A fracture mechanics study of stiff clays

Fissuration et rupture des argiles raides

A. S. SAADA, Professor and Chairman, Department of Civil Engineering, Case Western Reverse University, Cleveland, Ohio, USA
A. CHUDNOVSKY, Professor, Department of Civil Engineering, Case Western Reverse University, Cleveland, Ohio, USA
M. R. KENNEDY, Graduate Student, Department of Civil Engineering, Case Western Reverse University, Cleveland, Ohio, USA

SYNPOSIS Ideas and concepts about fracture and crack propagation in overconsolidated clays have been on the mind of geotechnical engineering researchers for years. The aim of this paper is to introduce the use of fracture mechanics concepts and methodology to the study of the strength of stiff clays. Tests to determine fracture toughness coefficients K_{IC} and K_{IIC} are described and results given. The stability of an overconsolidated clay slope is analyzed on the basis of the stability criterion derived from the principles of thermodynamics.

INTRODUCTION

Failure of civil engineering structures may occur by excessive deformation, yield or fracture. Failure criteria are usually based on the dominant failure mode. For yield dominant failures, the criteria of Mises, Tresca or Coulomb seem most appropriate. For fracture dominant failures several types of fracture must be considered. Cracks may be present in the material or may be initiated by high stress concentrations at low nominal stress levels. Crack propagation may follow any or a combination of the three modes shown in Fig. 1.

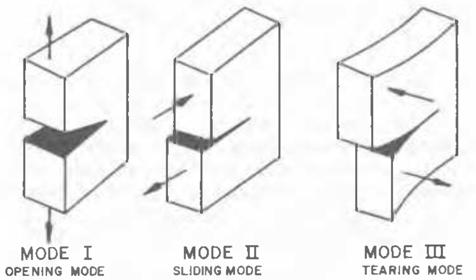


FIG. 1 Fracture modes.

The geotechnical engineering profession is quite familiar with stiff fissured and jointed clays where the discontinuities are found on a macroscopic scale. A preliminary classification of different types of discontinuities which constitute planes of weakness have been given by Skempton and his coworkers (1969). The importance of size effect on the undrained strength of stiff fissured clay was demonstrated by Bishop and Little (1976). The remarkable work of Lo (1970) on the operational strength of fissured clay has in it most of the elements on which classical fracture mechanics rests.

In his Terzaghi lecture Bjerrum (1967) states that in order to start a progressive failure there must be a discontinuity somewhere in the clay mass where failure can be initiated. The more overconsolidated the clay, the greater its content of recoverable strain energy and the greater the danger of progressive failure.

The mechanisms of propagation of a shear surface in a fissured clay involve concepts similar to those put forward by Griffith. These include energy absorbed in forming the surface and released by relieving the stresses in the locally highly stressed material adjacent to it. Once a slip surface has formed subsequent movements are mostly concentrated along it with appreciable particle reorientation. Morgenstern and Tchalenko (1967) have shown that the slip surface consists of a thin band in which the particles are more or less in the direction of the movement.

Slow crack propagation occurs through various damage accumulation mechanisms which result in the strength's degradation. Fracture mechanics should help answer the following questions:

1 - What is the strength as a function of the loading history and the environmental conditions?
2 - What size of crack can be tolerated at the expected service stresses (static or dynamic)? And what is the critical crack size?

Since failure generally appears in the form of a surface, one has to start by studying a single crack and its propagation. The components of the state of stress in the vicinity of a crack tip are proportional to a parameter known as stress intensity factors K_i. For a critical state at which instability takes place the values of the stress intensity factors are referred to as fracture toughness K_{iC}. Each of the 3 fracture modes has its own parameters. Those parameters are used in fracture based design methodologies. In this paper we are only interested in K_{IC} and K_{IIC}.

DETERMINATION OF THE FRACTURE TOUGHNESS COEFFICIENT K_{IC}

In this mode the displacements of the crack surfaces are perpendicular to the plane of the crack. The sample configuration used is that of a hollow disk with a precut notch or stress concentration cut in the inside edge. The dimensions are shown in Fig. 2. Static or dynamic pressure is applied to the inner surface through a rubber membrane.

