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INTRODUCTION

It is a well established experimental fact that the mechanical behaviour of a sample of natural clay depends on its orientation with respect to the loading direction. As a consequence relevant engineering parameters, such as undrained strength, are different for samples with different inclinations (Lo (1965)). In view of classical analyses by means of the limit equilibrium method, various expressions have been proposed to account for the shear strength variation (Bishop (1966)), which have been successfully used in practice and still hold their validity.

We want however to exploit the possibilities offered by the finite element method, we need a constitutive law that describes the behaviour of clay more realistically. Recently, many models have been proposed. While the philosophy which governs unloading-reloading is different, a large class of models (e.g. Mróz et al (1978), Dafalias and Herrmann (1982), Nova and Hueckel (1980)) make use of the hypothesis that behaviour of normally consolidated clay may be described by means of an elastic plastic strain hardening constitutive law. In general, however, it is assumed for the sake of simplicity that virgin soil behaviour is isotropic.

The aim of this paper is to show that this latter simplification may be removed so that anisotropy of normally consolidated clay may be modelled. Clay anisotropy is often considered to be either inherent or induced by the straining process. The distinction between the two is not neat since at least part of the inherent anisotropy of a clay is due to the horizontal orientation of clay particles during deposition and subsequent consolidation of the clay deposit under its own weight, which is a strain controlled process. However, this distinction may be convenient as a first approach to this topic. Indeed since it is generally difficult to substantially alter inherent anisotropy, we may consider first a case in which the anisotropy of the sample is fixed and cannot be altered by the straining process. We will further consider the case of a clay which acquires its anisotropy when one dimensionally consolidated from a slurry, which will be considered as a fully isotropic state.

THE MODELLING OF INHERENT ANISOTROPY

Clay is considered to be an elastic plastic strainhardening material. In triaxial conditions, i.e. the symmetry condition usually assumed to hold in a triaxial test, a convenient expression for the yield function is that proposed by Burland (1967) (modified Cam Clay):

$$ f = q^2 + M^2 (p' - p_c) = 0 $$

where

$$ p' = \frac{2}{3} (\sigma_1^2 + 2 \sigma_2^2) $$

is the isotropic effective stress, while

$$ q = \sigma_1^2 - \sigma_3^2 $$

is proportional to the square root of the second invariant of the stress deviator. The parameter $p_c$ gives the value of the intercept of the ellipse given by Eq. (1) with the hydrostatic axis, and coincides with the maximum isotropic pressure experienced by a clay sample, isotropically consolidated from a slurry. $M$ is the stress ratio $q/p'$ at critical state, which for a normally consolidated isotropic clay coincides with failure either for drained or undrained loading.

A simple extension of Eq. (1) to multi-axial stress conditions has been given by Nova and Sacchi (1982)

$$ f = \frac{1}{k^2} \delta_{ij} \delta_{ij} + 3 (p' - k) - 3k^2 = 0 $$

The tensor $\delta_{ij}$ is the stress deviator, while $k = \frac{1}{2} M$ and $k = \frac{1}{9} p_c$. Eq. (4) may be conveniently written as

$$ f = z_{ij} z_{ij} - 3k^2 = 0 $$

where

$$ z_{ij} = \frac{1}{3} \delta_{ij} + (p' - k) \delta_{ij} $$

is a generalized stress tensor. Eq. (5) is valid for an isotropic material and is formally identical to a Von Mises yield condition, which was generalized to the case of orthotropic materials by Hill (1950). In the same spirit, it is possible to define an orthotropic quadruple tensor $C_{ijrs}$ such that

$$ f = C_{ijrs} z_{ij} z_{rs} - 3k^2 = 0 $$

is the expression of the yield function for an orthotropic clay. From Eq. (7) it is possible to derive $k$ as a function...
of stresses. However, \( k \) is a hidden variable which depends on the plastic strains previously experienced by the clay sample. It will be assumed, in the line of other clay models, that hardening depends on volumetric strains only, that is

\[
k = c \exp \left( \frac{1}{3} \Gamma_0 \frac{p}{k} \delta_{kh} \right)
\]  

(8)

where \( \Gamma_0 \) and \( \mu_0 \) are the elastic-plastic and elastic bulk logarithmic compressibility, respectively. \( \lambda \) and \( \mu_0 \) are conceptually similar to the compression and the swelling indices, respectively. By comparing Eqs. 7 and 8 it is clear that a definite value of plastic volumetric strain is associated to a yield locus. By assuming that the normality rule holds and by imposing the Prager consistency rule it is possible to derive the expressions of plastic strain rates, which are given by:

\[
\frac{\dot{E}_{pq}}{\dot{E}_{rs}} = \frac{F_{pq} F_{rs}}{S (S + 3k)} \frac{\lambda}{\mu} \delta_{hk}
\]  

