A visco-elastic model for liquefaction of sands

Un modèle visco-élastique de la liquéfaction des sables

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SYNOPSIS An equivalent visco-elastic model consisting of 4 formulas for calculating the shear modulus, the damping ratio and the volumetric and deviatoric strains of sands during cyclic loading is proposed. The model, together with the effective stress method for dynamic response analysis, can be used to solve practical problems associated with seismic stability of sand masses.

INTRODUCTION

The author proposed an equivalent visco-elastic model several years ago (Shen & Wang, 1980), but it turned out to be a little complicated for practical use. In the following this model is revised and simplified, and a laboratory procedure is suggested to determine the related soil constants.

PROPOSED MODEL

Theoratically, the deformation characteristics of sands under cyclic loading as well as under static condition are better simulated by using elasto-plastic models. But the research of this kind has just begun, and the theories suggested by various authors seem far from being able to solve practical problems. A visco-elastic model aims to simulate the soil stress-strain behaviour in a cycle of loading as a whole but not in detail (Fig.1). It is said that the model gives equivelent hysteresis loops if the axis of inclination and the enclosed area of stress-strain ellipse are essentially equal to those of the actual stress-strain loops. A hysteresis loop can be characterized by the shear modulus G and damping ratio D, and the plastic deformation can be divided into the volumetric and deviatoric components ε^p and γ^p. Therefore,a complete set of relationships for the visco-elastic model must include 4 formulas.

For a given sand there are three fundamental factors affecting its deformation characteristics in a particular loading cycle, i.e. the initial static stress (σ_s, τ_s), the amplitude of cyclic stress τ_d or strain γ_d and the loading history charaterized by the accumulated shear straining $\xi = \sum_{i=1}^{r-1} (\gamma_d)_i$, where $\sigma_s = \frac{1}{2}(\sigma_1 + \sigma_3)$, $\tau_s = \frac{1}{2}(\sigma_1 - \sigma_3)$, $\gamma = \varepsilon_1 - \varepsilon_3$, σ_1, σ_3-principal stresses (in term of effective stress), ε_1, ε_3-principal strains, and r is the number of current cycle, with $(\gamma_d)_i$ shear strain amplitude of ith cycle of loading. The subscripts s and d refer to the static and dynamic states respectively. Taking these factors into account and using γ_d as an independant variable the proposed formulas are as follows

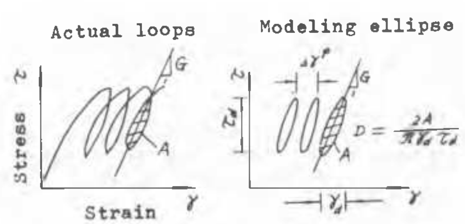

Fig.1. Visco-Elastic Model

$$G = \frac{k_2}{1 + k_1 \gamma_c} (\sigma_s)^{\frac{1}{2}} \quad (1)$$

$$D = D\max \frac{k_1 \gamma_c}{1 + k_1 \gamma_c} \quad (2)$$

$$\Delta\varepsilon^p = c_1 (\gamma_d)^{c_2} \exp(-c_3 Rs^2) \frac{1}{1 + Ne} \quad (3)$$

$$\Delta\gamma^p = c_4 (\gamma_d)^{c_5} Rs^2 \frac{1}{1 + Ne} \quad (4)$$

where

$$\gamma_c = (\gamma_d)^{\frac{3}{4}} (\sigma_s)^{-\frac{1}{2}} \quad (5)$$

is the corrected shear strain amplitude,

$$Ne = \frac{\xi}{\gamma_d} \quad (6)$$

is the effective number of loading cycles, and

$$Rs = \frac{\tau_s}{\sigma_s \sin\phi} \quad (7)$$

is the shear stress level, ϕ- angle of internal friction. Under undrained condition the pore pressure increment can be calculated by

$$\Delta u^p = Ku \, \Delta\varepsilon^p \quad (8)$$

where Ku is the unloading bulk modulus

$$Ku = k_u (\sigma_s)^{\frac{1}{2}} \quad (9)$$

In the abovementioned formulas c_1, c_2, c_3, c_4, c_5, k_1, k_2, k_u, Dmax and ϕ are soil constants.