In order to determine the fracture toughness of the material, two parameters must be known, namely the critical stress at which unstable fracture takes place and the critical crack length. Once these are known the numerical integration of a solution given by Tada (1973) allows one to obtain K_{IC}.

Sample Preparation and Testing Equipment

Kaolinite clay with liquid and plastic limits of 56.3 percent and 37.5 percent respectively was mixed with de-aired distilled water at twice the liquid limit. The slurry was consolidated in a large metal consolidometer under a series of pressures namely 414, 517, 620 and 723 kPa to provide a range of overconsolidation ratios. After equilibrium was reached the clay was allowed to rebound under atmospheric pressure leading to overconsolidation ratios of 4, 5.1, 6.1 and 7.1 respectively.

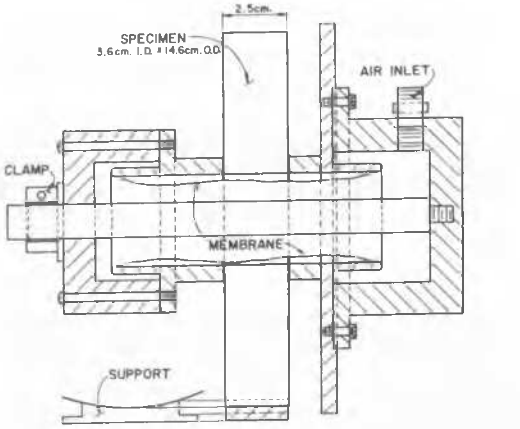


FIG. 2 Apparatus for the determination of K_{IC}.

The blocks of clay were sliced horizontally to yield specimens 25mm thick. A simple rig was used to punch a 35.6mm hole at the center of the disks and to cut a radial notch 10mm deep. They were covered with silicone oil wrapped and stored in a fog room. The specimens with a given overconsolidation ratio were mounted in the testing fixture as shown in Fig. 2 and sinusoidially varying air pressure sent through the air inlet. Under cyclic pressure the crack propagates in a stable manner as long as the pressure is moderate and the crack length below the critical. The crack was allowed to propagate half way through thickness to allow for the study of damage zones. Crack propagation was followed with a pointer attached to an LVDT. The crack opening displacement (COD) was measured with a transducer mounted on the specimen. The cyclic loading was then stopped and a sudden high pressure was applied inside the membrane which caused failure in a very short time. An X-Y plotter gave a graph of the inside pressure versus the COD. The maximum pressure is the critical pressure. The critical length was obtained from the examination of the fracture surface. There is an obvious difference in the surface morphology where the growth of the crack is stable and when it is unstable; with a distinct boundary line between the two regions. The distance between this line and the inside edge of the specimen is the critical length ℓ_c. Fig. 3 shows the shape of the fracture surface and the zones of stable and unstable crack propagation.

Calculation of K_{IC}

Assuming an elastic stress distribution the expression of the circumferential normal stress $\sigma_{\theta\theta}$ in a disk subjected to internal pressure is

$$\sigma_{\theta\theta} = \frac{p_i a^2}{b^2 - a^2} + \frac{1}{r^2}\,\frac{a^2 b^2}{b^2 - a^2}\,F_i \qquad (1)$$

where a and b are the inner and outer radii respectively, p_i the internal pressure and r the radial distance. Using this pressure distribution the solution given by Tada (1973) for the problem shown in Fig. 4 can be integrated to give an approximate value of K_{IC} for the disk. Tada

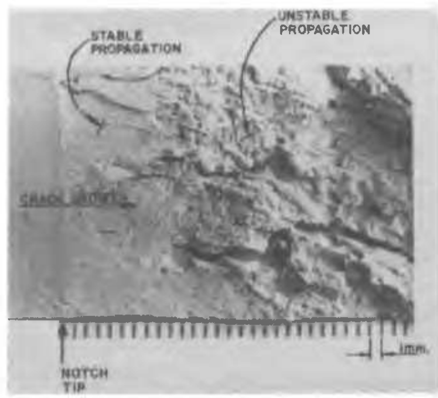


FIG. 3 Fracture surface.

