b	a	Em	Ea	σ_b	Vx
L P,S (S=0)	0.852	0.111	23.33	7.700	0.0609	0.1572
L P,S (S#0) (*)	0.897	0.069	13.24	4.720	0.0607	0.1166
L LogP,S(S=0)	0.854	0.110	22.81	7.671	0.0606	0.1567
L LogP,S(S#0) (*)	0.874	0.084	13.12	4.994	0.0664	0.1248
L P,LogS	0.908	0.062	10.00	4.160	0.0579	0.1122
L LogP,LogS	0.898	0.069	11.24	4.154	0.0606	0.1164
NL P,S (S=0)	0.853	0.110	23.42	7.655	0.0607	0.1568
NL P,S (S#0) (*)	0.901	0.066	12.81	4.553	0.0598	0.1151
NL LgP,S(S=0)	0.856	0.108	22.50	7.636	0.0602	0.1560
NL LgP,S(S#0) (*)	0.874	0.084	13.11	4.966	0.0663	0.1247
NL P,Log S	0.909	0.061	10.57	4.154	0.0576	0.1117
NL LogP,LogS	0.901	0.066	11.24	4.138	0.0597	0.1150
NL P,ThS(S=0)	0.936	0.048	13.97	4.926	0.0421	0.1174
NL P,ExS(S=0)	0.952	0.036	12.59	3.962	0.0369	0.1043
NL P,ThS(S#0) (*)	0.906	0.063	14.18	4.305	0.0586	0.1134
NL P,ExS(S#0) (*)	0.909	0.061	13.62	4.255	0.0578	0.1121

(*) Points lying in the plane Pa-Pw=0 are not considered

The best approximations found for void ratio of "Piñolén" clayey sand (test series No. 3) under confined compression is plotted in Fig. 4. The dependence of compressibility with suction variation and the transition from (little) swelling to collapse behaviour for increasing intensity of total stress are appreciated in the graph.

Fig. 5 is a plot of the optimum state surface of degree of saturation for the same test series on "Piñolén". The best fit was found with a function exponentially decreasing with suction and linearly dependent on total stress (σ-p_a).

Correlation coefficients between measurements and optimum predictions were computed for all of the test series mentioned previously. Their values are shown in Tables VI and VII for void ratio and degree of saturation respectively.

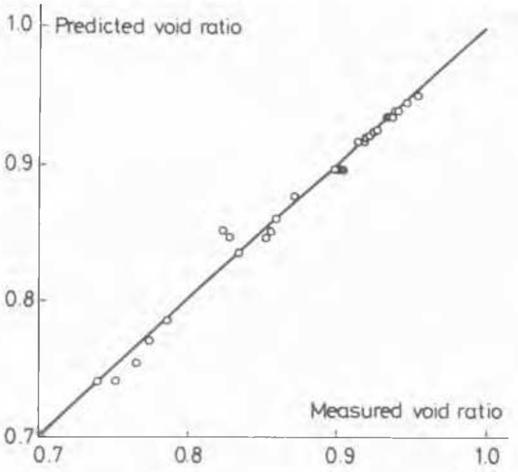

Fig. 2 Correlation between measured and predicted void ratios for test series No. 1 (Kaolin, confined compression). State surface model type NL P, Log S. (See Table III).

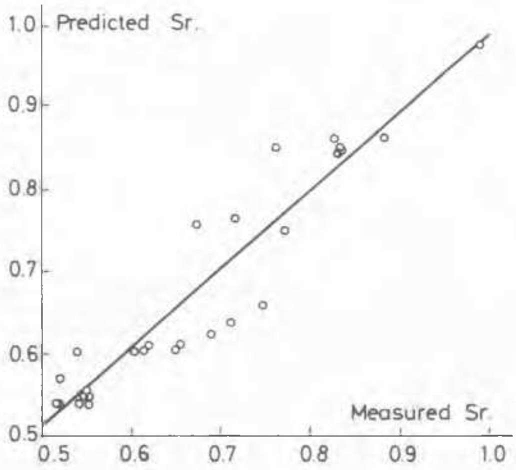

Fig. 3 Correlation between measured and predicted degrees of saturation for test No. 1 (Kaolin, confined compression). State surface model type NL P, Ex S.