DISCUSSION AND COMPARISION

Relationship (1) is essentially the same as that suggested by Hardin & Drnevich (1972). If it is assumed that the so-called hyperbolic strain γ_h

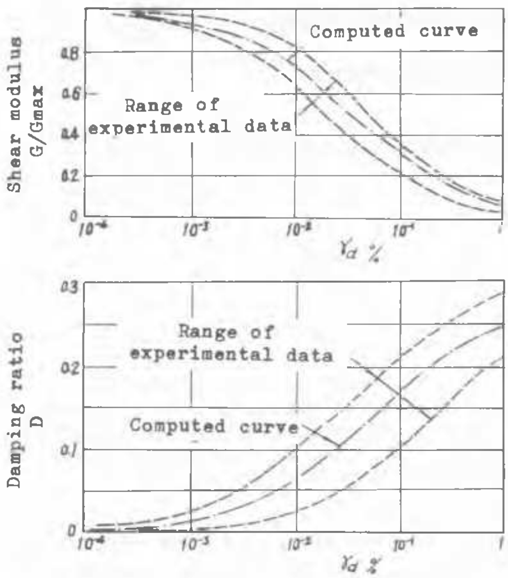

Fig.2 Shear Modulus and Damping

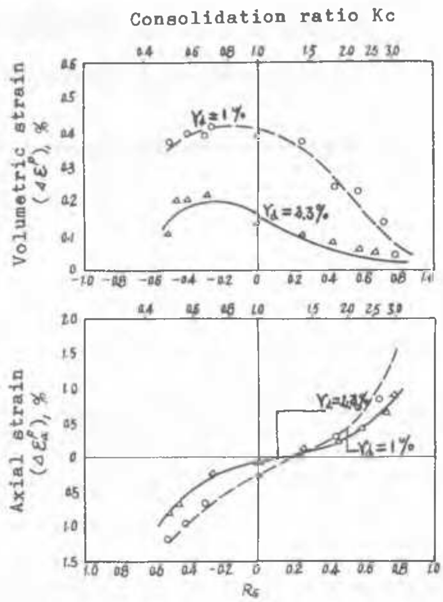

Fig.3 Residual Deformation

is simply equal to γ_d, then their formula can be written as follows

$$G = \frac{k(\sigma_s)^{\frac{1}{2}}}{1 + k/\sin\phi\, \gamma_d(\sigma_s)^{-\frac{1}{2}}} \qquad (10)$$

where $k = 325\,\frac{(2.973-e)^2}{1+e}$, e - void ratio and σ_s in kg/cm^2. Comparison of formulas (1) and (10) yields $k_1 = k/\sin\phi$, $k_2 = k$ and $(\gamma_d)^{\frac{1}{2}}$ in Eq.(1) instead of γ_d in Eq.(10). The last revision aims to make the calculated results better fitted to experimental data. The proposed formula (2) for damping ratio is also similar with that of Hardin and Drnevich. The curves of shear modulus and damping versus shear strain amplitude calculated from Eq.(1) and (2) for $\sigma_s = 1$ kg/cm^2, $k_1 = 10$ and Dmax = 0.27 are compared with the experimental data (Seed & Idriss, 1970) in Fig.2, where $G_{max} = k_2(\sigma_s)^{\frac{1}{2}}$. When σ_s increases formula (1) will give a little higher values of shear modulus than those in Fig.2.

