Stress analysis for layer system

Etude des contraintes dans un système bicouche

D. M. MILOVIĆ, Professor, Faculty of Technical Sciences, Novi Sad, Scientific Counsellor "Kosovoprojekt", Beograd, Yugoslavia

SYNOPSIS Conventional stress distribution theories for a homogeneous, elastic and isotropic half-space give excessively high stresses if applied to the two-layer soil system. In this study the problem of the determination of stresses and displacements, produced by strip foundation resting on the two layer soil system has been treated by the finite element method and by Fourier' s integrals. The thickness of the upper layer was H1 = 0,5 m; 1,0 m; 1,5 m; 2,0 m; and 2,5 m. For each thickness H1 the stress distribution was calculated for the ratio E1/E 2 = 5,0; 10;0 and 25, where E1 and E 2 are the deformation moduli of the upper and lower layer, respectively. Also, the stress distribution was calculated under the assumption that the upper layer had anisotropic properties. On the basis of the obtained results it was possible to analyze the effect of superposition or strip foundations.

INTRODUCTION

A rational analysis of the settlement and bearing capacity of a noncohesive fill over soft subsoil consists in the prediction of the stresses transmitted by the foundation system throught the fill into the underlying subsoil.

Theoretical analyses and field measurements indicate that the stresses and displacements in layered soil systems, where there is large difference in the properties of the layers, are considerably different from those developed in a homogeneous system. Elastic solutions for layered systems are based on the work of Burmister (1943). So, Fox (1948) used Burmister' s method to calculate vertical stresses at any point in the vertical axis of a two-layer system, arising from a uniformly distributed loading over a circular area at the top surface. Similar results have been obtained by Hank and Scrivener (1948) for two and three layered system.

Mitchell and Gardner (1971) have shown the solution of the load bearing fill problem, developed by application of the finite element method for axisymmetric layered soils and the characterization of nonlinear and stress-dependent material properties by means of hiperbolic stress-strain functions.

Egorov (1938) extended Marguerre' s solution to the determination of the verical stress between the layers under the center of a strip loading. Lemcoe (1961) has presented a solution for a two layer system which satisfies the conditions of plane strain, assuming that layer is elastic and isotropic.

Sundara and Alwar (1964) have given a general solution for the elastic stress distribution in a layered half-plane. The solution has been developed using Fourier intergrals and the results are shown for one thickness only of the upper layer.

In this Paper, the Author has calculated the stresses in an elastic two layer system, produced by uniformly distributed load over a strip foundation. The upper layer is assumed to be alternatively isotropic and anisotropic. The vertical stresses at any point under the loaded area and outside the loaded area have been calculated for the width of the strip foundation B =1,0 m, for the thickness of the upper layer H1= =0,50 m, 1,0 m; 1,5 m; 2,0 m; and 2,5 m. For each thickness H1 the stress distribution was calculated for the ratio E1/E2 = 5;10 and 25, where E1 and E2 are the deformation moduli for the upper and lower layer, respectively. The solutions have been obtained by the finite element method. The mesh of elements used in this study and the geometry of the problem is shown in Fig.1.

As shown inf Fig.1, a considered medium was divided in 483 elements.

STRESS ANALYSIS BY FINITE ELEMENT METHOD

In the case of two dimensional problem, assuming that the foundation soil is elastic and isotropic, the stress strain relationship can be expressed by:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xz} \end{Bmatrix} = \begin{vmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{11} & 0 \\ 0 & 0 & A_{55} \end{vmatrix} \begin{Vmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xz} \end{Vmatrix} \quad \text{....(1)}$$

where:

$$A_{11}=\frac{E(1-\mu)}{(1+\mu)(1-2\mu)}$$

$$A_{12}=\frac{E\mu}{(1+\mu)(1-2\mu)}$$

$$A_{55}=G=\frac{E}{2(1+\mu)}$$

with the stiffness matrix:

$$De=\frac{E}{(1+\mu)(1-2\mu)}\begin{vmatrix}1-\mu & \mu & 0\\ \mu & 1-\mu & 0\\ 0 & 0 & \frac{1-2\mu}{2}\end{vmatrix} \qquad ...(2)$$

