Elasto-plastic constitutive model for dense sand

Modèle constitutif élasto-plastique pour sable dense

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SYNOPSIS An elasto-plastic constitutive relationship has been developed for dense sand which accounts for the effect of stress-path and insitu stress condition. Axi-symmetric stress condition has been taken. Any stress-path is considered to consist of two components - consolidation and shear and the deformation due to both are summed up to obtain the total effect. The predictions of the results have been compared with experimental data available in the literature. It is shown that the constitutive relationship is capable of predicting the behaviour of dense sand satisfactorily.

INTRODUCTION

In the analysis of any geotechnical engineering problem using numerical methods, the accuracy of the results depends upon the assumptions made in modelling the soil behaviour. The behaviour of soil is influenced by a number of factors such as soil type, density, stress level, stress path, drainage condition, grain shape and size, stress history etc. A number of constitutive models are being developed considering the effect of one or more of the above mentioned factors. However, the effect of stress-path and insitu stress condition have not been adequately considered in these models, particularly with respect to dense sands. In this paper, an elasto-plastic constitutive model has been developed for dense sand which takes into account the effect of stress path and insitu stress condition. The model is restricted to axi-symmetric stress condition.

CONSTITUTIVE MODEL

In the development of the constitutive model, mean stress (p) - deviatoric space (q) has been used(Fig.1). The stress ratio η is defined as $\eta = q/p$.

Concept

Consider a sample consolidated to a stress level O_1 isotropically and then sheared along a stress-path O_1D (Fig. 1a). The volumetric strain (v) and shear strain (γ) in the sample due to the change of stress O_1A due to consolidation and effect of A'A due to shear. The same concept is extended to extension side as well (Fig. 1b). For the anisotropically consolidated samples, the reference line is the anisotropic consolidation line and the changes in p and q are calculated with respect to that line as shown in Fig. 1 c,d. The zones above and below consolidation lines are shown as zone A and zone B.

The volumetric and shear strains have elastic and plastic components. In pure shear test (p-constant) no elastic volumetric strain is induced and the elastic shear strain has been found negligible in the case of dense sand (Vermeer, 1978, Misra, 1981). In isotr-

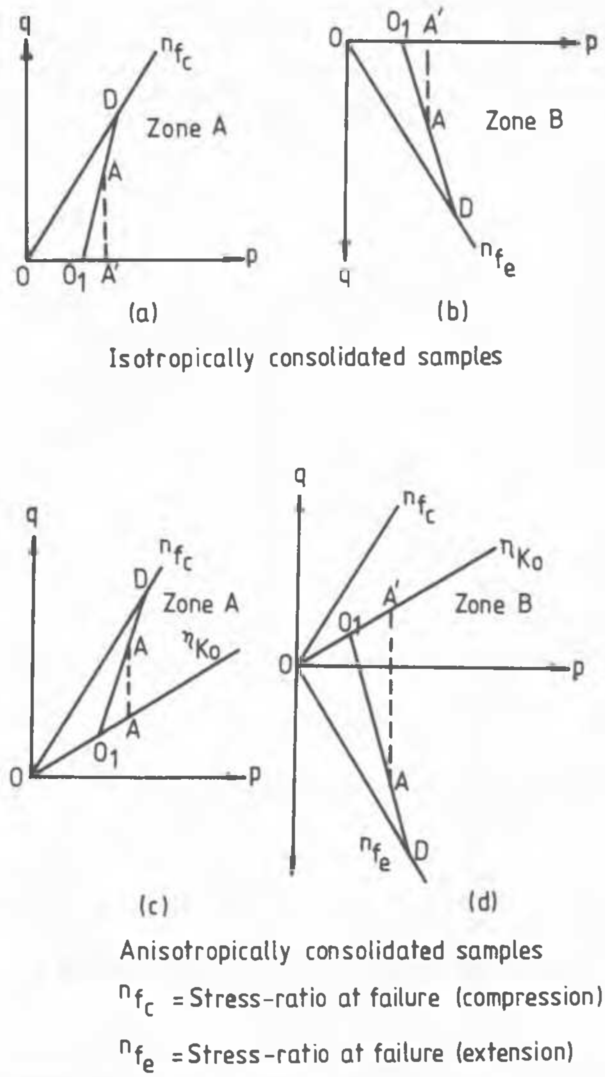

Fig. 1 Concept of Model.

opic consolidation test, the elastic and plastic shear strains are zero. Thus, in the incremental form, the strain relations can be written as

$$dv = dv_p^e + dv_p^p + dv_\eta^p \qquad (1)$$

$$\text{and} \quad d\gamma = d\gamma_\eta^p \qquad (2)$$

where dv and $d\gamma$ = increments in total volumetric and shear strains.

dv_p^e and dv_p^p = increments in elastic and plastic volumetric strains due to change in p, i.e. consolidation.

dv_η^p and $d\gamma_\eta^p$ = increments in plastic volumetric and shear strains due to change in η, i.e. shear.