As far as void ratio is concerned, the best approximation for tests in which the range of stress change has been relatively small is found with functions of the type NL P, Log S (see Table III for code definition). For larger stress ranges the best fit is found for models NL Log P, Log S. Nonlinear functions give, consistently, better results than their linear counterparts. However, differences are in general small, perhaps as a consequence of the limited range of variation of suction and total stress in the majority of test series analyzed.

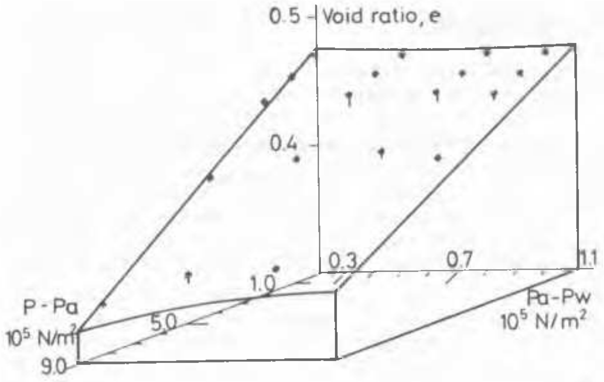

Fig. 4 State surface for void ratio of "Piñolén" clayey sand (test series No. 3, confined compression) as predicted by the model NL P, Log S.

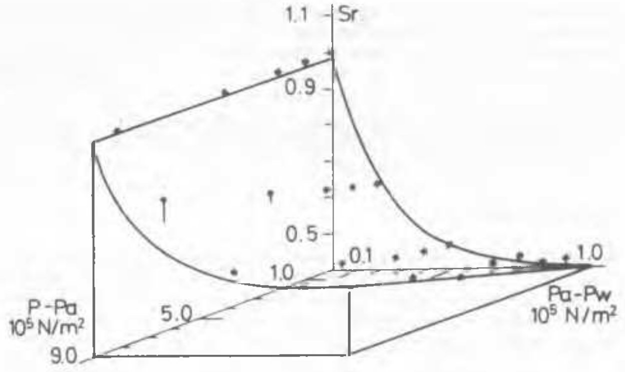

Fig. 5 State surface for degree of saturation of "Piñolén" clayey sand (test series No. 3, confined compression) as predicted by the model NL P, Ex S.

TABLE VI Regression Coefficients between Measured and Predicted Values of Void Ratio for Different Test Series and State Functions Approximations.

	TEST SERIES								
FUNCTION	1	2	3	4	5	6	7	MEAN	COEFFICIENT OF VARIATION
L P,S (S=0)	0.8596	0.9783	0.9745	0.8160	0.8904	0.5021	0.8894	0.8443	0.1917
L P,S (S#0) (*)	0.9717	0.9783	0.9822	0.8612	0.9312	0.6201	0.8894	0.8906	0.1437
L LogP,S(S=0)	0.6508	0.9376	0.5895	0.9370	0.8768	0.7109	0.8450	0.7925	0.1783
L LogP,S(S#0) (*)	0.6537	0.9376	0.5833	0.9443	0.9255	0.8345	0.8450	0.8177	0.1764
L P,LogS	0.9720	0.9738	0.9838	0.8614	0.9344	0.6107	0.8998	0.8908	0.1474
L LogP,LogS	0.6548	0.9401	0.5849	0.9452	0.9374	0.8295	0.8485	0.8201	0.1777
NL P,S (S=0)	0.9047	0.9860	0.9877	0.8191	0.9063	0.6190	0.9490	0.8817	0.1469
NL P,S (S#0) (*)	0.9863	0.9860	0.9875	0.8613	0.9337	0.7333	0.9490	0.9196	0.1018
NL LgP,S(S=0)	0.7302	0.9377	0.5993	0.9817	0.9649	0.8275	0.9090	0.8500	0.1660
NL LgP,S(S#0) (*)	0.6939	0.9377	0.5871	0.9732	0.9452	0.9484	0.9090	0.8564	0.1773
NL P,Log S	0.9878	0.9782	0.9905	0.8614	0.9352	0.7404	0.9628	0.9223	0.0997
NL LogP,LogS	0.6966	0.9419	0.5898	0.9753	0.9578	0.9501	0.9191	0.8615	0.1779

