A theory of consolidation with secondary compression

Une théorie de consolidation avec compression secondaire

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SYNOPSIS The possibility of an increase in the coefficient of earth pressure at rest, K_o, associated with the phenomenon of secondary compression, is discussed. A theory for the one-dimensional consolidation including secondary compression is also presented, in which the degree of consolidation, U, is a function of the relationship between final, σ'_{vf}, and initial, σ'_{vo}, vertical effective stresses, the Time Factor, T_v, of K_o and a parameter that controls secondary compression, θ.

INTRODUCTION

Among the many aspects not covered by Terzaghi and Frölich's theory maybe the most important one is that of secondary compression. Several authors have bypassed the problem by using rheological models, as can be seen in Gibson and Lo (1961), Wahls (1962) and Barden (1969). If on the one hand rheological models are helpful by providing a practical approach to engineering problems, on the other hand they do not unveil what's really behind the physical phenomenon itself (see, for example, Mitchell et al, 1968).

Recently, in reply to a question proposed by Schmertmann (1983), Kavanzajian and Mitchell (1984) presented a reasoning which, together with experimental evidence, shows that secondary compression is intimately linked with the increase of K_o with time, a possibility that Lacerda (1977) has shown to exist.

SIMPLIFIED MECHANISTIC VIEW OF SECONDARY COMPRESSION

According to Kavanzajian and Mitchell (1984), a soil element will undergo deformations as long as there are shear stresses between soil particles. The rupture of bonds will continue until the complete cessation of shear stresses. In the particular case of the oedometric test, at the end of primary consolidation and thereafter, the effective vertical stress, σ'_v, remains constant. If shear stresses tend to dissipate, this will have to happen due exclusively to the increase of the horizontal effective stress, σ'_h, which means that the coefficient of earth pressure at rest must increase. Upon reaching a value of $K_o = 1$, the shear stress will vanish, and the deformations will cease.

There are other arguments (Lacerda and Martins, 1984) that can be presented in favor of the increase K_o hypothesis, but it is important to point out that this increase is due to the dissipation of shear stresses, a phenomenon known as "stress relaxation". Thus, it might be hypothetized that the rate of dissipation of shear stresses, such as the deviator stress $(\sigma_1 - \sigma_3)$, is proportional to it's instantaneous value. According to this hypothesis it may be expected that the deviator stress decreases gradually, simultaneously with its rate of dissipation, until both are equal to zero. Then deformations will stop, which will occur, according to this mechanism, at an infinite time.

The two-dimensional figure presented by Bjerrum in 1972 to illustrate the phenomenon of secondary compression may then be expanded with the addition of a third axis, labeled K_o, as shown in figure 1.

At this point two important remarks must be made. The first is that if the phenomenon of secondary consolidation is due to the existence of shear stresses it must also occur during the primary stage. The second is that, with this approach, secondary consolidation is also viewed within the framework of the principle of effective stresses.

THEORY OF ONE-DIMENSIONAL CONSOLIDATION INCLUDING SECONDARY COMPRESSION

Let

$$p' = \frac{\sigma'_1 + \sigma'_2 + \sigma'_3}{3} \tag{1}$$

$$q = \frac{\sigma_1 - \sigma_3}{2} \tag{2}$$

In the case of oedometric consolidation, $\sigma'_1 = \sigma'_v$ and $\sigma'_3 = \sigma'_h$. By definition,

$$K_o = \sigma'_h / \sigma'_v \tag{3}$$

which allows

$$p' = \frac{\sigma'_v (1 + 2 K_o)}{3} \tag{4}$$

$$q = \frac{\sigma'_v (1 - K_o)}{2} \quad (5).$$

The hypothesis that are admitted in the following development are all those of Terzaghi and Frölich's theory (Taylor, 1948, p. 226) and:

- The soil is considered isotropic
- There is a relaxation of shear stresses with time. This relaxation is such that the rate of variation of the deviator stress with time is proportional to the instantaneous value of the deviator stress:

$$\frac{dq}{dt} = - \lambda q \quad (6)$$

Where λ is a coefficient that depends on soil type and that controls the velocity of the secondary compression.