In the case of the anisotropic soil, the coefficients A are given by (Zienkiewicz and Cheung, 1967):

$$A_{11}=\frac{E_v K(1-K\mu_v^2)}{(1+\mu_h)(1-\mu_h-2K\mu_v^2)}$$

$$A_{13}=\frac{E_v K\mu_v}{1-\mu_h-2K\mu_v^2}$$

$$A_{33}=\frac{E_v(1-\mu_h)}{1-\mu_h-2K\mu_v^2}\ ; \quad A_{ss}=mE_v$$

and the stiffness matrix:

$$De=\frac{E_v}{(1+\mu_h)(1-\mu_h-2K\mu_v^2)}\ *$$

$$*\begin{vmatrix}K(1-K\mu_v^2) & K\mu_v(1+\mu_h) & 0\\ K\mu_v(1+\mu_h) & 1-\mu_h^2 & 0\\ 0 & 0 & m(1+\mu_h)(1-\mu_h-2K\mu_v^2)\end{vmatrix} \qquad ....(3)$$

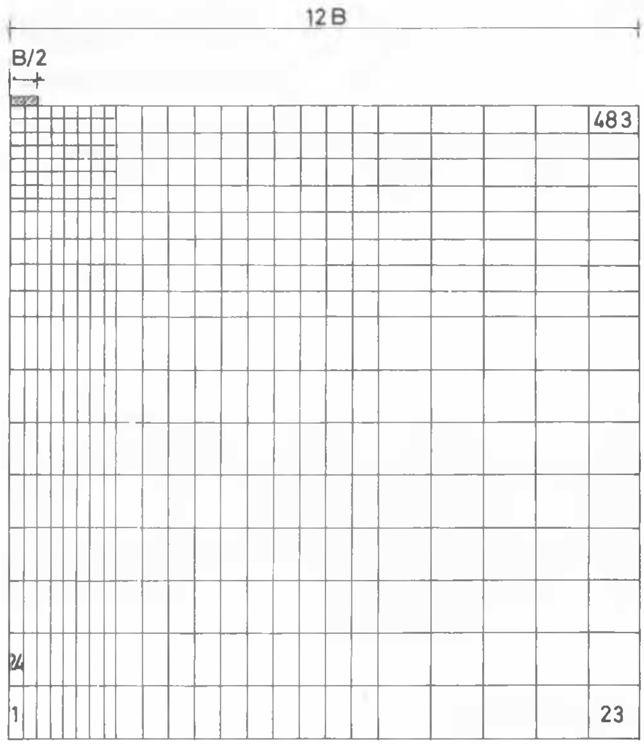

Fig.1 Finite element mesh

with:

$$K=\frac{E_h}{E_v}\ ; \quad m=\frac{G_v}{E_v}\ ; \quad n=\frac{E_v}{E_h}$$

where E_h and E_v are the moduli of elasticity in the horizontal and vertical direction, respectively.

The verical stresses in the middle of the strip $\sigma_z = \sigma_o I_z$, where σ_o is the conctact pressure beneath the foundation, have been calculated for the two layer system with the modulus of elasticity of the upper layer $E_1 = 50$ MN/m2 and the modulus of elasticity of the lower layer $E_2 = 2,0$; $5,0$ and $10,0$ MN/m2. The obtained curves for I_z are shown in Figs. 2-5.

The curves of I_z, obtained for five values of the thickness of the upper layer H_1, are shown in Figs. 6-8.