When a fresh sample is loaded, Ne = 0 and formulas (3) and (4) can be written as

$$(\Delta\varepsilon^p)_1 = c_1(\gamma_d)^{c_2}\exp(-c_3 Rs^2) \qquad (11)$$

$$(\Delta\gamma^p)_1 = c_4(\gamma_d)^{c_5} Rs^2 \qquad (12)$$

where subscript 1 means the first loading cycle. For isotropically consolidated samples, Rs = 0, and these folmulas become

$$(\Delta\varepsilon^p)_1 = c_1(\gamma_d)^{c_2}, \quad (\Delta\gamma^p)_1 = 0$$

In the abovementioned formulas the amplitude of shear strain is adopted as a main variable, as firstly suggested by Martin et al.(1975). Yokel et al.(1980) further argued about the advantage of shear strain approach over shear stress approach. However, the threshold strain is neglected and a simple power function is assumed for the relationships between $\Delta\varepsilon^p$ and γ_d as well as $\Delta\gamma^p$ and γ_d.

The multipliers with Rs in Eq.(11) and (12) account for static stress state before applying current cycle of loading. The assumption of error function and parabolic law of change of $(\Delta\varepsilon^p)$ and $(\Delta\gamma^p)$ versus Rs is supported by the experimental data of a medium sand (Shen & Wang, 1980) as shown in Fig.3, where the negetive Rs means that the lateral confined stress $(\sigma_r)_c$ is larger than the axial stress $(\sigma_a)_c$. The unsymmetrical shape of curves are probably due to the inherent anisotropy of samples introduced during their preparation, which results in more compressibility of samples in the radial direction than in the axial direction.

The multiplier $\frac{1}{1+Ne}$ in Eq.(3) and (4) accounts for decreasing rate of build-up of residual deformation with the increase of loading cycles. More complicated multipliers have been proposed by Ishibashi (1977) and others. Fig.4 shows the experimental data of rate of pore pressure generation of a saturated sand under undrained condition and uniform strain amplitude of various levels. Naturally,in this situation Ne is equal to actual cycle number N. It seems that the approximation of this simplest function is acceptable, although the more complicated one fits the experimental results a little better.

DETERMINATION OF SOIL CONSTANTS

A detailed discription of the proposed procedure to determine the soil constants in related formulas has been given (Shen, 1984). Its main points are outlined below.

i). The modulus numbers k_1 and k_2 can be determined by cyclic triaxial test accompanied with resonant column test. When the test data are not available it is recommended to use Rechart's formula (1977)

$$k_2 = 700\,\frac{(2.17-e)^2}{1+e} \qquad (13)$$

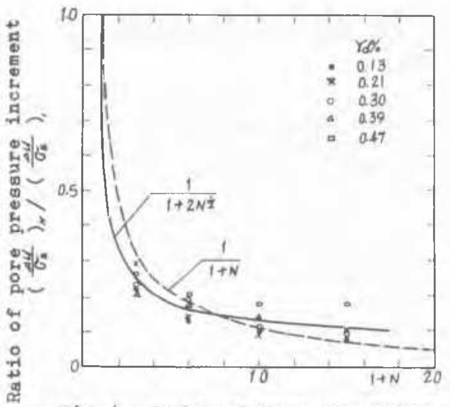

Fig.4 Rate of Pore Pressure Generation

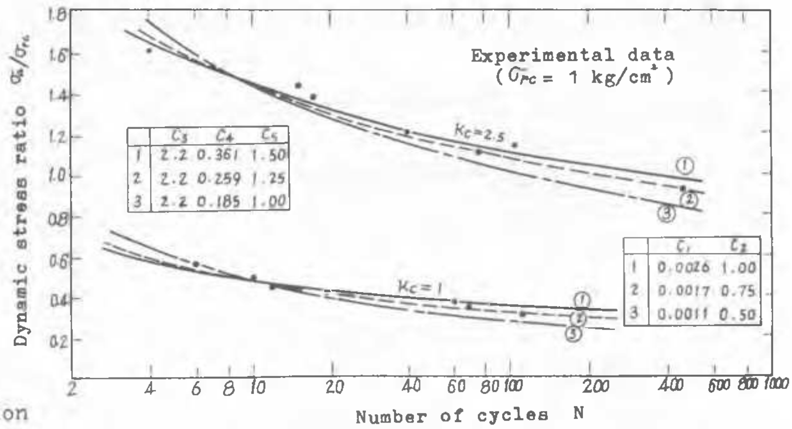

Fig.5 Cyclic Strengh of Chaihe Sand

and to take $k_1 = 10$ or $k_1 \approx 6$ when double amplitude γ_d is used in Eq.(5). Dmax is assumed equal to 0.27 in case of sands.