(9)

where the tensor \( E_{mn} \) is defined as

\[
E_{mn} = \frac{C_{ijrs} Z_{rs}}{S} (\delta_{mi} \delta_{nj} + \frac{1}{3} \delta_{mn} \delta_{ij} (\nu - 1))
\]  

(10)

\( S \) is the Kronecker symbol, and

\[
S = \frac{C_{ijrs} Z_{rs}}{S} \delta_{ij}
\]  

(11)

Assume for the sake of simplicity that clay can be considered as a transversely isotropic material, whose axis of symmetry coincides with the direction of consolidation. In that case \( C_{ijrs} \) is characterized by five independent constants. By comparing Eqs. (7,8) we readily derive that one of those may be put to be equal to unity without loss of generality. Moreover, under isotropic loading, yielding should occur for the same value of isotropic pressure, whatever the inclination of the sample might be. This implies from Eq. (7) and definition (6) that

\[
C_{ijrs} \delta_{ij} \delta_{rs} = 3
\]  

(12)

which reduces to three independent parameters.

In the following, for the sake of simplicity, one of these will be put equal to zero so that the tensor \( C_{ijrs} \) will be given, in a reference frame \( x_r, x_s, x_t \), where \( x_r \) coincides with the axis of symmetry, by the expression

\[
C_{ijrs} = \begin{pmatrix}
\frac{1-a}{2} & \frac{1-a}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\]  

(13)

The expression of \( C_{ijrs} \) in a different reference frame can be obtained by Eq. (13) by the usual tensor transformation rule

\[
C_{mnpq} = \frac{\delta_{rs}}{\delta_{rs}} C_{ijrs} \frac{\delta_{km}}{\delta_{pr}} \frac{\delta_{ln}}{\delta_{qr}}
\]  

(14)

Isotropy may be considered a special case for which \( a = b = 1 \). In triaxial conditions Eq. (7) specializes to

\[
A (\frac{3}{3p} - 2B p' - k) + 3p' (p' - 2k) = 0
\]  

(15)

where

\[
A = 4C_{1111} - 4(C_{1222} + C_{1333} + C_{2222} + C_{3333} + 2C_{2233})
\]  

(16)

\[
B = 2C_{2222} + C_{3333} + C_{2233} - C_{1111} - C_{1222} - C_{1333} + 2C_{1111}
\]  

(17)

The shape of the yield function for \( \theta = 0 \) (vertical specimen) is similar to that observed by various authors e.g. Tavenas and Leroueil (1971). From Eq. (19), we may see that the hardening modulus tends to zero; i.e. failure is attained, when

\[
S = 0
\]  

(18)

This implies that in triaxial conditions the failure locus in given by

\[
q = \frac{3a - 2k}{3a - 2a^2} \frac{3a - 2k}{9 p'}
\]  

(19)

which are the equations of two straight lines passing the origin. Once the constitutive parameters have been chosen, Eq. (9) may be integrated along any desired stress path and the plastic strains undergone by a specimen of any specified inclination under that loading condition may be calculated. However, to get total strains the elastic compliance of the soil should be taken into account. We shall assume henceforth an hypoelastic law such that the volumetric strain rate \( \dot{\varepsilon} \) is given by

\[
\dot{\varepsilon} = A \frac{\dot{p}}{p'}
\]  

(20)
and the deviatoric elastic rates are given by

$$\dot{\varepsilon}_{ij} = C_{ijrs} L_0 \eta_{rs}$$

(21)

where $L_0$ is a material constant and $\eta_{rs}$ is equal to the deviator stress divided by the isotropic pressure $p'$. The resulting elastic strain rates are given by

$$\dot{\varepsilon}_{pq} = \left[ C_{pqrs} (\delta_{hr} \delta_{ks} \frac{1}{3} \frac{1}{p'} \delta_{hk} + \frac{1}{9} \frac{B_{ps}}{p'} \delta_{hk} \delta_{rs} ) \right] \dot{e}_{hk}$$

(22)

In Fig. 3 are shown the calculated stress-strain laws in an undrained compression test after isotropic consolidation for different $\delta$ values. It is assumed that the consolidation process does not alter the inherent anisotropy of the clay and that the consolidation pressure is high enough so that clay behaves as normally consolidated. It is possible to note that the effective friction angle at failure only slightly depends on the inclination while much larger dependence is shown by $A_r$ Fig. 4.