On the basis of the obtained results, the following expression is suggested as a good approximation for the vertical stress increase $\Delta\sigma_z$, at the interface of the layers induced by applied stress σ_o:

$$\frac{\Delta\sigma_z}{\sigma_o}=\left(\frac{0.5B}{H_1}\right)^a\left(\frac{E_2}{E_1}\right)^b \qquad ....(4)$$

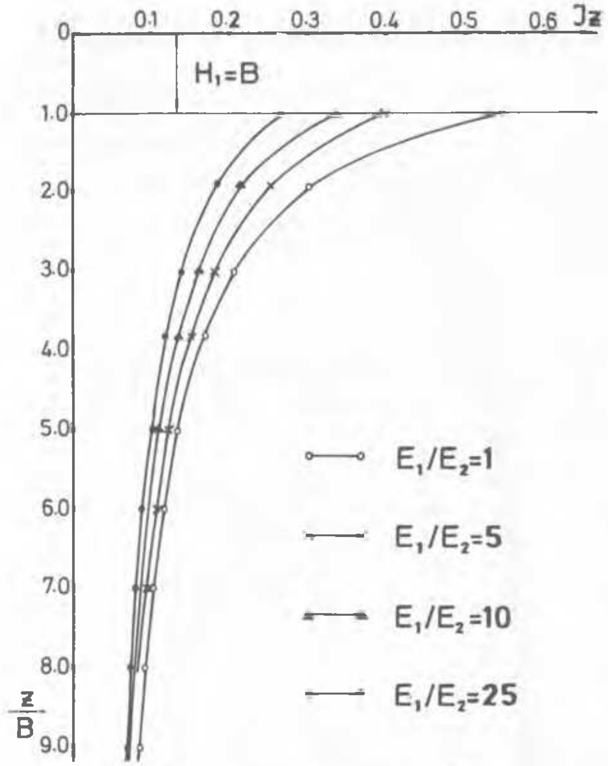

Fig. 2 Coefficients I_z ; fill H_1 = B

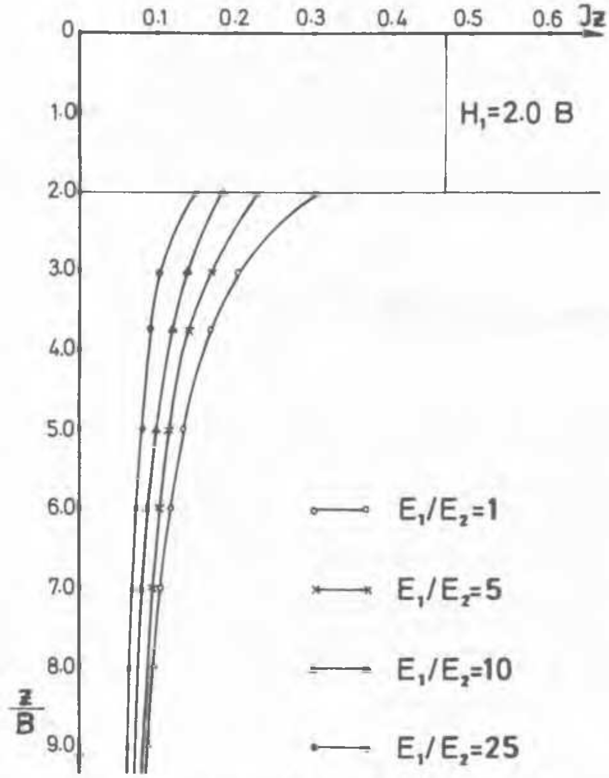

Fig. 4 Coefficients I_z ; fill H_1 = 2, 0 B

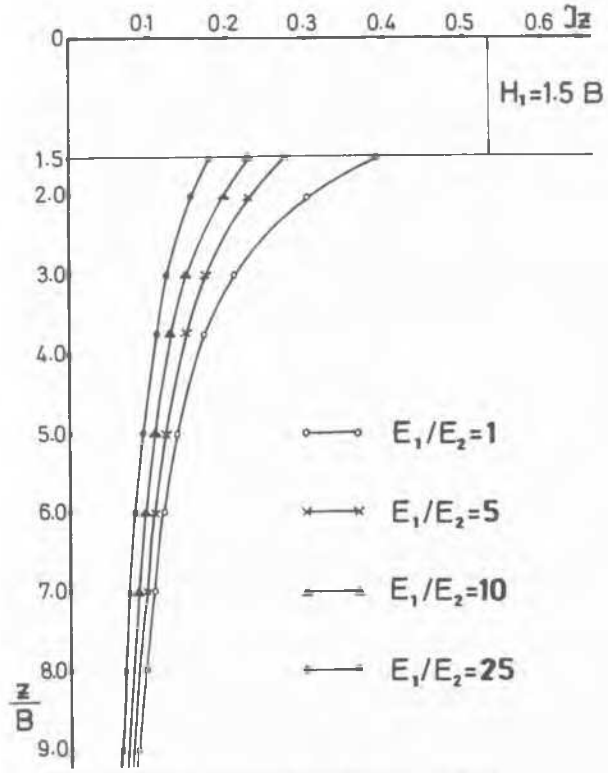

Fig. 3 Coefficients I_z; fill H_1 = 1, 5 B

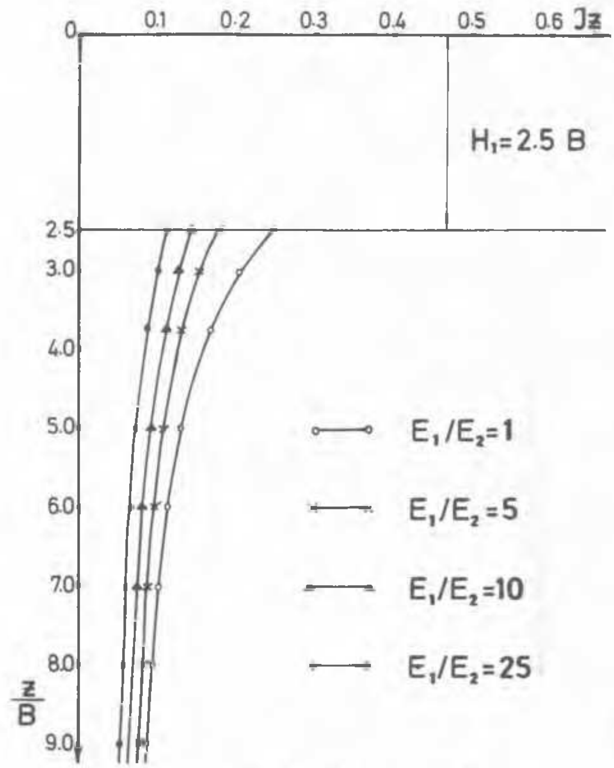

Fig. 5 Coefficient I_z; fill H_1 = 2, 5 B

where:

B = the width of the strip foundation
H_1 = the thickness of the upper layer
E_1, E_2 = the modulus values of the upper and lower layer, respectively.

So, for the strip foundation, the values of max $\Delta\sigma_z$ at the interface can be determined from:

$$\max\Delta\sigma_z = \sigma_o\left(\frac{0.5B}{H_1}\right)^{0.88}\left(\frac{E_2}{E_1}\right)^{0.20} \qquad \text{....(5)}$$

In Fig. 9. are shown the curves of max $\Delta\sigma_z$ for three values of the ratio E_1/E_2.

For comparison, the values of max $\Delta\sigma_z$ have been calculated for H_1 = 1,50 m by Lalaurie (1983) and for the other values of H_1 by the Author, using the Fourier's integrals (Sundara and Alwar, 1964).

On the basis of the results obtained in this study, it was possible to analyse also the effect of supperposition of stresses. For illustration, in Fig. 10 are shown the curves of vertical stresses σ_z for one of the considered cases.