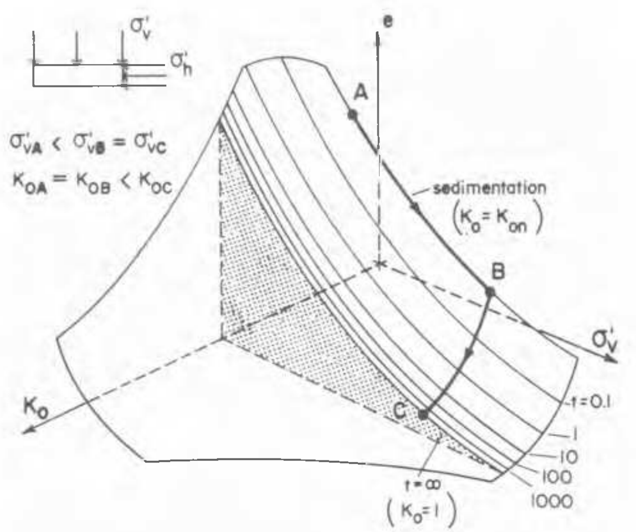

Fig.1. Relationship between void ratio (e), vertical effective stress (σ'_v) and coefficient of earth pressure at rest (K_o)

According to the principle of effective stresses, to each variation of volume there is a coupled variation of effective stresses. Then,

$$\varepsilon_v = \Delta p'/M' \quad (7).$$

Where the symbol Δ stands for variation, ε_v = volumetric strain, and M' = bulk modulus. In an oedometric test p' varies with time. Equation (4) then yields:

$$\frac{dp'}{dt} = \frac{\partial p'}{\partial \sigma'_v} \frac{d\sigma'_v}{dt} + \frac{\partial p'}{\partial K_o} \frac{dK_o}{dt} \quad (8).$$

Using equation (5) and taking into account that during consolidation $\sigma'_v = \sigma'_v(t)$ and $K_o = K_o(t)$, it may be written that

$$q\ (t) = \frac{\sigma'_v(t) \left[1 - K_o(t)\right]}{2} \quad (9).$$

The variation of q with time is given by

$$\frac{dq}{dt} = \frac{\partial q}{\partial \sigma'_v} \frac{d\sigma'_v}{dt} + \frac{\partial q}{\partial K_o} \frac{dK_o}{dt} \quad (10a)$$

which gives

$$\frac{dq}{dt} = \frac{1}{2}\left\{\left[1 - K_o(t) \cdot \frac{d\sigma'_v}{dt} - \sigma'_v(t) \cdot \frac{dK_o}{dt}\right]\right\} \quad (10b).$$

Now, writing Eq. (6) = Eq. (10b), one gets:

$$\frac{1}{\sigma'_v} \cdot \frac{d\sigma'_v}{dt} + \lambda = \frac{1}{1 - K_o} \cdot \frac{dK_o}{dt} \quad (11)$$

which yields

$$\frac{d(ln\ \sigma'_v)}{dt} + \lambda = - \frac{d\left[ln(1 - K_o)\right]}{dt} \quad (12)$$

which, integrating, gives:

$$-\left\{ln\left[1 - K_o(t)\right] - ln\left[1 - K_o(0)\right]\right\} =$$

$$= ln\ \sigma'_v(t) - ln\ \sigma'_v(0) + \lambda t \quad (13).$$

Calling $K_o(0) = K_{on}$ and $\sigma'_v(0) = \sigma'_{vo}$, Eq. (13) can be written as

$$\frac{\sigma'_v(t) \left[1 - K_o(t)\right]}{\sigma'_{vo} \left[1 - K_{on}\right]} = \exp(-\lambda t) \quad (14).$$

Then, at $t = \infty$, $K_o = 1$.

In order to solve Eq. (8) it is necessary to determine dK_o/dt, which can be done through Eq. (14). Then,

$$\frac{dK_o}{dt} = \lambda\left[1 - K_o(t)\right] + \frac{\left[1 - K_o(t)\right]}{\sigma'_v(t)} \cdot \frac{d\sigma'_v}{dt} \quad (15).$$

Equation (8) can be rewritten as

$$\frac{dp'}{dt} = \frac{1}{3}\left[(1 + 2K_o) \frac{d\sigma'_v}{dt} + 2\sigma'_v \cdot \frac{dK_o}{dt}\right] \quad (16).$$

Substituting Eq. (15) into Eq. (16),

$$\frac{dp'}{dt} = \frac{d\sigma'_v}{dt} + \frac{2}{3} \cdot \lambda \cdot \sigma'_{vo}(1 - K_{on}) \exp(-\lambda t) \quad (17)$$

which, by integration, yields:

$$p'(t) - p'(0) = \sigma'_v(t) - \sigma'_v(0) + \frac{2}{3} \sigma'_{vo}(1 - K_{on})\left[1 - \exp(-\lambda t)\right] \quad (18)$$

going back to Eq. (7) the volumetric strain can now be expressed as a function of time:

$$\varepsilon_v(t) = \frac{\sigma'_v(t) - \sigma'_{vo}}{M'} + \frac{2}{3} \frac{\sigma'_{vo}}{M'} \cdot (1 - K_{on})\left[1 - \exp(-\lambda t)\right] \quad (19).$$