(*) Points lying in the plane Pa-Pw=0 are not considered

TABLE VII Regression Coefficients between Measured and Predicted Values of Degree of Saturation for Different Test Series and State Function Approximation.

	TEST SERIES								
FUNCTION	1	2	3	4	5	6	7	MEAN	COEFFICIENT OF VARIATION
L P,S (S=0)	0.8520	0.9854	0.7827	0.7426	0.9441	0.9619	0.8698	0.8769	0.1052
L P,S (S#0) (*)	0.8972	0.9854	0.8339	0.9514	0.9637	0.9617	0.8698	0.9233	0.0614
L LogP,S(S=0)	0.8538	0.9811	0.7649	0.7460	0.9450	0.9745	0.8722	0.8768	0.1095
L LogP,S(S#0) (*)	0.8740	0.9811	0.7274	0.8955	0.8900	0.9804	0.8722	0.8887	0.0958
L P,LogS	0.9077	0.9842	0.9418	0.9691	0.9397	0.8897	0.9755	0.9439	0.0375
L LogP,LogS	0.8977	0.9729	0.8353	0.9174	0.9505	0.9786	0.9753	0.9325	0.0567
NL P,S (S=0)	0.8533	0.9867	0.7854	0.7434	0.9461	0.9619	0.8742	0.8787	0.1047
NL P,S (S#0) (*)	0.9009	0.9867	0.8361	0.9572	0.9645	0.9769	0.8742	0.9281	0.0624
NL LgP,S(S=0)	0.8560	0.9836	0.7661	0.7461	0.9458	0.9755	0.8752	0.8783	0.1096
NL LgP,S(S#0) (*)	0.8741	0.9836	0.7295	0.9230	0.8957	0.8414	0.8752	0.8746	0.0896
NL P,Log S	0.9087	0.9853	0.9442	0.9737	0.9519	0.7811	0.9777	0.9318	0.0765
NL LogP,LogS	0.9011	0.9734	0.8378	0.9424	0.9553	0.9793	0.9769	0.9380	0.0554
NL P,ThS(S=0)	0.9360	0.9904	0.9949	0.9956	0.9600	0.9684	0.9890	0.9763	0.0230
NL P,ExS(S=0)	0.9516	0.9910	0.9938	0.9962	0.9594	0.9684	0.9893	0.9785	0.0187
NL P,ThS(S#0) (*)	0.9057	0.9904	0.9784	0.9554	0.9675	0.9634	0.9890	0.9643	0.0300
NL P,ExS(S#0) (*)	0.9090	0.9910	0.9840	0.9611	0.9651	0.9633	0.9893	0.9661	0.0292

(*) Points lying in the plane Pa-Pw=0 are not considered

More marked differences between linear and non-linear approximations have been found for the degree of saturation. The best fits are obtained for models NL P, Th S and NL P, Ex S (see Table III). It is also observed that the best fit is associated with the smallest dispersion error. This tendency is reflected in Tables IV and V.

CONCLUSIONS

Based on the analysis performed on a number of confined and isotropic compression tests of two types of partially saturated soils as well as other sets of results available in the literature, the following conclusions are advanced:

1. For a limited variation range of total external stress a suitable analytical expression for the state surface of void ratio is

$$e = a + b(\sigma - p_a) + c \log (p_a - p_\omega) + d(\sigma - p_a)\log(p_a - p_\omega)$$

2. If the range of significant stress variation is larger a more suited state function for void ratio is given by

$$e = a + b \log(\sigma - p_a) + c \log (p_a - p_\omega) + d \log (\sigma - p_a) \cdot \log(p_a - p_\omega)$$

3. Regarding the state surface for degree of saturation, excellent results are obtained with anyone of the following two models

$$S_r = a - \mathrm{Th}\,[b(p_a - p_\omega)]\,[c + d(\sigma - p_a)]$$

$$S_r = a - \{1 - \exp[-b(p_a - p_\omega)]\}\,[c + d(\sigma - p_a)]$$

In this case a significant part of the deviations observed is probably explained by the experimental difficulty is measuring very small water volume changes.