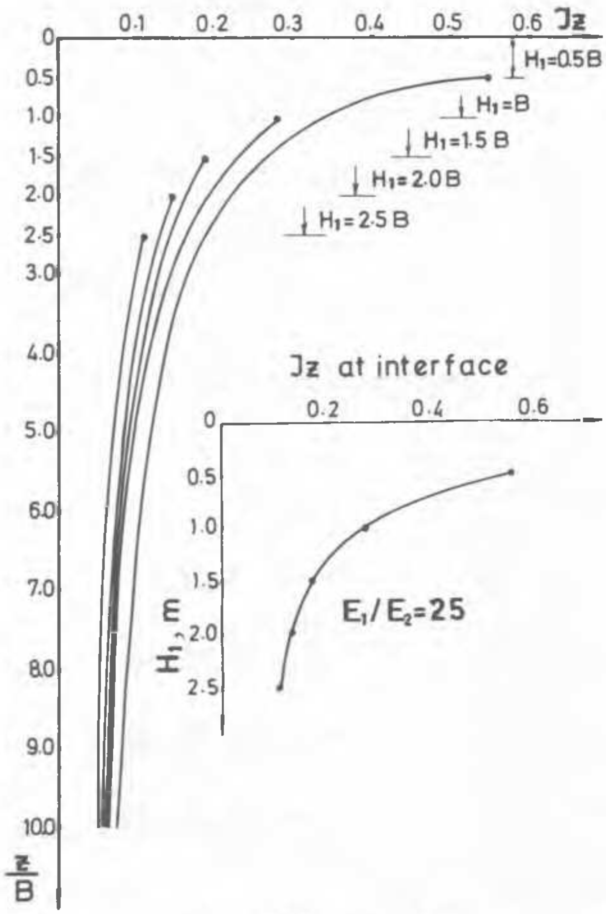

Fig. 6 Coefficients I_z

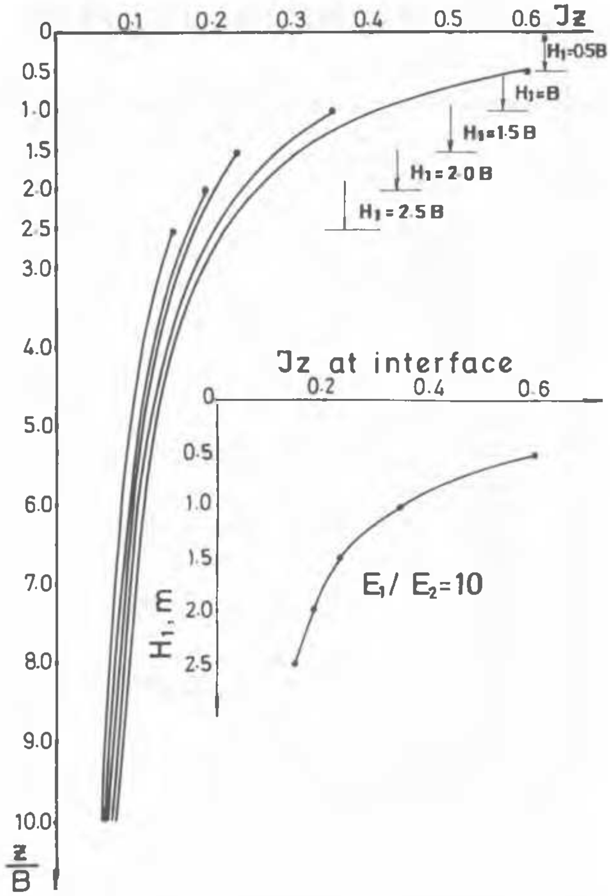

Fig. 7 Coefficients I_z

Curve 1 reffers to the σ_z distribution for the single strip, whereas curves 2 and 3 refer to the strip foundations with the stresses caused by supperposition. Curve 2 has been obtained for the value l_o = 5 B and curve 3 for the value l_o = 4 B.

The effect of the assumed anisotropic properties of the upper layer has been studied for two values of the degree of anisotropy n = 0,25 and 0,50. For illustration, some of the obtained results are shown in Fig. 11.