expresses K_{IC} in terms of an influence function F of the geometry and location of the load:

$$K_{IC} = \frac{2P}{\sqrt{\pi \ell_c}}\,F(\gamma,\xi) \qquad (2)$$

where $\gamma = x/\ell_c$ and $\xi = \ell_c/(b-a)$. Writing K_{IC} in differential form, replacing P by $\sigma_{\theta\theta}\,dx$ and integrating one obtains

$$K_{IC} = 2p_i\sqrt{\frac{\ell_c}{\pi}}\left(\frac{a^2}{b^2 - a^2}\right)\int_0^1 \left[1 + \frac{(1+\beta)^2}{(\beta+\gamma\xi)^2}\right] F(\gamma,\xi)\,d\gamma \qquad (3)$$

where $\beta = a/(b-a)$. Eq. 3 can be normalized with respect to a factor K_{I0} given by

$$K_{I0} = \frac{a^2 + b^2}{b^2 - a^2}\,p_i\,\sqrt{\pi \ell_c} \qquad (4)$$

leading to

$$\frac{K_{IC}}{K_{I0}} = \frac{2}{\pi}\left[\frac{\beta^2}{2\beta^2 + 2\beta + 1}\right]\int_0^1 \left[1 + \frac{(1+\beta)^2}{(\beta+\gamma\xi)^2}\right] F(\gamma,\xi)\,d\gamma \qquad (5)$$

The function $F(\gamma,\xi)$ has been tabulated by Tada. Numerical integration leads to the curve shown in Fig. 4. For every value of the critical length ℓ_c and a critical pressure p_i one can obtain a value of K_{IC}. For the tests conducted on disks this value as a function of the overconsolidation ratio is shown in Fig. 5. K_{IC} is a basic parameter to be used in design when failures such as those occurring during hydraulic fracturing are studied.

DETERMINATION OF THE FRACTURE TOUGHNESS COEFFICIENT K_{IIC}

In this mode the displacement of the crack surfaces are parallel to the plane of the crack. The sample configuration used is that of a thin long hollow cylinder with the dimensions shown in Fig. 6. Once the length of the crack and the critical shearing stress are known a solution due to Erdogan and Ratwani (1972) allows one to compute K_{IIC}.

Sample Preparation and Testing Equipment

Clay slurries consolidated at approximately 280 kPa were first cut in a shape of a hollow cylinder. A special lathe was used to reduce them to the size shown in Fig. 6. Two diametrically opposite horizontal notches were cut in the specimens and 2 layers of teflon tape inserted in them to prevent hydrostatic pressure from closing and healing the crack. The specimens were then placed in a special cell where they were one dimensionally consolidated under a given cell pressure. Three cell pressures were used leading to vertical consolidation pressures of 1394, 1174 and 954 kPa; K_o for this clay being 0.47. The samples were then allowed to rebound at a cell pressure of 207 kPa leading to vertical overconsolidation ratios of 6.7, 5.7 and 4.6.

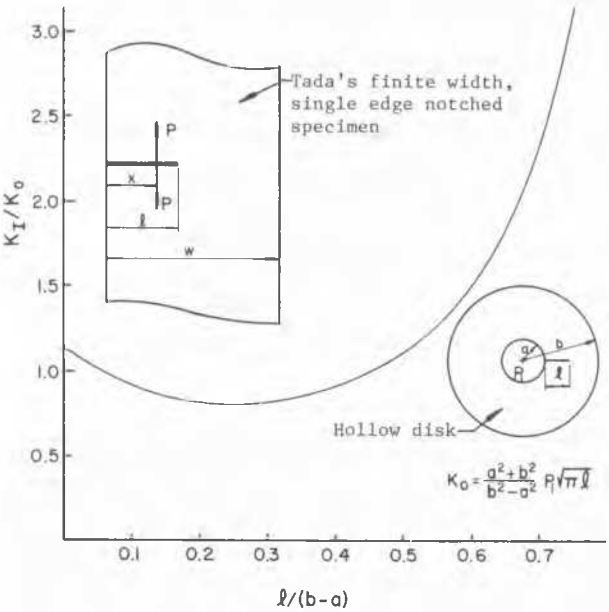


FIG. 4 Determination of K_I for a hollow disk.