THE MODELLING OF STRAIN INDUCED ANISOTROPY

During one dimensional consolidation, clay platelets tend to rearrange themselves so that the direction of the normal to the plane of the platelets is oriented as the direction of consolidation (Quigley and Thompson (1972)). Available experimental evidence for less simple loading paths does not allow definite conclusions on the influence of strain path on platelets orientations. However several authors (e.g. Morganstein and Tchalenko (1967), Barden (1971), Mitchell (1972)) agree on the following qualitative points:

a) the platelets change continuously their orientation as straining proceeds;

b) however, if a proportional strain path is followed the type of anisotropy does not change, i.e. anisotropy is influenced by the relative amount of strain components and not by their absolute value;

c) if a specimen which enjoys a certain type of anisotropy is subjected to a continuous strain process, this tends to erase the initial anisotropy and to gradually orient the clay particles in a different direction. For example a sample one-dimensionally loaded in a direction orthogonal to that of consolidation (horizontal sample) will initially show a different compressibility from a sample oriented as the consolidation direction (vertical sample). After prolonged straining however, the platelets of the horizontal sample will become oriented perpendicularly to the new direction of loading and the compressibility of the horizontal sample will become identical to that of the vertical one (Wesley (1975)). In particular an anisotropic specimen subjected to isotropic loading will tend to become isotropic.

To model this complex behaviour, we have to assume that the anisotropy tensor $C_{ijrs}$ is a function of the previous history of the sample. Since only plastic strain may affect the intrinsic structure of the sample, we may write

$$C_{ijrs} = C_{ijrs} (e^P)$$

(23)

We face several possibilities in the choice of the functional dependence (23). However, a reasonable starting hypothesis may be that a clay slurry is initially isotropic and that it remains so if it is isotropically loaded. A possible choice which obeys at the same time the mathematical conditions on $C_{ijrs}$ discussed in the previous section and that is able to reproduce the experimental findings $a, b$ and $c$ is the following

$$C_{ijrs} = \frac{1}{2} (\delta_{ir} \delta_{js} + \delta_{is} \delta_{jr}) + \frac{\eta_{ir}}{p} \frac{\eta_{rs}}{p'} \delta_{hk}$$

(24)
The parameter $\gamma$ is a material constant which gives the sensitivity to induced anisotropy. Indeed Eq. (26) splits the tensor $C_{ijrs}$ into two components: one isotropic and the other deviatoric. Since during isotropic straining $e_{ii}^P = 0$, an initially isotropic clay sample remains isotropic under this type of loading process.

It is easy to verify that the tensor transformation rule is obeyed and that condition (12) is satisfied for every $e_{ij}^P$. If a sample is one-dimensionally loaded from a slurry, assuming that anisotropy affects in the same way elastic and plastic strain and that the reference axes coincide with the axes of principal strains, $e_{11}^P = e_{11}^P$, $e_{22}^P = e_{22}^P$, $e_{33}^P = e_{33}^P$, $e_{ij}^P = 0$ if $i \neq j$. Thus the anisotropy tensor $C_{ijrs}$ becomes

$$C_{ijrs} = \begin{pmatrix} 1 + 4/9\gamma & -2/9\gamma & -2/9\gamma \\ 1 + 1/9\gamma & 1/9\gamma & 1 + 1/9\gamma \\ 1/2 & 1/2 & 1/2 \end{pmatrix}$$

which is independent of $e_{ij}^P$.

Finally it is evident from Eq. (25) that the relevance of the deviatoric component of $C_{ijrs}$ decreases when the specimen undergoes large volumetric but limited deviatoric

![Fig. 5 Effective stress paths and stress-strain relationships in undrained test for ideal N.C. clay samples cored at different inclinations after $k_o$ consolidation with variable anisotropy.](image1)

![Fig. 6 Effective stress path and stress-strain relationships in undrained triaxial test for two ideal N.C. clay samples consolidated either isotropically or in $k_o$ conditions to $\sigma'_v = 1.00$.](image2)
strains. Thus the material tends to demise its acquired anisotropy if it is subjected to a prolonged isotropic loading. It can be shown that plastic strain rates are given by

\[ \dot{\varepsilon} = \frac{\mathbf{P}}{\mathbf{J}^2} \mathbf{E} : \mathbf{h} \]

where

\[ \mathbf{P} = \mathbf{R} \mathbf{P} \mathbf{R}^{-1} \]

(26)

Assume now to take a set of samples from a layer one dimensionally consolidated from a slurry in a consolidated pressure equal to the preconsolidation pressure \( p_c \). Since peak for \( \theta = 90^\circ \) this model allow to take account of the kind of consolidation, e.g. either isotropic or \( K_0 \), on the subsequent behaviour of the specimen. Fig. 6 shows a comparison between calculated behaviour of two ideal samples consolidated from a slurry, the former isotropically the latter laterally confined, and then sheared to failure in undrained compression. Qualitatively the behaviour is similar to that observed by Jamiołkowski et al. (1980) on Porto Tolle silty clay. In particular the ultimate friction angle is the same, as experimentally observed.

\[ \dot{\varepsilon} = \frac{\mathbf{P}}{\mathbf{J}^2} \mathbf{E} : \mathbf{h} \]

(27)

REFERENCES


Tavenas, F., Leroueil (1977). Effects of stresses and time on yielding of clays. 9th ISRM Tokyo 1, 319-326.