The curves of stress coefficients I_z, as shown in Fig. 11, indicate the decrease of stresses for the values of n used in this study, in comparison with the isotropic properties of the upper layer.

ANALYSES OF THE RESULTS

In the sudy of stresses and displacements in two layer system the simplest assumption has been made i.e. the soil has linear and elastic properties.

Mitchell and Gardner (1971) have shown some results of the stress distribution study, obrained by application of the finite element method but for axisymmetric layered solids and the sharacterization of nonlinear and stress-dependent material properties by means of hyperbolic stress-strain functions. According to the obtained results, stresses by linear two-layer elastic theory are greater, in general, than those determined by nonlinear soil properies.

Some neasurement have shown that simple elastic theory can be used to predict stresses in thick strata with a satisfactory accuracy even in the case of nonlinear and stress-dependent media properties (Huang, 1968), Some results of field load tests (Milović, 1979) shown in Figs.12 and 13, could be useful in these considerations.

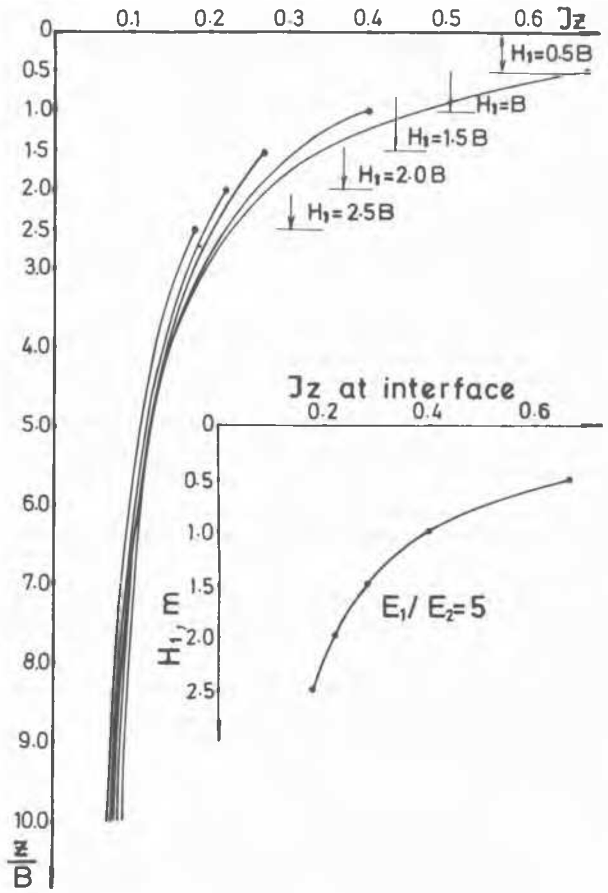

Fig. 8 Coefficinets I_z

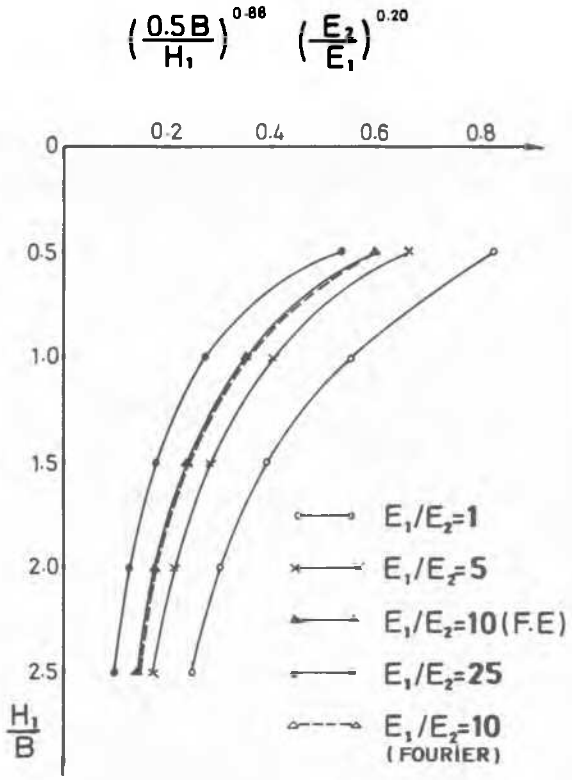

Fig. 9 Curves of max $\Delta\sigma_z$

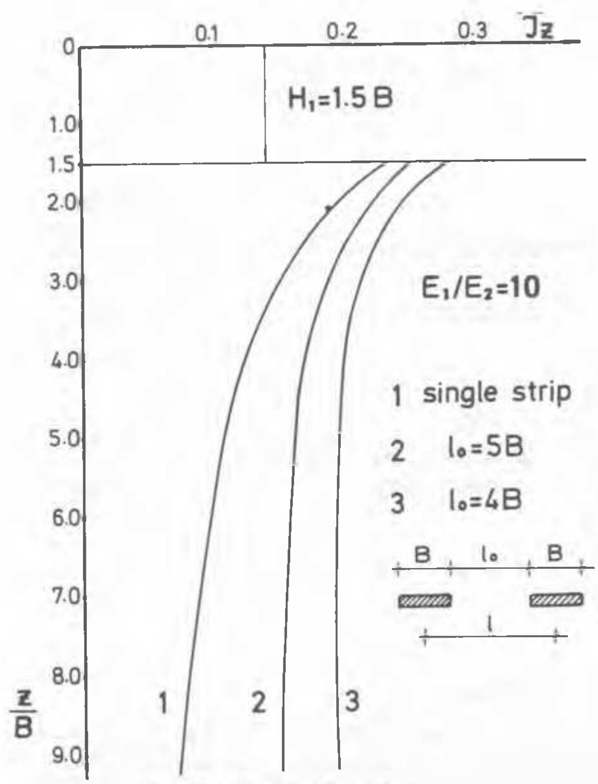

Fig.10 Coefficinets I_z

In Figs.12 and 13 are shown the load-setlement curves for circular footing of diameter D = 60 cm. Curve 1 in Fig.12 reffers to the homogeneous clay and curve 2 to the two-layer system, where the upper gravel layer is of thickness H_1 = 45 cm. Curve 1 in Fig.13 reffers to the thick sand layer and curves 2 and 3 to the two layer system, where the upper gravel layer is of Hl = 30 cm and Hl = 45 cm. Having in mind that in some cases stess-deformation relationship is almost linear, what is confirmed by presented load settlement curves, on may say that Huang's finding concerning the applicability of linear elastic theory in some cases is justified.