From equations (3) and (4) it is seen that the elastic strain components are due to consolidation only whereas plastic strains are caused by both consolidation and shear. The elastic strain components are calculated using elasticity equations. For determining plastic strains, two different yield functions with hardening laws are considered - one for consolidation and the other for shear. The yield surfaces are shown in Fig.2.

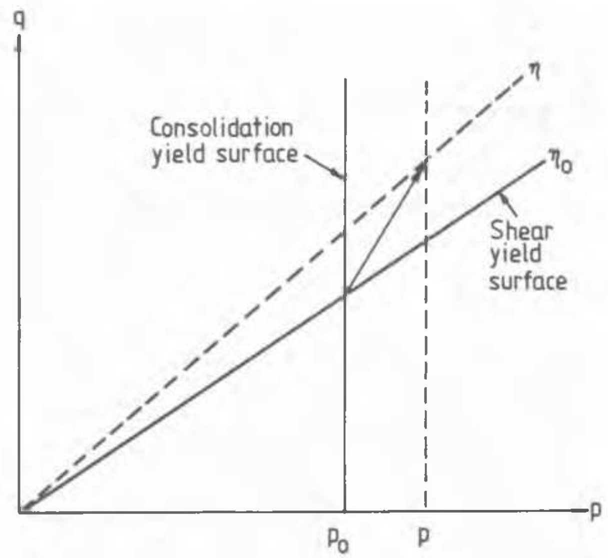

Fig. 2 Yield Surfaces.

The yield function for consolidation(Fig. 2) is

$$F_p = p - p_o = dp \qquad (3)$$

where p_o is the mean stress corresponding to the previous position of yield surface.

The incremental volumetric plastic strain is given by

$$dv_p^p = (\lambda - k)\, dp/p \qquad (4)$$

in which λ and k are the slopes of loading and unloading curves of p - v relationship.

The incremental plastic work done (dw^p) during shear deformation can be expressed as

$$dw^p = p\, dv^p + q\, d\gamma^p \qquad (5)$$

Moroto (1976) showed that the plastic work done in the case of sand largely depends on stress-path. He introduced a new parameter S_s such that

$$S_s = \int \frac{dw^p}{p} \qquad (6)$$

and showed that S_s bears a unique relationship with and shear strain irrespective of the stress-paths followed. Mishra (1981) observed similar behaviour in case of three Indian sands. Also they observed a linear relationship between S_s and γ_η^p which was independent of mean stress in p - constant test, i.e.

$$S_s = \eta_w \gamma_\eta^p \qquad (7)$$

where η_w is a constant.

The relationship between S_s and η can be approximated by a hyperbola :

$$S_s/\eta = a + b\, S_s \qquad (8)$$

in which a and b are constants. The reciprocal of b gives the asymptotic value of η.

The yield function for shear (Fig. 2) is

$$F_\eta = \eta - \eta_o = d\eta \qquad (9)$$

where η_o is the previous value of yield function. Moroto (1976) derived the equations for plastic volumetric and shear strains using non-associated flow rule as

$$dv_\eta^p = h\frac{1}{p}\left(1 - \frac{\eta}{\eta_w}\right) d\eta \qquad (10)$$

$$d\gamma_\eta^p = h\frac{1}{p\eta_w} d\eta \qquad (11)$$

$$h = ap/(1 - \eta b)^2 \qquad (12)$$

SOIL PARAMETERS AND PREDICTIONS

The elasto-plastic model discussed above can be applied for all stress-paths in the Zones A and B both for isotropically consolidated and anisotropically consolidated samples. For the purpose of verification of the working of this model, the triaxial test results of Jamuna sand in dense condition (Mishra, 1981) have been used.

