On the drained and undrained analyses of poroelastic media

A propos des analyses drainées et non-drainées sur milieux poro-élastiques

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SYNOPSIS A parametric study using plane strain analysis is presented to obtain an insight of the validity of the drained and undrained analyses resulting from the reduction of Biot's consolidation equations for wave-induced offshore foundations. The results show that the drained analysis can be used, provided that the common drained assumptions are met. On the other hand, the validity of the undrained analysis is limited to a few cases only, even if the common undrained assumptions are satisfied.

INTRODUCTION

Biot's(1941) consolidation equations have been extensively used in the wave-induced response analysis of offshore foundations. In general, the equations are applied to an anisotropic, nonelastic and nonlinear porous media representing the foundation soil. However, results of recent model tests (Lindenberg et al., 1982) and field measurements of offshore gravity structure foundations (Mizikos et al., 1982) indicate that the wave-induced response can be predicted with reasonable accuracy if the foundation soil is assumed to be elastic.

In the analysis of wave-induced response of offshore foundations, Biot's consolidation equations for elastic soils is often reduced to two extreme conditions, i.e. the drained and the undrained conditions. The drained analysis utilizing Laplace equation is commonly used when the hydraulic conductivity and the soil skeleton stiffness are relatively high. On the other hand, if the hydraulic conductivity is relatively low, the analysis is often simplified to an undrained analysis using an equivalent elastic equation.

This paper presents the results of a parametric study on the wave-induced response of linear elastic offshore gravity structure foundations. The study was conducted in order to obtain a better insight of the basic assumptions and the validity of the drained and undrained analysis for a range of typical soil properties.

THEORETICAL BACKGROUND

Biot's consolidation equations for an isotropic poroelastic media saturated with fluid can be written as follows

$$\frac{K}{\gamma_f}\nabla^2 p = n\beta\frac{\partial p}{\partial t} + \frac{\partial \varepsilon}{\partial t} \tag{1}$$

$$G\nabla^2 u + (\lambda + G)\frac{\partial \varepsilon}{\partial x} = \frac{\partial p}{\partial x} \tag{2a}$$

$$G\nabla^2 v + (\lambda + G)\frac{\partial \varepsilon}{\partial y} = \frac{\partial p}{\partial y} \tag{2b}$$

$$G\nabla^2 w + (\lambda + G)\frac{\partial \varepsilon}{\partial z} = \frac{\partial p}{\partial z} \tag{2c}$$

where K is the hydraulic conductivity, γ_f is the unit weight of fluid, p is the pore pressure, n is the porosity, β is the fluid compressibilty, ε is the volumetric strain, G is the shear modulus, λ is the Lame's constant, and u, v and w are the displacements in the x, y and z directions, respectively. If the problem considered is harmonic, Equations 1 and 2 can be simplified by allowing $p = P'\exp(i\omega t)$, $u = U'\exp(i\omega t)$, $v = V'\exp(i\omega t)$ and $w = W'\exp(i\omega t)$; where P', U', V' and W' are the amplitudes of p, u, v and w, respectively. Furthermore, the governing equations can be nondimensionalized by letting $x' = x/a$, $y' = y/a$, $z' = z/a$, $P = P'/P_o$, $U = U'G/(aP_o)$, $V = V'G/(aP_o)$ and $W = W'G/(aP_o)$ where x', y', z', P, U, V and W are all dimensionless, a and P_o are the reference values for length and stress, respectively. The nondimensionalized equations can be written as

$$- i\kappa\nabla^2 P = \xi P + \varepsilon' \tag{3}$$

$$\nabla^2 U + \frac{1}{1 - 2\nu}\frac{\partial \varepsilon'}{\partial x'} = \frac{\partial P}{\partial x'} \tag{4a}$$

$$\nabla^2 V + \frac{1}{1 - 2\nu}\frac{\partial \varepsilon'}{\partial y'} = \frac{\partial P}{\partial y'} \tag{4b}$$

$$\nabla^2 W + \frac{1}{1 - 2\nu}\frac{\partial \varepsilon'}{\partial z'} = \frac{\partial P}{\partial z'} \tag{4c}$$

in which κ is $KG/(\gamma_f a^2\omega)$, ξ is nβG, ν is the Poisson's Ratio, ε' is the nondimensionalized volumetric strain, and ω is the angular frequency of the harmonic motion.

Equations 3 and 4 show that the wave-induced response of a poroelastic seabed is controlled by the soil properties through κ, ξ and ν. The dimensionless parameter κ is defined as the poroelasticity factor, since it contains the elastic and the porous properties of the soil-fluid matrix. In addition, κ also includes a geometric factor through the parameter a as well as a wave parameter through ω. The parameter ξ is defined as the fluid compressibility factor because it delineates the contribution of the fluid compressibility to the total response. Toha (1983) and Edil and Toha (1984, 1985) showed that the use of κ and ξ can also be used for the experimental measurements of the elastic properties of a wide range of soil types.