ii). The residual deformation coefficients c_1, c_3 and c_4 can be determined from cyclic triaxial test data under certain prescribed values c_2 and c_5. Meanwhile, c_1 is obtained by testing isotropically consolidated samples, and c_3 and c_4 - samples consolidated under stress ratio Kc = 2 or 2.5. A computer program SCUDA has been developed to determine these coefficients. Fig.5 shows the cyclic strengh curves of Chaihe sand (a medium sand with ralative density 60%) computed with 3 different values of c_2 and c_5 for isotropically and anisotropically consolidated samples respectively. The failue criterion for the anisotropically consolidated samples is 5% residual axial strain. The experimental results are also shown in the Figure. It seems that the influence of c_2 and c_5 on the form of curves is not very large, and it is recommended to take $c_2 = 3/4$ and $c_5 = 1$ for most of sands.

iii). An all-round static compression and rebound test is needed to determine the unloading modulus number k_u. Attention must be paid to correct the menbrane penetration effect in such a test. The loading modulus number k_b can also be deduced from this test. The another shear modulus number k_d used in static stress analysis as well as angle of internal friction ϕ can be determined from static triaxial test results.

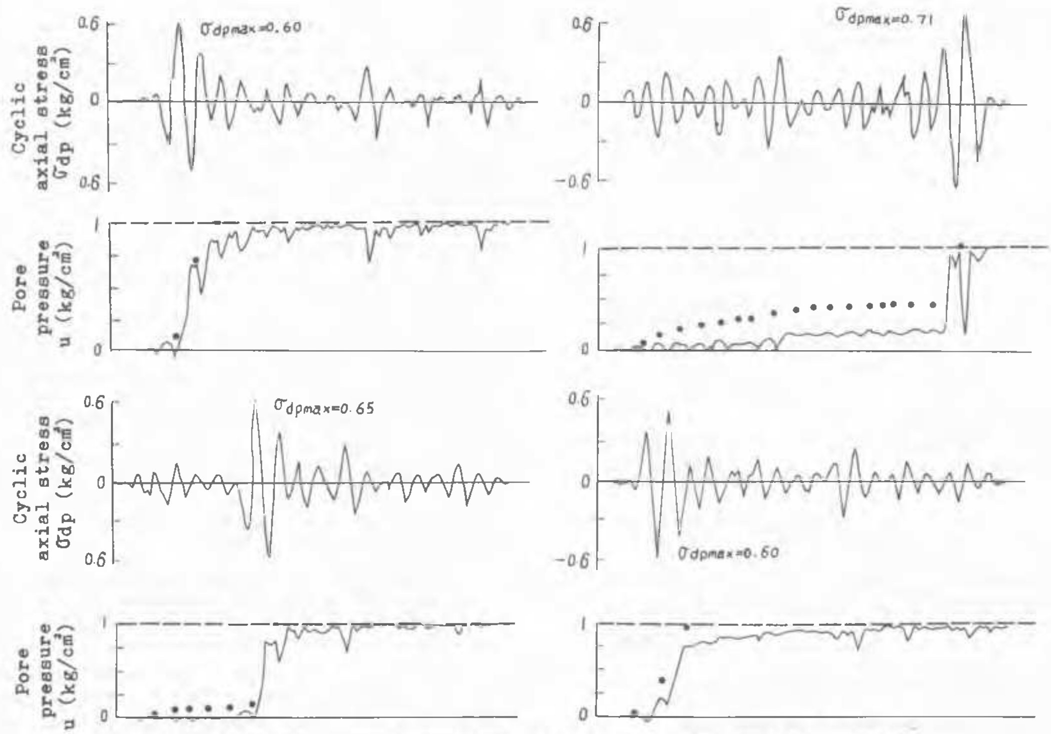

Fig.6 Verification of Model for Irregular Loading

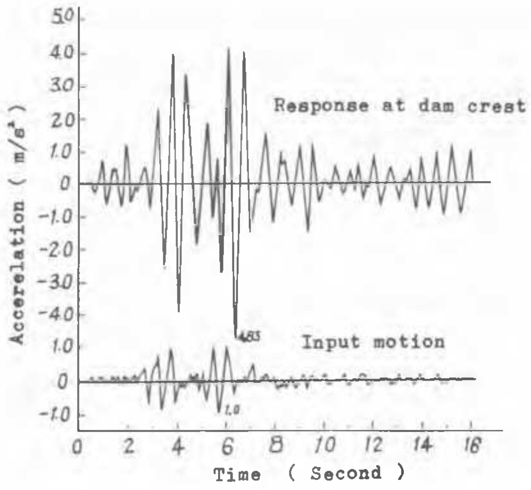

Fig.7 Input and Response Accerelation

VERIFICATION