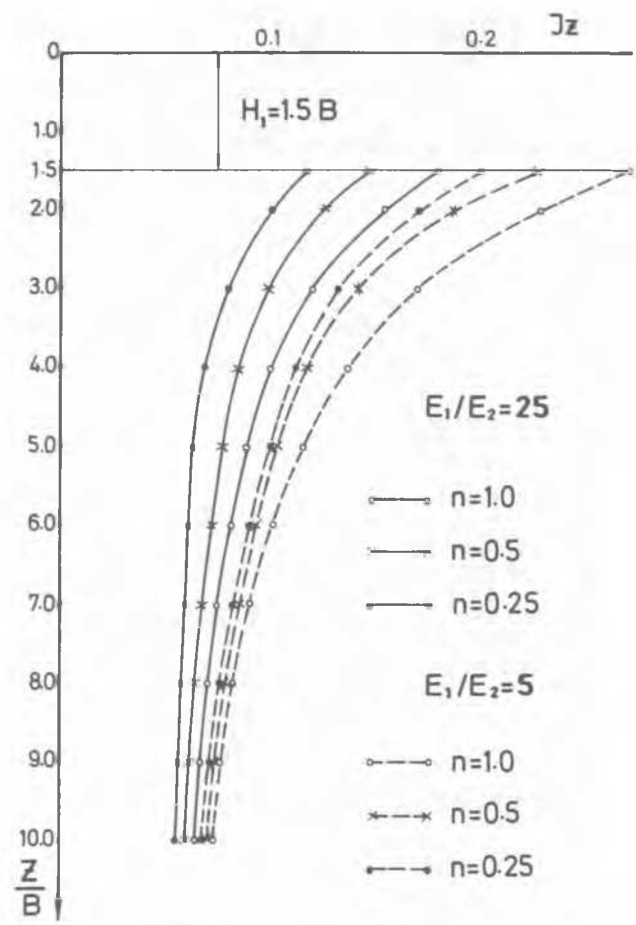

Fig. 11 Coefficients I_z

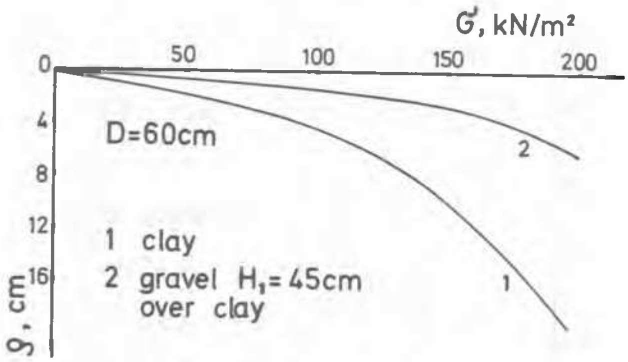

Fig. 12 Load Settlement curves

CONCLUSIONS

Conventional stress distribution theories for a homogeneous, elastic and isotropic half-space give excessively high stresses if applied to the two-layer soil system.

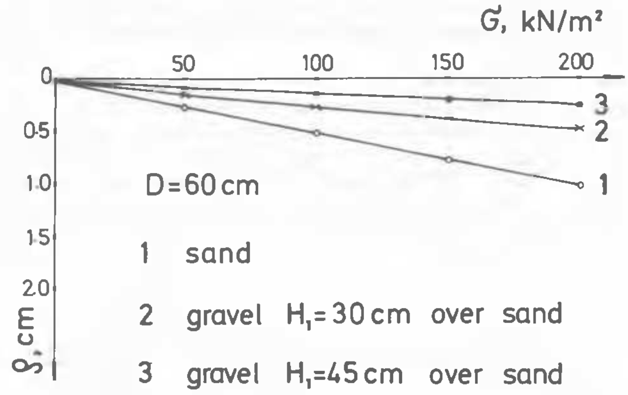

Fig. 13 Load Settlement curves

Stess distribution analyis in the two-layer soil system, where the upper layer is stronger that the lower one, shows a considerable decreas in stresses induced in the weak subsoil.

The stiffer and thicker the upper fill layer, relative to the weak subsoil, the greater the reduction in vertical stresses transmitted to the weaker layer.

The values of vertical stresses at interface in the two-layer system, obtained by the finite element method and the Fourier' s intergrals are practically identical.

Expression (5) can be used to provide approximate estimates of vertical stresses in weak soil underlying cohesionless fill.

Anisotropy of the upper layer can be taken into account. This property can reduce the vertical stresses induced in the weak subsoil.

The results obtained in this study make possible the complete evidence in the problem of the supperposition of strip foundations placed at any distance.

REFERENCES

Huang, J.H. (1968): "Stesses and displacements in non-linear soil media." Journal of the Soil Mechanics and Foundation Division, ASCE, Vol. 94, No SM1, pp.1-19.

Lalaurie, P. (1983): Personal communications. OTH, Paris.

Mitchell, J.K. and W.S. Gardner (1972): "Analyis of load-bearing fills over soft subsoils". Journal of the Soil Mech. and Found. Div. ASCE, Vol. 97, No SM 11.