Dynamic shear modulus of soils

Module de cisaillement dynamique des sols

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SYNOPSIS The dynamic shear modulus of soils is one of the important quantities required for the prediction of the dynamic response of the foundations. This paper presents the results on the variation of the dynamic shear modulus with dynamic and static loads, area of foundation and the soil type. The dynamic shear modulus was calculated from the resonance frequency obtained on a field vibration test. Experimental investigation carried out by the authors and the published results were made use off to calculate the modulus. Further, a new method of calculating the spring constant viz., dynamic load divided by the corresponding displacement amplitude at resonance was used and the computed spring constant was correlated with the spring constant obtained from resonance frequency. It was observed that a good correlation could be achieved.

INTRODUCTION

The importance of dynamic shear modulus in the design of a foundation - soil system to resist dynamic loads was stressed by many in the past (e.g. Hardin and Black, 1968; Richart et al., 1970). Several experimental methods have been evolved from time to time to determine the dynamic shear modulus in the laboratory and the field as well. These can be classified as static and dynamic. Of these, the field vibration test on model footings is a more realistic method of determination of the shear modulus since they are essentially performed on undisturbed soil under field conditions. Also, these tests provide the average value of G for the soil tested. In this paper, an attempt is made to study in detail the significant factors affecting the dynamic shear modulus under different test conditions. Also, a new method has been proposed to obtain the dynamic spring constant and the same has been compared to the conventionally obtained spring constant from frequency at resonance.

EXPERIMENTAL PROGRAMME

A number of field tests were conducted at the Indian Institute of Science to study the effect of amplitude on shear modulus under varying test conditions using a Lazan oscillator. The tests were conducted with a plain concrete footing of size 0.45m x 0.45m x 0.15m. The static load on the soil was varied by changing the surcharge over the top of the oscillator. Four eccentric moments were applied under each static load. The vibrations were picked up by means of an electrodynamic pick-up and were recorded by a vibration meter. The shear modulus / spring constant was computed from the resonance frequency considering the rigid base pressure distribution as

$$k_f = \frac{4\pi^2 W (f_r)^2}{g} \qquad (1a)$$

$$G = \frac{\pi^2 W (f_r)^2 (1-\mu)}{g r_o} \qquad (1b)$$

where f_r = Frequency at resonance
W = Total static load
k_f = Spring constant at f_r
r_o = Equivalent radius
g = Acceleration due to gravity
μ = Poisson's ratio for the soil

Tests were also conducted with embedded footings without soil friction by providing for an air gap around the footings. These footings were cast-in-place at depths equal to 0.6m and 1.2m and to the same heights. The behaviour on different soils and under different modes of vibration was also studied from the data available in literature. Only a few of these results are discussed in the following text for want of space as the behaviour was broadly the same in the other cases also.

Analysis

Figs. 1 and 2 present the variation of shear modulus with respect to the displacement amplitude at resonance under different static loads for red earth at the I.I.Sc. site and silty clay at the Vicksburg site (Fry, 1963) respectively. Using the test results of Fry (1963), Fig.3 shows the variation of shear modulus with displacement amplitude for different contact areas, the total static load being constant. Similar results obtained for loess loam are shown in Fig.4 using the published data of Novak (1970). The effect of static load and contact area can be easily