The test data of random loading provided kindly by Shen C.K. (1978) are used to verify the validity of the proposed model. For Ottawa sand of Dr = 65% and e = 0.61, k_2 = 1050 is computed from Eq.(13). Assuming k_1 = 6 and k_4 = 840, c_1 = 0.0025 is obtained from cyclic strengh data when c_2 = 3/4 is used. c_3 and c_4 are not needed for the isotropically consolidated condition. The comparision between computed results and measured data is shown in Fig.6. It can be seen that the model gives some higher pore pressure values for low stress amplitude range.

APPLICATION

The previously developed computer program EFESD (Shen, 1980) is revised accordingly. Its capability of dealing with the seismic stability problems of earth structures is demonstrated by the following example.

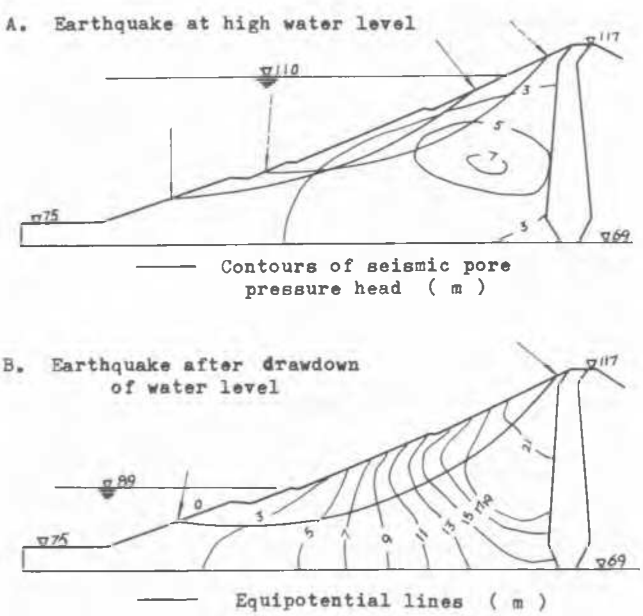

Fig.8 Seismic Pore Pressure and Potential Sliding Surfaces

Chaihe earth dam with sand shell and clay core is located in the north of Liaoning province. The stability of its upstream slope during an hypothetical earthquake is evaluated. The soil constants used in computation are: k_1= 6, k_2= 1000, k_4= 780, c_1= 0.0017, c_2= 0.75, c_3= 2.2, c_4= 0.18, c_5= 1 and $\sin\phi$ = 0.7. The input bedrock motion and response accerelation at the crest of dam are given in Fig.7. The computed pore pressure distribution and potential sliding surfaces with factor of safety equal to 1 at the end of earthquake for two particular cases of reservior water level are shown in Fig.8. It is concluded from the analysis that the dam may suffer moderate but not catastrophical damage, probably like that which happened in the Shimenling earth dam located in the south of this province during Haicheng earthquake on Feb. 4, 1975 (Shen, 1981). The suggestion is that the reservior should not be emptied in a hurry before a coming earthquake.

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