Equation (19) has two components of volumetric deformation. The first one is associated with primary consolidation, $\varepsilon_{vp}(t)$:

$$\varepsilon_{vp}(t) = \left[\sigma'_v(t) - \sigma'_{vo}\right]/M' \qquad (20)$$

which can be calculated by Terzaghi and Frölich's theory, and can be written as (see, for example, Taylor (1948), pp. 208-249):

$$\varepsilon_{vp}(t) = \frac{\sigma'_{vf} - \sigma'_{vo}}{M'}\left\{1 - \sum_{n=0}^{\infty} \frac{4}{(2n+1)\pi}\left[\sin(2n+1)\frac{\pi}{2}\cdot\frac{z}{H}\right]\cdot \exp\left(-\pi^2 (2n-1)^2 \cdot T_v/4\right)\right\} \qquad (21)$$

where (see fig. 2)

σ'_{vf} = final effective vertical stress

z = distance to the top of the consolidating layer

$2H$ = thickness of the consolidating layer

T_v = Time Factor = $c_v \cdot t/H^2$ (22)

c_v = coefficient of consolidation of Terzaghi and Frölich's theory.

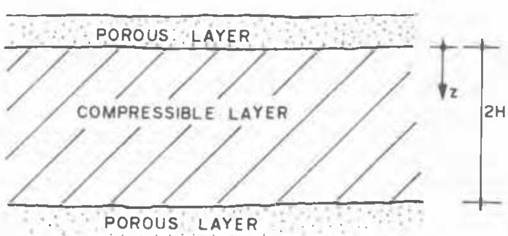

Fig.2. Consolidating layer

The second component, which does not depend on z, is associated with the so called secondary compression, and can be written as

$$\varepsilon_{vs}(t) = \frac{2}{3}\cdot\frac{\sigma'_{vo}}{M'}(1 - K_{on})\left[1 - \exp(-\lambda t)\right] \qquad (23).$$

The unfolding of Eq. (19) can be better understood by observing Figure 3.

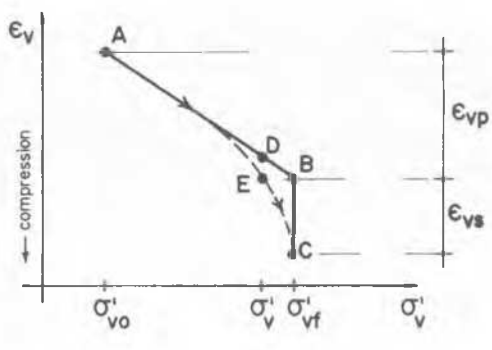

Fig.3. Components of volumetric strain

In reality both components act simultaneously, and the path, followed by a soil element is given by AEC and not ABC in that figure.

The total volumetric strain due to primary consolidation, ε_{vp}, and that due to secondary compression, ε_{vs}, are represented in Figure 3, and are given, respectively, by

$$\varepsilon_{vp} = (\sigma'_{vf} - \sigma'_{vo})/M' \qquad (24)$$

and

$$\varepsilon_{vs} = \frac{2}{3}\cdot\frac{\sigma'_{vo}}{M'}(1 - K_{on}) \qquad (25)$$

at any given time, t, of the process, represented by point E on Figure 3, the degree of primary consolidation, U_{zp} can be defined as

$$U_{zp} = \frac{\overline{AD}}{\overline{AB}} = \frac{\sigma'_v(t) - \sigma'_{vo}}{\sigma'_{vf} - \sigma'_{vo}} = \frac{\varepsilon_{vp}(t)}{\varepsilon_{vp}} \qquad (26).$$

In a similar way, the degree of secondary consolidation U_{zs} will be:

$$U_{zs} = \frac{\overline{DE}}{\overline{BC}} = \frac{\varepsilon_{vs}(t)}{\varepsilon_{vs}} \qquad (27)$$

which, according to Eqns. (23) and (25), is given by

$$U_{zs} = 1 - \exp(-\lambda t) \qquad (27b)$$

The global degree of consolidation, U_z, is defined as

$$U_z = \varepsilon_{vp}(t) + \varepsilon_{vs}(t) \qquad (28)$$

or, yet:

$$U_z(z,t) = \frac{(\sigma'_{vf} - \sigma'_{vo})\left[1 - \sum_{n=0}^{\infty} \cdot \frac{2}{N}(\sin\frac{Nz}{H}) \cdot \exp \cdot (-N^2 T_v)\right]}{\sigma'_{vf} - \sigma'_{vo} + \frac{2}{3}\sigma'_{vo}(1 - K_{on})} + \frac{\frac{2}{3}\sigma'_{vo}(1 - K_{on})\left[1 - \exp(-\lambda t)\right]}{\sigma'_{vf} - \sigma'_{vo} + \frac{2}{3}\sigma'_{vo}(1 - K_{on})} \qquad (29),$$

where $N = \frac{\pi}{2}(2n + 1)$.