Firstly, the soil parameters have been evaluated from consolidation - rebound tests and p-constant tests. The parameters thus evaluated for isotropically consolidated and anisotropically consolidated samples are given in Table 1. The parameters are different for Zone A and Zone B for isotropic and anisotropic consolidation cases. λ ($= 1.084\times10^{-2}$) and k($= 0.4039\times10^{-2}$) are same for all the cases. Secondly, a number of test results with different stress -paths in the two zones have been compared with the predictions of this model. Three stress paths in zone A and three in Zone B have been used. Two initial consolidation pressures, 137.30 Kpa and 216.30 Kpa have been used for all the stress-paths. For anisotropic consolidation $\eta_{ko} = 1.1$ has been adopted.

TABLE 1

Parameters Used

ZONE	a_4 x 10^{-4}	b	η_W
Isotropic Initial Stress			
A	6.17	0.588	1.107
B	6.59	0.614	1.050
Anisotropic Initial Stress			
A	-3.40	1.643	-0.066
B	15.42	0.360	2.150

Numerical analyses of triaxial tests along various stress-paths with the present model have been carried out by the computer program named CPICR(a computer program for integration of constitutive relations), Sharma(1980). In this program, the stress-condition at a single point is considered and the strains due to stress increments are calculated.

RESULTS AND DISCUSSION

From the components of strain vector, the volumetric strain (v) and shear strain (γ) have been plotted against stress ratio.

The comparison of the predicted results with experimental results are shown in Figs. 3 to 6 for isotropically consolidated samples and Figs. 7 to 10 for anisotropically consolidated samples.

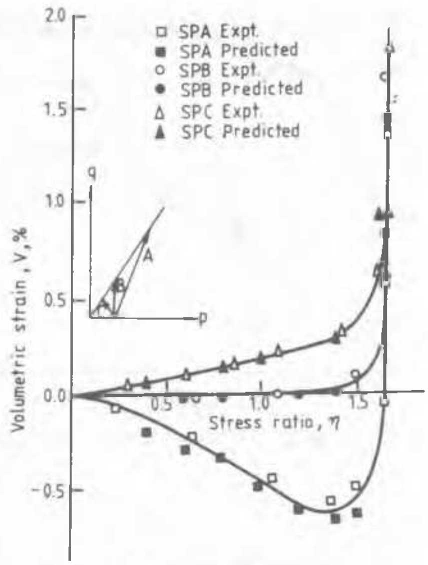

Fig. 4 v - η relationship for zone A

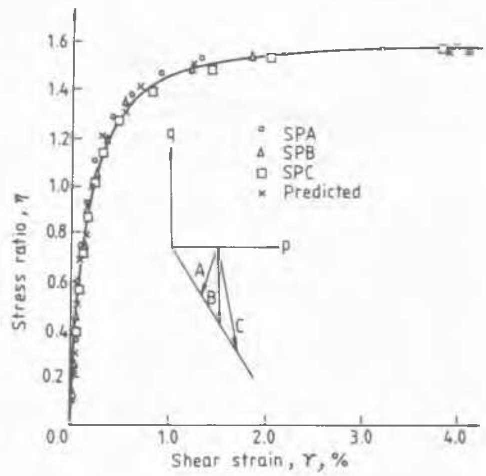

Fig. 5 η- γ relationship for Zone B.

