Undrained stability of Ko-consolidated clays

La stabilité non drainée des argiles naturellement consolidées

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SYNOPSIS The undrained strength of Ko-consolidated clays sheared under plane strain condition is formulated as a function of the direction of the major principal stress on the basis of an elasto-plastic constitutive equation. The equation for the anisotropic undrained strength is applied to the theory of characteristic line field as well as to the conventional circular slip line method. The bearing capacity factor and the factors of safety of embankments at failure calculated based on the theory are shown together with the experimental and field data. Laboratory and field data are in good agreement with the theoretical values.

INTRODUCTION

The anisotropy of the undrained strength of clays has been paid attention in connection with the analysis of undrained stability. Bjerrum (1972, 1973), Ladd and Foott (1974) proposed new experimental procedures taking the effect of the anisotropy of undrained strength into account.

The undrained shear strength must be obtained by combining two equations, one of which represents the constraint on the volume change (undrained condition) and the other is the failure condition under various boundary condition such as plane strain condition or axi-symmetric stress state. In this paper, the undrained strength of the anisotropically consolidated clay is derived as a function of the direction of the major principal stress from a constitutive equation proposed by Sekiguchi and Ohta(1977), Ohta and Sekiguchi (1979). Thus obtained anisotropic undrained strength is applied to the theory of the characteristic line as well as to the conventional method of stability analysis.

TABLE I Stress and Material Parameters

MATERIAL PARAMETERS	λ	$= 0.434Cc$ (Cc; compression index)
	κ	$= 0.434Cs$ (Cs; swelling index)
	D	Dilatancy coefficient proposed by Shibata(1963)
	Λ	$= 1-\kappa/\lambda$; Irreversibility ratio
	M	$= \frac{\lambda-\kappa}{D(1+e_o)} = \frac{6\sin\phi'}{3-\sin\phi'}$; critical state parameter
	Ko	Coefficient of earth pressure at rest
	β	$= \frac{\sqrt{3}\eta_o\Lambda}{2M}$
STRESS PARAMETERS	η^*	$= \sqrt{\frac{3}{2}\left(\frac{s_{ij}}{p'}-\frac{s_{ijo}}{p'_o}\right)\left(\frac{s_{ij}}{p'}-\frac{s_{ijo}}{p'_o}\right)}$; Normarized shear stress
	p'	effective mean principal stress
	s_{ij}	$= \sigma'_{ij} - p'\delta_{ij}$ (δ_{ij}; Kronecker's delta) ; deviatoric stress tensor
	σ'_v	effective overburden pressure
	τ	undrained shear resistance along slip line
	η_o	$= \frac{3(1-Ko)}{1+2Ko}$
	Note: Subscript o specifies the value at the time of completion of Ko-consolidation. Subscript i specifies the value at the initial state prior to undrained loading.	

OUTLINE OF THEORY

The outline of the theory is shown as a form of flow chart in Fig.1. The theory starts with the description of the plastic and elastic volumetric strains as functions of stress parameters p' and η^*, see Table I. Based on the equation for the plastic volumetric strain, the yield function is defined. The yield function together with the associated flow rule results in the plastic strain increment. Substitution of definition of the critical state into plastic strain increment yields the failure condition Eq.(1) in Fig.1. The undrained condition is given by Eq.(2), the condition of no volume change. The plane strain condition is approximated by zero increment of intermediate principal plastic strain. Eq.(3) is thus obtained plane strain condition. The combination of Eqs(1) and (3) gives Eq.(4) as the plane strain failure condition. The reduction of Eq.(4) needs an approximation. The combination of Eqs.(2) and (3) produces Eq.(5) as the plane strain undrained condition.

From Eqs.(4) and (5), the undrained strength of Ko-consolidated clays under plane strain conditions is derived as given by Eq.(6) after some approximation which cancells the error arising from preceding approximation needed in deriving Eq.(4). Eq.(6) has a few percents of error if compared with the rigorous plane strain undrained strength. Eq.(6) indicates that the undrained strength varies with the change in the direction of principal stress θ. The undrained strengths of KoPUC (plane strain undrained compression test on Ko-consolidated clay), DSS (direct simple shear test) and KoPUE (plane strain undrained extension test on Ko-consolidated clay) are derived from Eq.(6) by substituting $\theta=0$, $\pi/4$ and $\pi/2$ respectively.