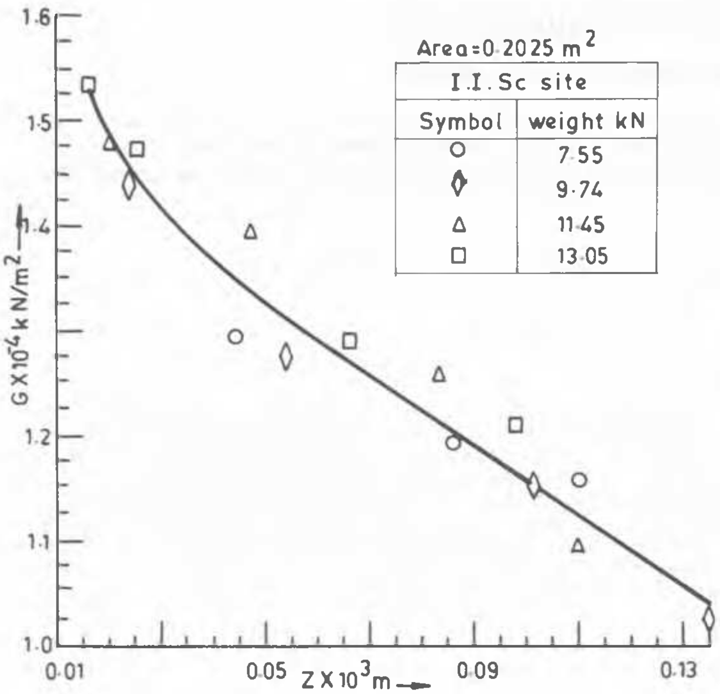

Fig.1 Resonance Amplitude vs Dynamic Shear Modulus for Red Earth

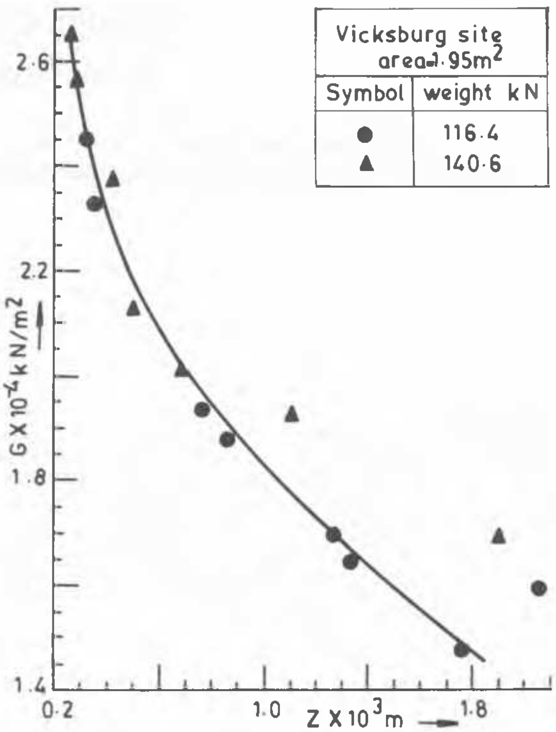

Fig.2 Resonance Amplitude vs Dynamic Shear Modulus for Silty Clay for ConstantArea

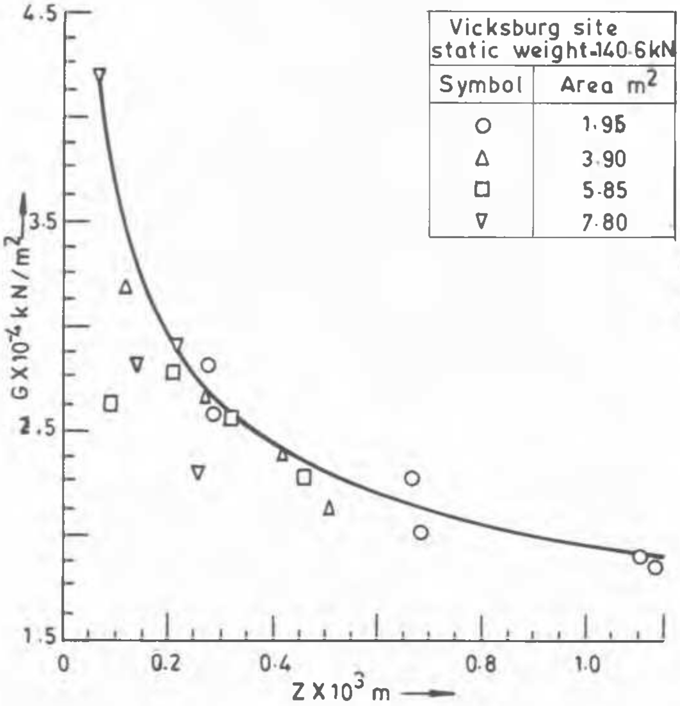

Fig.3 Resonance Amplitude vs Dynamic Shear Modulus for Silty Clay for Equal Equal Weight of Foundation