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REFERENCES

Aitchison, G.D. (1965). Discussion in Proc. 6th. Int. Conf. Soil Mech. and Found. Eng. Vol.3 318 - 321, Montreal.

Aitchison, G.D. and Bishop, A.W. (1960). Discussion in "Pore pressure and suction in soils", 150. London: Butterworths.

Aitchison, G.D. and Woodburn, J.A. (1969). Soil suction foundation design. Proc. 7th Int. Conf. on Soil Mech. and Found. Eng. Vol 2, 1-8. Mexico.

Barden, L., Madedor, A.O. and Sides, G.R. (1969). Volume change characteristics of unsaturated clays. J. Soil Mech. Fnds. Div. A.S.C.E. 95, SM1, 33-51.

Bishop, A.W. (1959). The principle of effective stress. Teknik Ukeblad. 39. 859-863.

Bishop, A.W. and Blight, G.E. (1963). Some aspects of effective stress in saturated and unsaturated soils. Geotechnique 13, N°3, 177-197.

Bishop, A.W. and Donald, I.B. (1961). The experimental study of partly saturated soil in the triaxial apparatus. Proc. 5th Int. Conf. Soil Mech. and Found. Eng. Vol. 1, 13-21, Paris.

Blight, G.E. (1965). A study of effective stresses for volume change, in Moisture equilibria and moisture changes in soils beneath covered areas, 259-269. Sidney:Butterworths.

Burland, J.B. (1965). Some aspects of mechanical behaviour of partly saturated soils, in Moisture equilibria and moisture changes in soil beneath covered areas, 270-278, Sidney: Butterworths.

Casteleiro, M. (1975). Mathematical model of on-edimensional consolidation and desiccation of dreged materials. Ph. D. Thesis. Northwestern University.

Coleman, J.D. (1962). Stress strain relations for partly saturated soils. Correspondence. Geotechnique 12, N°4, 348-350.

Escario, V. (1967). Measurement of the swelling characteristics of a soil fed with water under tension. International Co-operative Research on the Prediction of Moisture Content under Road Pavements; Working Group under the auspices of OCDE Madrid Meeting.

Fredlund, D.G. (1979). Second Canadian Geotechnical Colloquium: Appropiate concepts and technology for unsaturated soils. Can. Geotech. J. 16, N°1, 121-139.

Fredlund, D.G. and Morgenstern, N.R. (1976). Constitutive relations for volume change in unsaturated soils. Can. Geotech. J.13, N°3, 261-276.

Fredlund, D.G. and Morgenstern, N.R. (1977). Stress state variables for unsaturated soils. J. Geotech. Div. A.S.C.E. 103, GT5. 447-466.

Jennings, J.E.B. and Burland, J.B. (1962). Limitations to the use of effective stress in partly saturated soils. Geotechnique 12, N°2, 125-144.

Lloret, A. (1982). Comportamiento deformacional del suelo no saturado bajo condiciones drenadas y no drenadas. D. Thesis. Universidad Politécnica de Barcelona.

Lloret, A. and E.E. Alonso (1980). Consolidation of unsaturated soils including swelling and collapse behaviour. Geotechnique 30, N°4, 449-477.

Matyas, E.L. and Radhakrishna, H.S. (1968). Volume Change Characteristics of partially saturated soils. Geotechnique, 18. N°4, 432-448.

Moore, P.J. and Millar, P.V. (1981). The collapse of sands upon saturation. Proc. 1st Australia-New Zealand Conf. on Geomechanics 54-60. Melbourne.