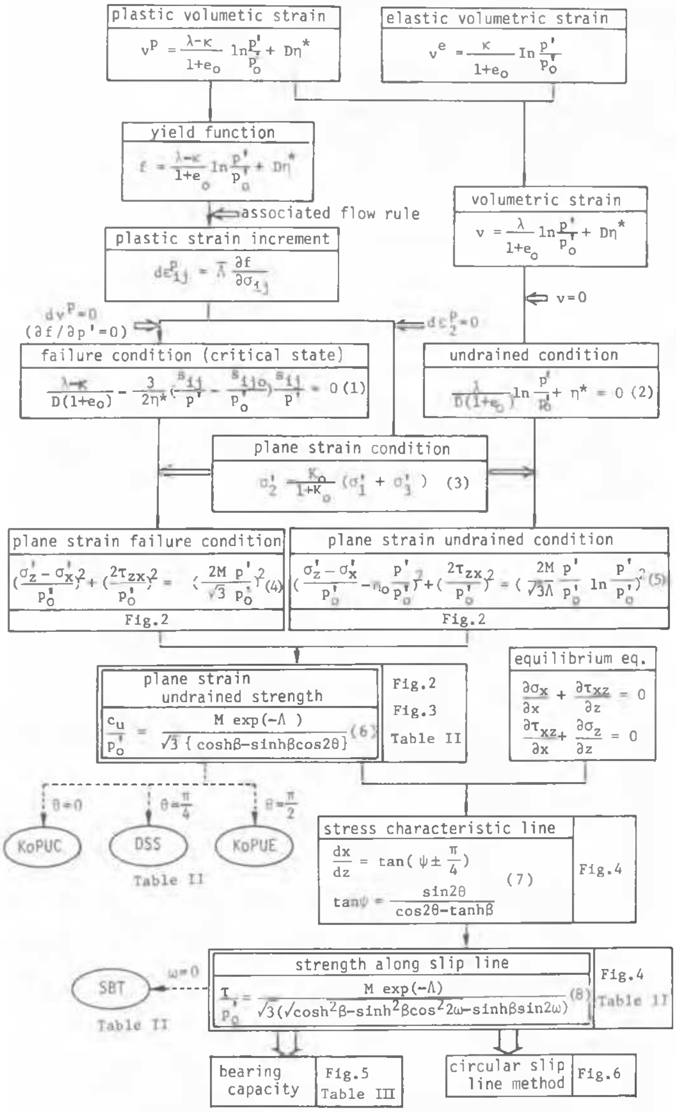


Fig.1 Outline of Theory (for Normally Ko-Consolidated Clay)

Substitution of Eq.(6) into equations of equilibrium gives the direction of the characteristic line of stress as shown by Eq.(7). This is identical with the direction of the characteristic line of velocity since the material is assumed to be incompressible. From Eq.(7), the shear strength mobilized on a slip line is obtained as a function of the direction of the slip line ω, see Eq.(8). The undrained strength obtained from the SBT (shear box test) may be calculated by substituting $\omega=0$ into Eq.(8). The slip line theory together with the strength as a function of the direction of the slip line gives solutions to some of the stability problems. Applying Eq.(8) to the conventional method of stability analysis such as circular slip line method, the effect of stress induced anisotropy of undrained shear strength of Ko-consolidated clay is taken into account in the stability analysis.

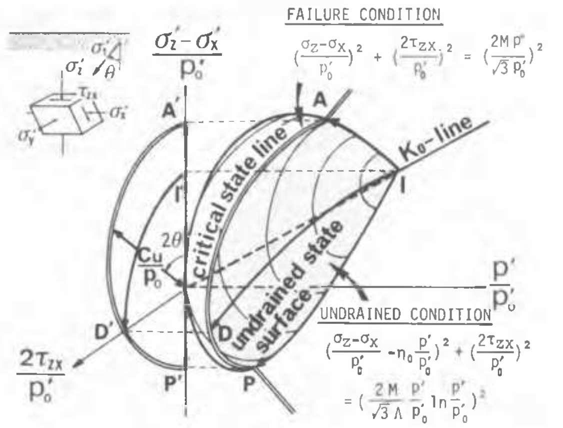


Fig.2 Plane Strain Failure and Undrained Condition

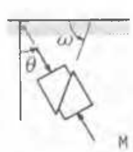
Fig.2 gives an idea how the failure condition and the undrained condition look like in the stress space. The plane strain failure condition is a cone with an apex at the origin of the coordinate system. The plane strain undrained condition is in a skewed bullet shape, the cross section of which with p'=constant planes are the circles with the centres on Ko-line. The cross section made by the failure condition and the undrained condition gives the critical state line (plane strain undrained failure condition). The projection A'D'P' of the critical state line is a curve described by Eq.(6).