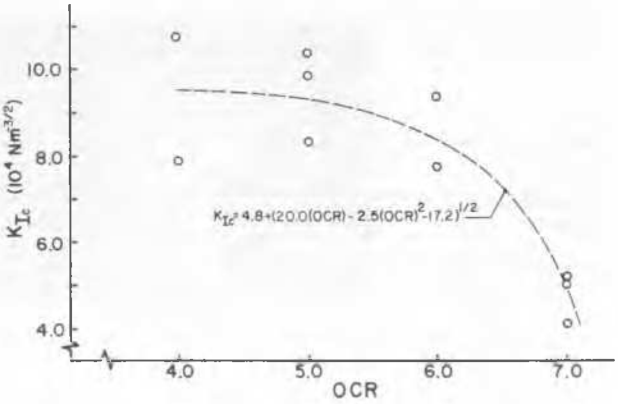


FIG. 5 K_{IC} for overconsolidated clay.

A system of pullies and actuator was used to apply rapidly a very high torque to the piston; while transducers inside the cell measured the actual torque transmitted to the sample, and the angle of rotation. These values were recorded on an X-Y plotter. The maximum torque is the critical value needed to compute K_{IIC}. Fig. 6 shows a series of failure surfaces in MODE II for 3 of the hollow cylinders tested.

Calculation of K_{IIC}

Knowing the geometry of the sample and the crack as well as the critical torque, Erdogan's solution yields the value of K_{IIC}. Fig. 7 shows the relation between K_{IIC} and the overconsolidation ratio for the kaolinite tested.

APPLICATION TO SOIL MECHANICS

Thermodynamic Criteria of Stability

For an irreversible process the second law of thermodynamics states that the rate of entropy production S_i is such that

$$\dot{S}_i = \sum_k j_k X_k \geq 0, \tag{6}$$

where j_k is a generalized flux and X_k is a generalized force. The rate of entropy production for a propagating crack can be written as (Chudnovsky, 1983)

$$T\dot{S}_i = \dot{D} + \dot{\ell}(J_1 - \gamma^* R_1) \geq 0 \tag{7}$$

where, T is the absolute temperature, $\dot{D}$ is the portion of the irreversible work spent on the development of microdamage in the vicinity of the crack tip, ℓ is the crack length, $\dot{\ell}$ is the rate of crack growth; $J_1 = -\partial\pi/\partial\ell$ is the energy release rate, π is the potential energy also called Gibbs potential; γ^* is the specific enthalpy of damage. It can be looked upon as the surface energy if the damage consists of microcracks only. R_1 is a value integral characterizing the damaged zone around the crack tip (Chudnovsky, 1983). Where as $\gamma^* R_1$ represents the amount of energy needed for the crack and its surrounding damaged zone to advance, J_1 characterizes the amount of accumulated potential energy available for crack propagation. Thus $(\gamma^* R_1 - J_1)$ represents the energy barrier which must be overcome for the crack advance. Within the framework of irreversible thermodynamics, $(J_1 - \gamma^* R_1)$ appears as a generalized force reciprocal to the generalized flux $\dot{\ell}$.

When $(\gamma^* R_1 - J_1) \leq 0$, the crack propagates spontaneously. When $(\gamma^* R_1 - J_1) > 0$, the crack can propagate only if $\dot{D} \geq \dot{\ell}(\gamma^* R_1 - J_1)$, i.e., if the dissipation compensates for the negative entropy production $\dot{\ell}(J_1 - R_1)$. This implies that the crack propagation is controlled by dissipative processes like irreversible deformation, diffusion, chemical reaction, etc. Apparently, the critical condition for instability (stability) is