In using Biot's consolidation equations, very often the governing equations are reduced to one of its extreme conditions. The drained condition is usually assumed when the lefthand term of Equation 3 is relatively large compared to the righthand terms. This condition can be achieved if κ is large and/or if $\xi P + \varepsilon'$ is small. Physically this means that the drained condition can be achieved if the porous media is highly permeable and the soil skeleton is very stiff. The undrained condition, on the other hand, is usually assumed when κ gets very small since the term "undrained" implies that there are no relative flow between the fluid and the soil skeleton. The undrained condition is usually assumed when the hydraulic conductivity is small.

The drained and undrained conditions offer a great simplification to the analysis. When the drained condition is achieved, Equation 3 can be reduced to

$$\nabla^2 P = 0 \tag{5}$$

which is nothing more than the Laplace equation. This also means that the pore pressure can be solved independently from Equation 5. The undrained condition dictates that $\kappa \to 0$, thus Equation 3 reduces to

$$P = -\frac{\varepsilon'}{\xi} \tag{6}$$

and if Equation 6 is substituted into Equation 4, the results will be

$$\nabla^2 U + \frac{1}{1-2\nu'}\frac{\partial \varepsilon'}{\partial x'} = 0 \tag{7a}$$

$$\nabla^2 V + \frac{1}{1-2\nu'}\frac{\partial \varepsilon'}{\partial y'} = 0 \tag{7b}$$

$$\nabla^2 W + \frac{1}{1-2\nu'}\frac{\partial \varepsilon'}{\partial z'} = 0 \tag{7c}$$

where ν', the equivalent Poisson's Ratio, satisfies

$$\frac{1}{1-2\nu'} = \frac{1}{1-2\nu} + \frac{1}{\xi} \tag{8}$$

It is clear that Equation 7 is an equivalent elastic equation with a modified Poisson's Ratio. This would enable one to solve Equation 7 using classical theory of elasticity.

Zienkiewicz et al. (1980) explored the possible simplifications resulting from the drained and undrained assumptions. An inspection of the range for the validity of the drained and undrained assumptions was presented for a simple case. The analysis showed that the drained or undrained behavior was mainly governed by a single parameter very similar to κ. In a more advanced application, Mei and Foda (1981a) showed that another parameter, also similar to κ, remained sufficiently small for most of the domain in wave-induced offshore foundation problems, hence, the problem could be reduced to an equivalent elastic problem. A correction for pore pressure was applied afterwards to a thin boundary layer. This approach was named the boundary layer theory and was lateron applied to several specific cases (Mei and Foda, 1981b, Mynett and Mei, 1982, and Mei, 1982). A similar approach was also reported by Verruijt (1982).

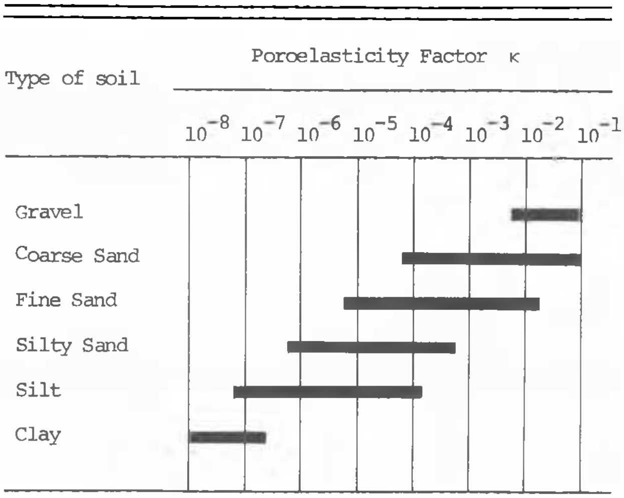
The aforementioned studies show that the drained undrained condition is met when a parameter very similar to κ approaches infinity or zero, respectively. There is no further inspection of the terms $\kappa\nabla^2 P$ and $\xi P + \varepsilon'$ when κ gets large or small. It should be noted that the term $\kappa\nabla^2 P$ may become small even if κ is large, since the pore pressure is not necessarily large compared to ε'. Oppositely, $\kappa\nabla^2 P$ may remain relatively large compared to $\xi P + \varepsilon'$ at low values of κ since P may get large. Moreover, when Biot's consolidation equations are reduced, the order of the governing differential equations for the drained and undrained analyses is also reduced. Therefore, the number of boundary conditions required will be less than what is available from the original problem. The study presented here explores the behavior of Equation 3 when κ is large and small, particularly for a plane strain analysis of a two dimensional offshore gravity structure foundation.