APPLICATION TO STABILITY PROBLEMS

Eqs.(6) and (8) in Fig.1 are easily extended for overcosolidated clays. In addition to this, the preconsolidation pressure p_0' (effective mean principal stress) in Eqs.(6) and (8) may be converted into σ_{vi}' which is the initial effective overburden pressure prior to undrained loading (OCR= $\sigma_{vo}' / \sigma_{vi}'$). Table II summarizes the results of theoretical calculation. At the top of Table II, Eqs.(9) and (10) are shown giving the undrained strength as a function either of the direction of the major principal stress or the direction of slip surface. By using either of these equations, the undrained strength corresponding to various types of testing methods are derived as listed in the lower part of Table II. The equations for KoUC and KoUE are derived directly from Eqs.(1) and (2) together with the axi-symmetric stress condition. On the right hand side of Table II, the experimental data reported by Ladd (1973) are compared with theoretical values calculated by using the reported parameters ϕ' and Ko. The theoretical values are generally in good agreement with the experimental values.

Bjerrum (1973) demonstrated the undrained strength obtained from direct simple shear tests on the samples taken from the ground at various angles to the horizontal plane. Fig.3 shows the undrained shear strength obtained by Bjerrum plotted against the direction of shear.

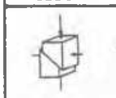
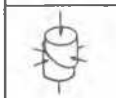
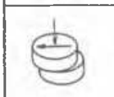
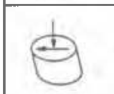
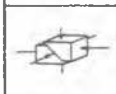
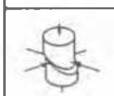
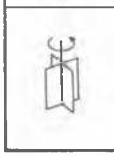
TABLE II Undrained Strengths for Various Testing Methods



$$\frac{C_u}{\sigma'_{vi}} = \frac{OCR^{\Lambda}(1+2K_o)\,M\exp(-\Lambda)}{3\sqrt{3}(\cosh\beta - \sinh\beta\cos 2\theta)} \quad (9)$$

$$\frac{\tau}{\sigma'_{vi}} = \frac{OCR^{\Lambda}(1+2K_o)\,M\exp(-\Lambda)}{3\sqrt{3}(\sqrt{\cosh^2\beta-\sinh^2\beta\cos^2 2\omega} - \sinh\beta\sin 2\omega)} \quad (10)$$

$$M = \frac{6\sin\phi'}{3-\sin\phi'} \quad \Lambda = 1 - C_s/C_c \quad \eta_o = \frac{3(1-K_o)}{1+2K_o} \quad \beta = \frac{\sqrt{3}\eta_o\Lambda}{2M}$$

Type of test	Reduced equation for specified test on normally consolidated clay	Blue marine clay PI=20 φ'=33° Ko=0.5 measured	predicted
	Ko-consolidated Plane strain Comp. **KoPUC** (A) $\frac{C_u}{\sigma'_{vo}} = \frac{(1+2K_o)M\exp(-\Lambda)}{3\sqrt{3}(\cosh\beta - \sinh\beta)}$	0.34	0.347
	Ko-consolidated Triaxial Comp. **KoUC** $\frac{C_u}{\sigma'_{vo}} = \frac{1+2K_o}{6} M\exp(\frac{\Lambda\eta_o}{M} - \Lambda)$	0.33	0.318
	Shear Box Test **SBT** $\frac{\tau}{\sigma'_{vo}} = \frac{(1+2K_o)M\exp(-\Lambda)}{3\sqrt{3}}$	—	0.239
	Direct Simple Shear **DSS** (D) $\frac{\tau}{\sigma'_{vo}} = \frac{(1+2K_o)M\exp(-\Lambda)}{3\sqrt{3}\cosh\beta}$	0.20	0.224
	Ko-consolidated Plane strain Ext. **KoPUE** $\frac{C_u}{\sigma'_{vo}} = \frac{(1+2K_o)M\exp(-\Lambda)}{3\sqrt{3}(\cosh\beta + \sinh\beta)}$	0.19	0.165
	Ko-consolidated Triaxial Ext. **KoUE** (P) $\frac{C_u}{\sigma'_{vo}} = \frac{1+2K_o}{6} M\exp(-\frac{\Lambda\eta_o}{M} - \Lambda)$	0.155	0.135
	Field Vane **FV** $\frac{\tau_h}{\sigma'_{vo}} = \frac{(1+2K_o)M\exp(-\Lambda)}{3\sqrt{3}}$ $\frac{\tau_v}{\sigma'_{vo}} = \frac{1+2K_O}{3\sqrt{3}}\sqrt{(\frac{M}{\Lambda}\frac{p}{p_0}\ln\frac{p}{p_0})^2 - (1-\frac{p}{p_0})^2\eta_0^2}$	0.19	0.182

(Test data by Ladd,1973)

The theoretical curve given in Fig.3 are calculated based on the assumption that the direction of the major principal stress is at an angle of 45 degrees to the direction of shear, i.e., shear stress along the direction of shear is the maximum shear stress in the specimen. In calculating the theoretical curve shown in Fig.3, the values of M (function of ϕ') are determined to fit the theoretical value in with the experimental value at α =0 since the values of ϕ'-parameter were not reported. Karube's (1975) empirical equation, $\Lambda = M/1.75$, is also used. Fig.4 shows the relationship between the slip line (which happens to be identical with the characteristic line of stress under undrained conditions) and the direction of the major principal stress.