$$J_1 - \gamma^* R_1 = 0 \tag{8}$$

which is Griffith's criterion if $\gamma^* R_1$ is taken as the surface energy. This quantity in the fracture mechanics literature is referred to as γ^{eff} or the critical energy release rate J_{1C}. While Eq. 8 is a necessary condition for instability, the sufficient condition results from the universal criterion of evolution. For a stable process, the following inequality holds:

$$\frac{d\dot{S}_i}{dt} = \sum_k j_k \frac{dX_k}{dt} \leq 0 \tag{9}$$

where j_k is assumed to remain constant. For an elastic crack $\dot{D} = 0$ and Eqs. 7 and 9 yield

$$T\frac{d\dot{S}_i}{dt} = \dot{\ell}^2 \frac{\partial(J_1 - \gamma^* R_1)}{\partial \ell}\Bigg|_{\substack{\sigma = const \\ T = const}} \leq 0 \tag{10}$$

where σ is the stress tensor. If the damaged zone does not change as the crack length increases one can assume that $\gamma^* R_1 = \gamma^{eff}$ is constant. The sufficient condition of stability (10) reduces to

$$\frac{\partial J_1}{\partial \ell}\Bigg|_{\substack{\sigma = const \\ T = const}} \leq 0 \tag{11}$$

Therefore, at constant temperture T, the necessary and sufficient conditions of instability are

$$J_1 = \gamma^{eff} \quad (a) \qquad \frac{\partial J_1}{\partial \ell}\Bigg|_{\sigma = const} > 0 \quad (b) \tag{12}$$

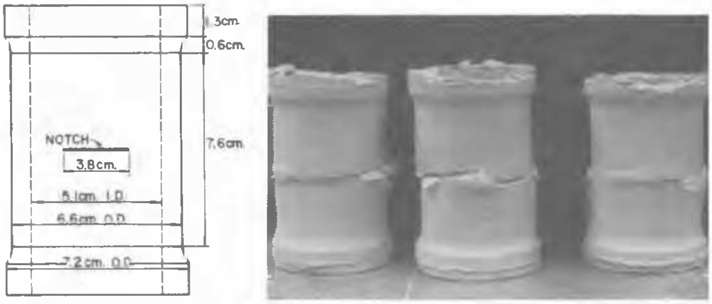


FIG. 6. Failure of hollow cylinders in MODE II.

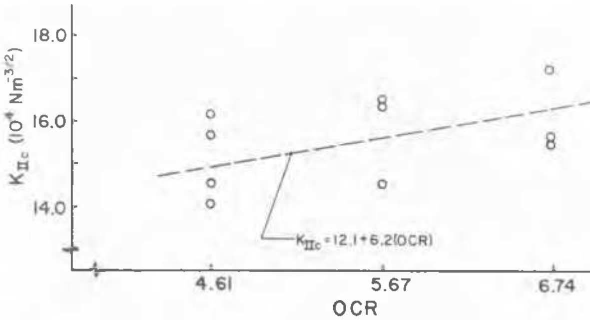


FIG. 7 K_{IIC} for overconsolidated clay.

Application to the Stability of Infinite Slopes

Consider an infinite slope with a crack of length 2ℓ parallel to the surface and at a depth h. It will be assumed that there is no shearing resistance along the crack and that for saturated overconsolidated undrained clay, compressive stresses do not change the shear response. A MODE II fracture is under consideration. The shearing stress on any plane parallel to the surface at a depth h is given by $\tau = \rho gh \sin(i) \cos(i)$, where i is the inclination of both slope and crack. Let us now consider a MODE II crack in an infinite plane under the uniform shearing stress and use the principle of superposition (see Broek, 1982). If $K_{II\infty}$ is the fracture toughness of the plane and K_{IIS} is that of the problem at hand then $K_{IIS} = K_{II\infty}$ because of the symmetries involved in an infinite slope. Thus,

$$K_{II\infty} = K_{IIS} = \tau\sqrt{\pi\ell} = \frac{1}{2}\rho gh\sqrt{\pi\ell}\sin(2i) \qquad (13)$$