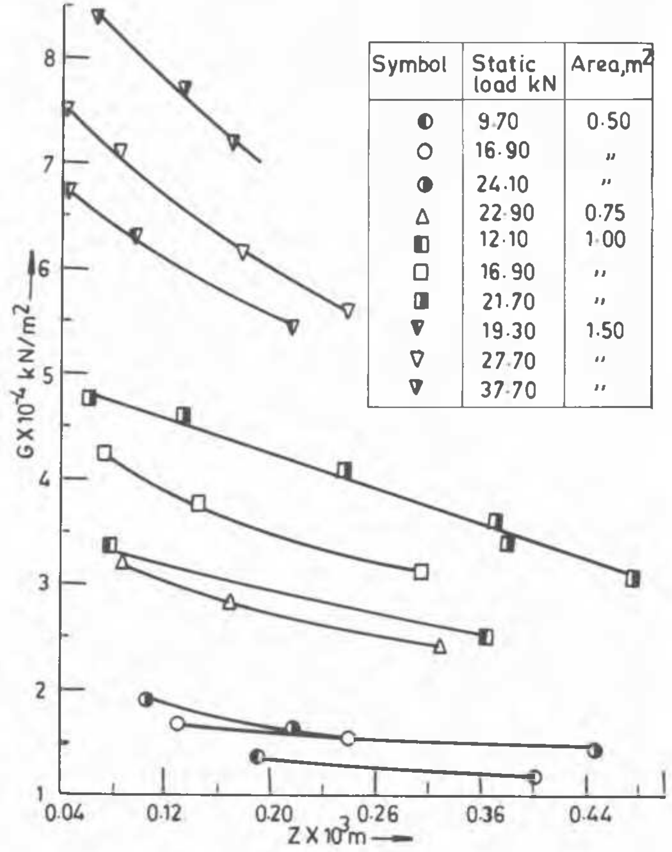

Fig.4 Resonance Amplitude vs Dynamic Shear Modulus for Loess Loam

seen in this figure. All the results presented in figures 1 to 4 are for vertical vibration.

From these results it could be seen that the shear modulus significantly decreases with the increase of amplitude at resonance. For the red earth (I.I.Sc. site) and silty clay, the influence of static load on the shear modulus is negligible. From fig.3 it is also seen that the area effect is marginal. This shows that the resonance amplitude which is a function of area and static load can be taken as a single parameter influencing the shear modulus. Similar conclusion could be made from the analysis of the test results of Fry (1963) on uniform fine sand at the Eglin site, and tests on beach sand by Chae (1969). Contrary to the above, the results illustrated in fig.4 show that the effect of static load and area cannot be fully represented by displacement amplitude. However, all the above results clearly reveal the dependence of shear modulus on the displacement level. This is but naturally attributed to the nonlinear behaviour of soil.

The shear modulus can be related to the resonance amplitude by means of an exponential curve of the form $G = A\ e^{Bz}$ where A and B are constants and z is the amplitude at resonance. The coefficients A and B from statistical fits are presented in Table I.

TABLE I

Values of the Regression Coefficients and Correlation Coefficient for the equation of the form $G = A.\ e^{Bz}$ for different soils

Soil Type	Regression Coefficients		Correlation Coefficient
	$A \times 10^{-4}$ kN/m^2	B	r
Red earth (Fig.1)	1.535	-0.263	0.962
Silty clay (Fig.2)	2.634	-0.029	0.915
Silty clay (Fig.3)	2.670	-0.023	0.789
Loess loam (Fig.4)	3.354	-0.280	0.750

In Fig.5 are presented results showing the variation of shear modulus with respect to resonance amplitude for different depths of embedment of foundation in red earth at the I.I.Sc. site. The increase in shear modulus with an increase in the depth of embedment is evident from the results. Also, it can be seen that the increase is not proportional when the depth of embedment is doubled. Analysis of the results of Ananda Krishna and Krishnaswamy (1973) and Novak and Beredugo (1971) for tests on silty clay with soil friction for varied embedments also show an increase of one and a half to two times in the value of G with the increase of embedment by two to five fold.

Fry's (1963) test results from Vicksburg site in rocking (vertical displacement only) and torsional modes were also analysed and presented in Table II. The decrease of G with the increased resonance amplitude could be seen for these modes. The difference between the G values for the two modes of vibration is significantly large for the same range of angle of rotation especially, at lower static and dynamic intensities. The shear modulus obtained in torsional mode is almost 10 times that of rocking mode.