Fig.5 shows the bearing capacity factor Nc as a function of the coefficient of earth pressure at rest Ko. The bearing capacity factor is obtained based on Eq.(10) assuming that the strength of soil is uniform through the depth. The bearing capacity factor Nc in Fig.5 should be multiplied by the undrained shear strength obtained from shear box tests to give the ultimate bearing capacity q. Table III gives the ultimate bearing capacities which were

Clay	Soil Properties	Experiment	Theory
SKA-EDEBY	PI=48 Ko=0.75 OCR=1.2 (ϕ'=20°)	●	———
PLASTIC DRAMMEN	PI=29 Ko=0.60 OCR=1.5 (ϕ'=25°)	⊙	—·—
LEAN DRAMMEN	PI=11 Ko=0.55 OCR=1.2 (ϕ'=18°)	○	-----

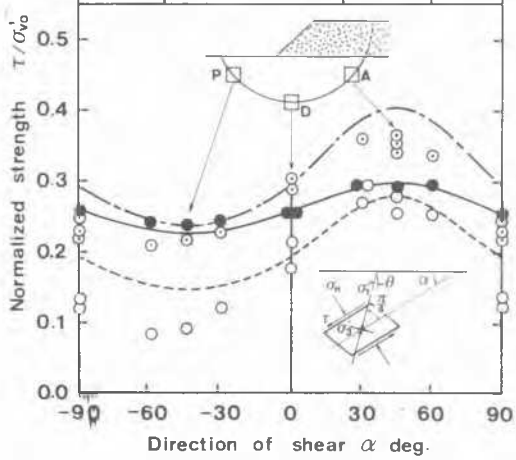


Fig.3 Undrained Strength-Direction of Shear Relations
(Test data by Bjerrum, 1973)

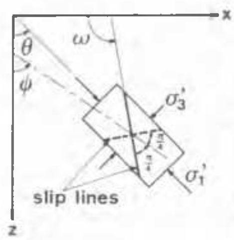


Fig.4 Direction of Slip Line Relative to Direction of Major Principal Stress

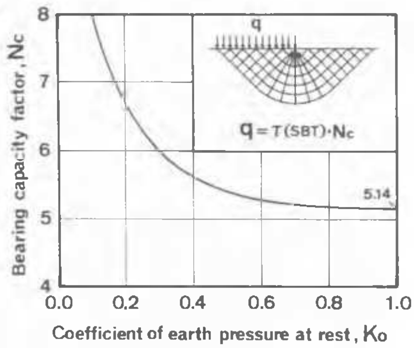


Fig.5 Bearing Capacity Factor

experimentally obtained by Kinner and Ladd (1973) for clays with overconsolidation ratio of 1, 2 and 4. In Table III, the theoretical values (Theory) of ultimate bearing capacity are also listed. They are obtained by using the bearing capacity factor given in Fig.5. The ultimate bearing capacities calculated with Nc=5.14 using the undrained strength obtained from KoPUC, IUC, DSS and UC (Kinner and Ladd, 1973) are listed as well. ADP and Theory are calculated by the authors. In the authors' calculation, Ko and ϕ' values reported by Kinner and Ladd (1973) are used together with the Karube's empirical equation Λ =M/1.75. The ultimate bearing capacity obtained by using the present theory is in the best agreement with the experimental value.