The average degree of consolidation will be given by:

$$U = \frac{\int_o^{2H} \varepsilon_v(z,t) \cdot dz}{\int_o^{2H} \varepsilon_v(z,\infty) \cdot dz} \qquad (30)$$

Substituting into Eq. (30) the expressions for $\varepsilon_v(z,t)$ and $\varepsilon_v(z,\infty)$ for the particular case where $(\sigma'_{vf} - \sigma'_{vo})$ is constant along the thickness of the consolidating layer and integrating,

$$U = \frac{(\sigma'_{vf} - \sigma'_{vo})\left[1 - \sum_{n=0}^{\infty} \frac{2}{N^2} \cdot \exp\ (-N^2 \cdot T_v)\right]}{\sigma'_{vf} - \sigma'_{vo} + \frac{2}{3}\sigma'_{vo}\ (1 - K_{on})} +$$

$$+ \frac{\frac{2}{3} \cdot \sigma'_{vo}(1 - K_{on})\left[1 - \exp.\ (-\lambda t)\right]}{\sigma'_{vf} - \sigma'_{vo} + \frac{2}{3}\sigma'_{vo}\ (1 - K_{on})} \qquad (31).$$

Remembering that $T_V = c_V \cdot t/H^2$, let $\lambda t = \Theta T_V$, and substituting into Eq. (31) and dividing by σ'_{vo} throughout, finally:

$$U = \frac{(\frac{\sigma'_{vf}}{\sigma'_{vo}} - 1)\left[1 - \sum_{n=0}^{\infty} \frac{2}{N^2} \cdot \exp\ (-N^2 \cdot T_v)\right]}{(\frac{\sigma'_{vf}}{\sigma'_{vo}} - 1) + \frac{2}{3}\ (1 - K_{on})} +$$

$$+ \frac{\frac{2}{3}\ (1 - K_{on})\left[1 - \exp\ (-\Theta T_v)\right]}{(\frac{\sigma'_{vf}}{\sigma'_{vo}} - 1) + \frac{2}{3}\ (1 - K_{on})} \qquad (32)$$

It can be observed that the secondary compression contribution to the total deformation decreases as the ratio $(\sigma'_{vf}/\sigma'_{vo})$ increases and the nearer K_{on} is of unity. In Figure 4 curves of $U \times T_V$ are shown for four values of $(\sigma'_{vf}/\sigma'_{vo})$, for a given pair of values of Θ and K_{on}. In Figure 5, curves of $U \times T_V$ are shown for varying values of Θ, maintaining K_{on} and $(\sigma'_{vf}/\sigma'_{vo})$ constant.

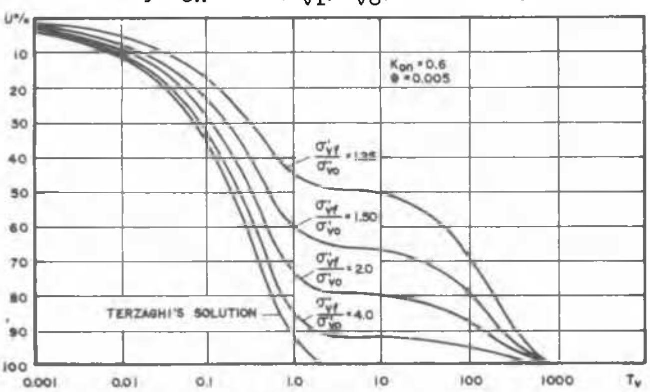

Fig.4. $U \times T_V$, Θ = constant

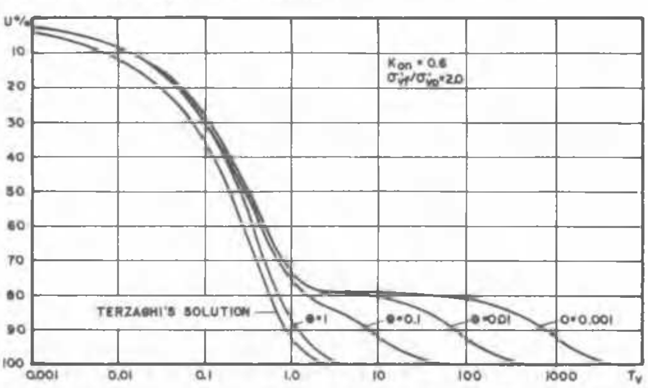

Fig.5. $U \times T_V$, $\sigma'_{vf}/\sigma'_{vo}$ = constant

CONCLUSION

Although more well documented experimental evidence is necessary, it seems, from the preceding reasoning, based on physical behavior, that the mathematical model developed explains available experimental data (see for example Felix, 1981) quite well.

An experimental research program is now under way at COPPE to fully explore the potential of this theory.

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