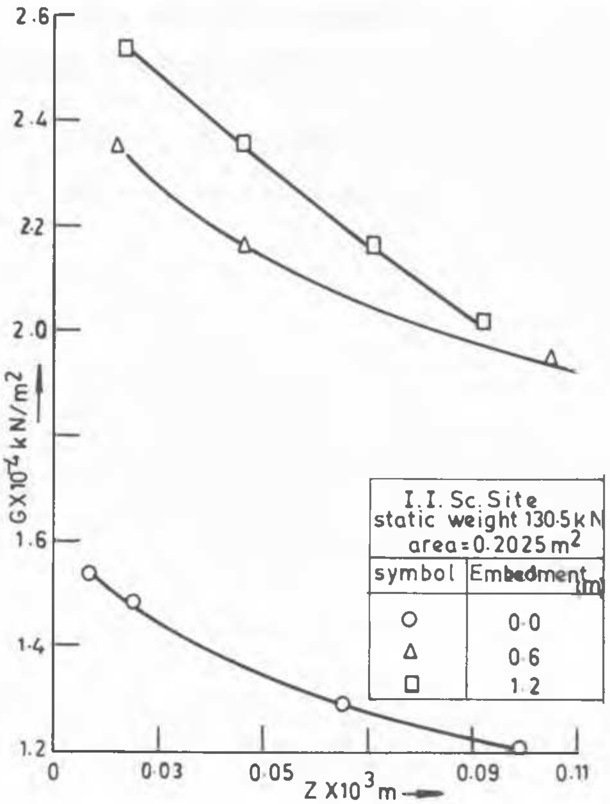

Fig.5 Resonance Amplitude vs Dynamic Shear Modulus for Red Earth

TABLE II

Comparision of G in Rocking and Torsional Modes from Fry's (1963) Test Results for Vicksburg site

r_o	Static intensity	eccentricity	Rocking		Torsional	
			$A \times 10^{6}$	$G \times 10^{-4}$	$A \times 10^{6}$	$G \times 10^{-4}$
m	kN/m^2	θ °	Radians	kN/m^2	Radians	kN/m^2
1.11	21.65	3	0.18	4.33	0.31	0.46
		6	0.36	3.21	0.52	0.36
		9	0.61	2.60	0.81	0.32
		12	0.77	2.22	1.01	0.30
0.79	71.71	3	0.11	1.95	0.26	1.04
		6	0.42	1.18	0.54	0.98
		9	0.84	0.79	0.81	0.77
		12	1.18	0.73	0.95	0.64

DYNAMIC SPRING CONSTANT

As a simple method of determining the dynamic spring constant, a new procedure is proposed with following definition. The dynamic spring constant is the ratio of the dynamic load at the resonance frequency to the corresponding amplitude. This spring constant has been designated as k_A as it is obtained from the resonance amplitude and is related to the one obtained conventionally from resonance frequency from eqn.1. Fig.6 presents typical results of spring constant obtained from resonance amplitude. It can be seen that there is an unique relationship irrespective of static load or area of foundation. Assuming a linear relationship between k_A and k_f, i.e. $k_f = A + Bk_A$, A and B coefficients are obtained from statistical fits and are tabulated in Table III. Considering the vagaries in the vibration tests, the correlation between the two constants are reasonably good.
Table III includes results analysed from Fry (1963) and Novak (1970).

TABLE III

Values of Regression Coefficients and Correlation Coefficient for the equation of the form $k_f = A + Bk_A$ for different soils

Soil Type	Regression Coefficients		Correlation Coefficient
	$Ax10^{-5}$ kN/m	B	r
Red Earth	0.12	0.425	0.888
Silty clay (constant area tests,Fry,1963)	0.62	0.977	0.787
Silty clay (equal weight tests,Fry,1963)	0.93	0.452	0.971
Loess Loam (Novak,1970)	0.23	1.345	0.935

CONCLUSIONS

1. The dynamic shear modulus is strongly dependent on the displacement amplitude and its behaviour could be explained with an exponential relationship with a reasonable accuracy.
2. The shear modulus - displacement relationship is marginally influenced by the static load level, and their relationship can be taken as unique.
3. Except for loess loam the effect of area on the shear modulus is marginal.
4. G increased significantly with an increase in the depth of embedment.
5. The dynamic spring constant defined as the ratio of dynamic load to amplitude at resonance can be correlated to that obtained conventionally from frequency with a fair degree of accuracy.

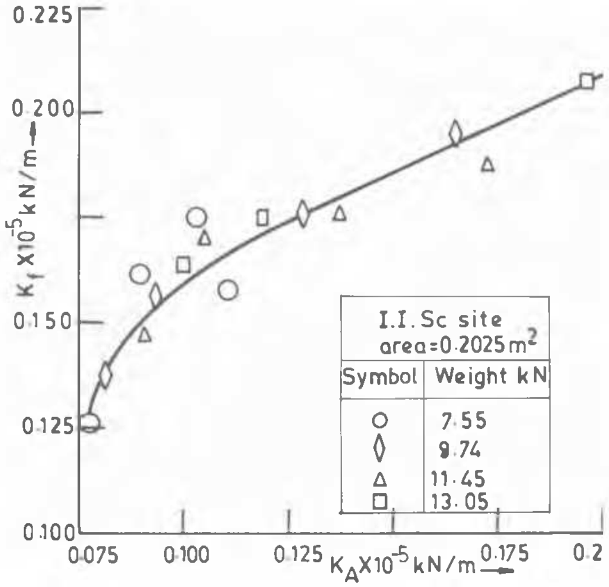

Fig.6 Relationship between k_A and k_f

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