Conventionally the energy release rate for plane strain and MODE II is written $J_1 = (1 - \nu^2)K_{II}^2/E$, where E and ν are Young's modulus and Poisson's ratio respectively. Thus,

$$\frac{\partial J_1}{\partial \ell} = \frac{2(1 - \nu^2)}{E} K_{II} \frac{\partial K_{II}}{\partial \ell} > 0 \qquad (14)$$

and condition (12b) is always satisfied. Condition (13a) can be rewritten in the following equivalent form

$$K_{IIS} = K_{IIC} \qquad (15)$$

where K_{IIC} is the fracture toughness coefficient. It can be obtained for instance from tests conducted on hollow cylinders in MODE II fracture. Substituting (13) into (15) we obtain the expression for the critical crack length $2\ell_c$. Thus,

$$2\ell_c = \frac{8K_{IIC}^2}{\pi(\rho gh \sin 2i)^2} \qquad (16)$$

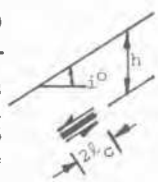
The values of K_{IIC} for a range of overconsolidation ratios can be obtained from Fig. 7. Table 1 gives values of the critical lengths of crack which would cause catastrophic failure for various depths h and slope inclinations i. The value of ρg is 17.3 kN/m^3 and that of K_{IIC} is 156×10^3 $Nm^{3/2}$. The dimensions are in meters. As one can notice the results seem to be quite reasonable.

CONCLUSION

A framework for the use of fracture mechanics in the study of the stability of soil structures with cracks has been established. Tests have been proposed and then conducted for the determination of the fracture toughness coefficients K_{IC} and K_{IIC}. An example of the use of K_{IIC} for a limited range of depths and slope inclinations has been given, and reasonable results obtained. Perhaps the most important contribution of K. Terzaghi to soil mechanics was his bringing together in one treatise many of the branches of the mechanics of continuous media; and casting them in a form easily used by engineers. Fracture mechanics is a relatively young science which has its place in the interpretation of the behavior of soil masses. It is hoped that this research will help develop new testing and analysis techniques leading to more rational design in fissured soils.

TABLE I
Critical length of cracks for various h and i.

h \ i^o	15.00	20.00	25.00	30.00	35.00
5.0	33.13	20.05	14.11	11.04	9.38
6.0	23.01	13.92	9.80	7.67	6.51
8.0	12.94	7.83	5.51	4.31	3.66
10.0	8.28	5.01	3.53	2.76	2.34



ACKNOWLEDGEMENT

The authors wish to express their gratitude to the U.S. Army Research Office for its support.

REFERENCES

Bishop, A.W. and Little, A.L. (1967). The Influence of the Size and Orientation of the Sample on the Apparent Strength of the London Clay at Maldon, Essex. Proc. Geotechnical Conf., Oslo 1, 89-96.

Bjerrum, L. (1967). Progressive Failure in Slopes of Overconsolidated Plastic Clay and Clay Shales. ASCE J. Soil Mech. Found. Engg., 93, SM5, Part I, 3-49.

Broek, D. (1982). Elementary Engineering Fracture Mechanics, 3rd Ed., pp. 1-469. M. Nijhoff, Boston.

Chudnovsky, A. (1983). Crack Layer Theory. NASA Report, Grant NAG-3-23.

Erdogan, F. E. and Ratwani, M. (1972). A Circumferential Crack in a Cylindrical Shell Under Torsion, Int. J. Fract. Mech., 8, 87-95.

Lo, K.Y. (1970). The Operational Strength of Fissured Clay. Geotechnique 20, No. 1, 57-74.

Morgenstern, N.R. and Tchalenko, J.S. (1967). Microstructural Observations on Shear Zones from Slips in Natural Clays. Proc. Geotechnical Conf., Oslo 1, 147-152.

Skempton, A.W., Schuster, R.L. and Petley, D.J. (1969) Joints and Fissures in London Clay at Wraysbury and Edgware. Geotechnique 19, No. 2, 205-217.

Tada, H., Paris, P.C. and Irwin, G.R. (1973). The Stress Analyses of Cracks Handbook. Del Research Corporation, 2-27.