The anisotropic undrained strength given by Eq.(8) is employed in the conventional circular slip line analysis of the stability of embankments on soft clay foundations. The undrained strength to be mobilized on each piece of the slip line is calculated as a function of the

TABLE III Predicted and Measured Ultimate Bearing Loads (Test Data by Kinner and Ladd, 1973)

OCR	METHOD	Cu/σ_{vi}^{*}	ultimate load q/σ_{vi}^{*}		$\frac{\text{predicted}}{\text{measured}}$
			predicted	measured	
1	KoPUC	0.34	1.75 *	1.34	1.31
	IUC	0.31	1.59 *		1.19
	DSS	0.20	1.03 *		0.77
	UC	0.19	0.98 *		0.73
	ADP	0.24	1.25 §		0.94
	Theory	--	1.29 †		0.96
2	KoPUC	0.57	2.93 *	2.43	1.21
	IUC	0.55	2.82 *		1.16
	DSS	0.37	1.90 *		0.87
	UC	0.36	1.85 *		0.76
	ADP	0.44	2.24 §		0.92
	Theory	--	2.29 †		0.94
4	KoPUC	0.95	4.88 *	4.20	1.16
	IUC	0.91	4.67 *		1.11
	DSS	0.61	3.14 *		0.75
	UC	0.60	3.08 *		0.73
	ADP	0.74	3.82 §		0.91
	Theory	--	4.05 †		0.97

Note; UC = unconfined compression test
IUC = isotropically consolidated triaxial compression test
* calculated by $q = 5.14Cu$
§ calculated by $\begin{cases} q = 5.14Cu \\ Cu = (KoPUC + DSS + KoPUE)/3 \end{cases}$
† calculated by eq.(10) in Table II using reported values of Ko and M

direction of slip line ω and then integrated along the slip line shown in Fig.6 resulting in the average shear resistance which happens to be identical with the strength obtained from a shear box test on a specimen sheared along the horizontal plane, $\omega=0$ (see the equation for SBT in Table II). A curve shown in Fig.6(a) is the theoretically expected average shear resistance calculated by using the equation for SBT with the parameter Ko and ϕ' (and hence M) estimated based on PI values through empirical relations proposed by Massarsch (1975) and by Kenney (1959). Karube's empirical equation $\Lambda = M/1.75$ is also employed in deriving the parameter Λ. Thus obtained theoretical curve in Fig.6(a) may act as a rough guide line of the average of plane strain shear resistance of normally consolidated clays sliding along the circular slip lines such as shown at the top of Fig.6. In Fig.6(a) the undrained strengths obtained from field vane, unconfined compression, direct simple shear and shear box tests on a number of clays are shown in forms of the most likely ranges.

Provided that the theoretical curve in Fig.6(a) is correct, the use of the shear resistance indicated by the theoretical curve gives the factor of safety of 1.0 for actually failed embankments. Therefore, the ratio of strength obtained from each of FV, UC, DSS and SBT tests to theoretically calculated shear resistance must give the calculated factor of safety which should have been obtained, for actually failed embankments, based on the undrained strength from each of these testing methods. The bands shown in Fig.6(b) are thus predicted ranges of the possible factor of safety for actually failed embankments. Four kinds of plots in Fig.6(b) indicating the factor of safety of actually failed embankments backcalculated by using reported FV, UC, DSS and SBT strengths are generally in the ranges for each testing methods predicted from the bands shown in Fig.6(a).

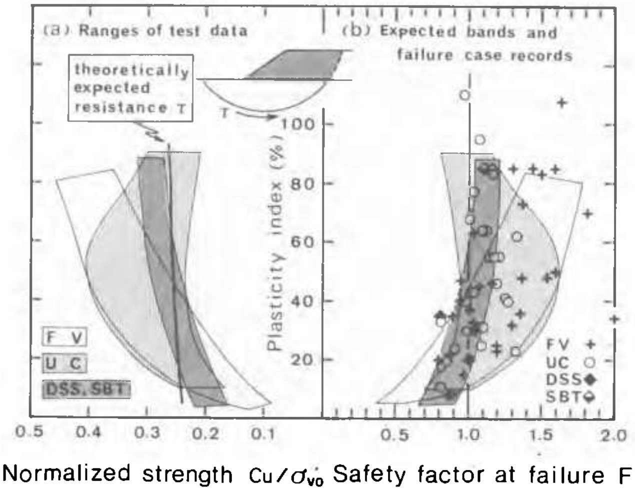


Fig.6 Undrained Stability of Embankments

CONCLUSIONS

The undrained failure of clays under plane strain conditions is described in terms of strain as (i) infinite shear strain, (ii) zero volumetric strain, (iii) zero intermediate principal strain. These three requirements are converted into three stress conditions by employing an elasto-plastic constitutive equation. Simultaneous solution of these three stress conditions gives the undrained strength as a function of the major principal stress direction. Employment of the equation for anisotropic undrained strength as the failure condition to be substituted into the equation of stress equilibrium leads to the anisotropic undrained shear resistance to be mobilized on a piece of slip line inclining at an angle to the horizontal plane. The applications of these theoretical equations to various kinds of laboratory tests, model footing tests and case studies of actually failed embankments on soft clays indicate that the theory is in good agreement with observed laboratory and field data.

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