PARAMETRIC STUDY

A parametric study was conducted in order to gain better understanding of the behavior of Equation 3 at high or low values of κ. The range of the parameters was selected based on several published data. The weight, size and other geometric dimensions of 13 concrete gravity platforms in the North Sea were given by Eide et al. (1979). Wave properties were determined from the data presented by Thrasher and Aagaard (1969), Bjerrum (1973), Clausen et al. (1975), Lee and Focht (1975) and Rahman et al. (1977). Based on the available data and taking the half base width of the structure as the reference length a, a range of κ for the parametric study was established as shown in Table I.

Table I

Range of Poroelasticity Factor (κ) for Offshore Gravity Platform Foundations



The parametric study was conducted on a plane strain problem as shown in Figure 1. A numerical analysis using finite difference method was performed on 144 cross sections with varying values of L (wave length), a (half base width), H (sea bed thickness), ν, ξ and κ.

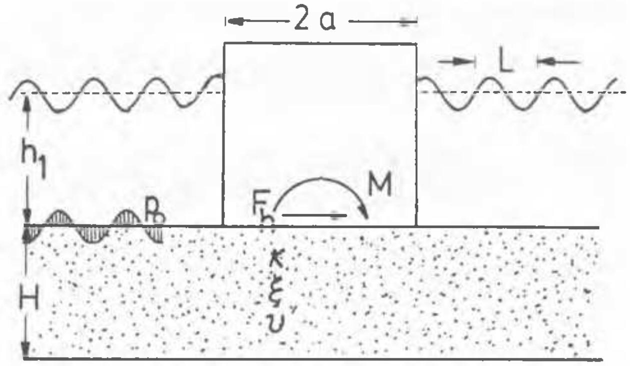


Figure 1 Plane Strain Problem

The reference stress of the numerical analysis is taken from the pore pressure amplitude at the sea bottom, shown as P_o in Figure 1.

RESULTS

The results of the analysis on the selected 144 sections were presented elsewhere by Toha (1983). A typical curve of the ratio of the pore pressure amplitude along the base of the structure (P) to the reference stress (P_o) is shown in Figure 2. The results of the drained analysis was obtained by solving the Laplace equation $\nabla^2 P = 0$ using finite difference analysis.

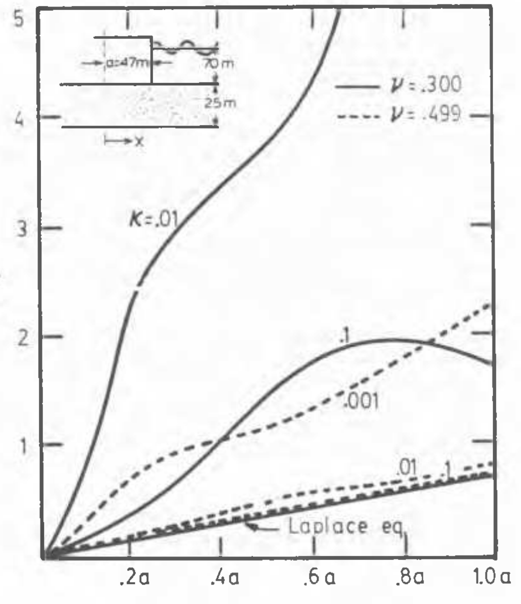


Figure 2 Pore Pressure Ratio along Base of Structure

The results in Figure 2 show that the pore pressure ratio P/P_o from Biot's consolidation equations approaches the P/P_o from the drained analysis when κ = 1 for ν = 0.300 and when κ = 0.1 when ν = 0.499. This implies that the higher pore pressures from Biot's consolidation equation are due to the elastic volumetric strain produced by the wave-induced structural forces exerted on the base of the structure, the smaller the volumetric strain, the smaller the pore pressure increase would be. In a more refined analysis where the wave-induced forces were considered separately, P/P_o from Biot's consolidation equations approached the P/P_o of the drained analysis at values of κ as low as 0.01 if the sea bottom pressure penetration is the dominant force. Comparing the above results to the range of κ in Table I, it is clear that the drained condition would prevail if κ is larger than 1 and ν is close to 0.5, i.e. if the seabed is highly permeable and stiff so that the volumetric strains remain small. This is identical to the common requirements for the drained condition, therefore the drained analysis seems justified once the drained assumptions are met.

A comparison of pore pressures at low values of κ from Biot's consolidation equations to those from the undrained analysis is not presented in Figure 2. As mentioned previously, when Biot's consolidation equations are reduced to the equivalent elastic equations, the governing differential equations are actually reduced to lower order differential equations. Therefore, since Biot's consolidation equations must satisfy six boundary conditions in a plane strain problem, whereas the equivalent elastic equations only need four, there will two boundary conditions that cannot be satisfied by the undrained analysis. The difference between the results of Biot's consolidation equations and the undrained analysis is highly dependent on which four of the six boundary conditions are satisfied in the undrained analysis. The results of such study are not presented here due to practical reasons, however, it can be reported that there were no definitive pattern observed during the course of the parametric study. Mei and Foda (1981a) obtained comparable results from their undrained analysis to the results of Yamamoto (1978) due to the favorable boundary conditions of the particular case. In this particular case, the boundary conditions produced a decoupling between the pore pressure and the strain equations. Hence, the displacements could be calculated first by solving the equivalent elastic equation and then followed by pore pressure computations. The pore pressures were corrected afterwards using a boundary layer correction.

The validity of the drained and undrained analyses for high or low values of κ was investigated in the parametric study by evaluating the terms of Equation 3 after the solutions were obtained. Figure 3 illustrates a typical result of such evaluation. The ratio of the argument of $i\kappa\nabla^2 P$ to the argument of $\xi P + \varepsilon'$ was evaluated for a range of κ values.

The results in Figure 3 show that the ratio of $i\kappa\nabla^2 P$ to $\xi P + \varepsilon'$ increases significantly as κ approaches unity. This was explained previously, since for ν = 0.49 and κ ≒ 1 the response approaches the response of the drained analysis. On the other hand, $i\kappa\nabla^2 P$ does not become small quickly as κ decreases. Taking the values of κ from Table I, the particular case shown in Figure 3 would not exhibit negligible magnitudes of $i\kappa\nabla^2 P$ until a clay characteristic is found. This is due to the fact that the pore pressure also increases as κ decreases. When the value of κ is reduced by decreasing the hydraulic conductivity, or increasing the width of structure, or increasing the wave frequency, the ability of the pores to dissipate the pore pressure decreases. In fact, when the Poisson's Ratio is low, the pore pressures may become even higher. It is clear from this typical result that a low value of κ does not necessarily reduce Equation 3 into the equivalent elastic equations of the undrained analysis.

Another interesting aspect of the behavior of Equation 3 was observed during the parametric study. At very low values of κ, the finite difference equations started to become unstable. This could be caused by the significant difference in order of magnitude of the

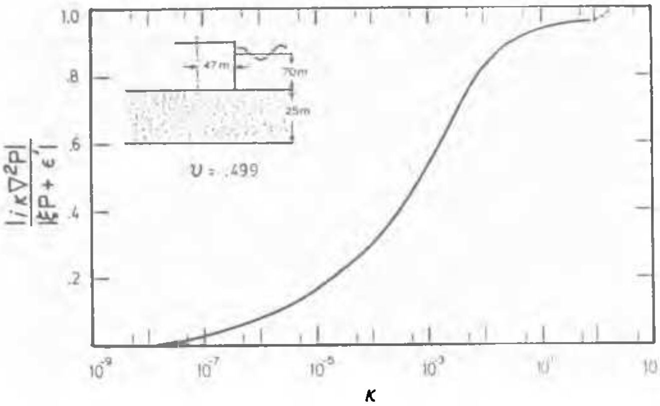


Figure 3 Behavior of Equation 3 at Various κ Values

coefficients in the system of finite difference equations, but more likely due to the fact that the finite difference equations were attempting to approach Equations 6 and 7. This means that there were two boundary conditions in excess of the four required boundary conditions. The extra two boundary conditions formulated in the numerical analysis caused the coefficient matrix of the finite difference equation to behave as a singular matrix. In the case of the high κ values, the observed instablity was more likely caused by round off errors. A method to cure the instablity of the finite difference equations is discussed in a pending publication.

CONCLUSIONS

The results of a parametric study of wave-induced response of poroelastic offshore gravity foundations using plane strain analysis indicate that the pore pressures can be estimated using the drained analysis if the poroelasticity factor κ is high, and the volumetric strain is low. When κ is high, a complete decoupling of Biot's concolidation equations occur so that the pore pressures can be solved from the Laplace equation while the deformations are obtained from the equivalent elastic equations.

The undrained analysis is applicable for a very limited cases only. It was shown that even if κis low, the term $\kappa\nabla^2 P$ may not be small due to the accompanying pore pressure increase. Furthermore, the undrained analysis requires fewer boundary conditions which generally provides different results than those of Biot's consolidation equations at low values of κ. The undrained analysis is applicable only if the boundary conditions are such that complete decoupling occurs.

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