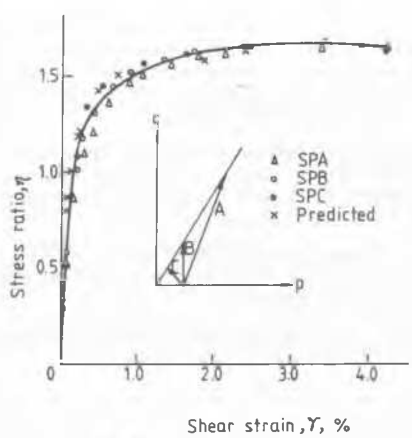

Fig. 3 η-γ relationship for zone A

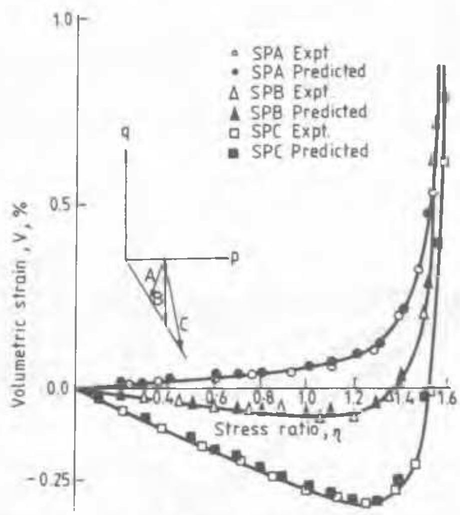

Fig. 6 V - η relationship for Zone B

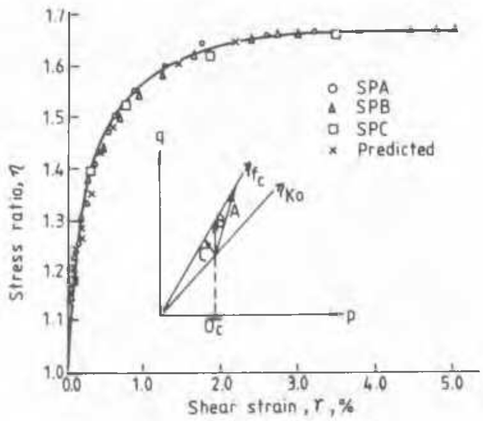

Fig. 7 η-γ Relationship for Zone A

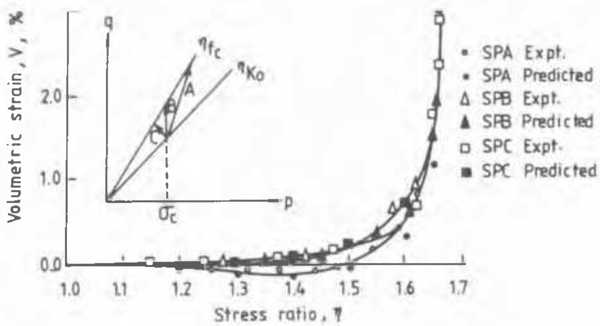

Fig. 8 V - η Relationship for Zone A

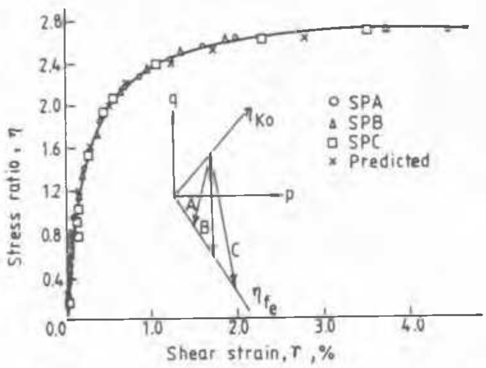

Fig. 9 η- γ Relationship for Zone B

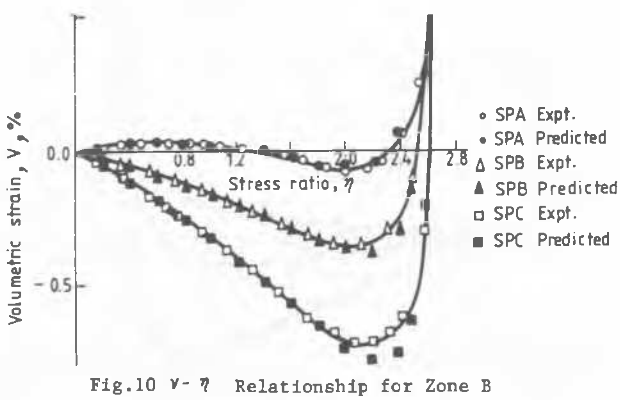

Fig.10 V- η Relationship for Zone B

It is clearly seen in the figures that the constitutive behaviour of sand is significantly influenced by the stress-path and initial stress condition. Though the nature of shear strain variation with stress ratio is similar for all the stress-paths analysed, the magnitudes are very different.

The volumetric strain-stress ratio relationship is very much affected by the stress-path. At high value of η all stress-paths show volume expansion. But at low levels of η, volume contraction occurs for some stress -paths. As one moves in the counterclockwise direction above the isotropic consolidation line, the volume contraction is maximum for Stress path A and it reduces for Stress path B and in case of Stress path C volume expansion is noticed (Fig. 4). For stress-paths directed below the isotropic consolidation line, volume expansion is seen for Stress path A and as one moves in the counterclockwise direction, the amount of volume expansion reduces and volume contraction is observed for Stress path B and the volume contraction is found to increase for Stress path C (Fig. 6). Similar observations have been made in the case of anisotropically consolidated sample.

It is clear that the predicted results of volumetric and shear strains compare well with the experimental data.

CONCLUSIONS

The elastoplastic constitutive model developed in this paper takes into account the effect of stress-path and insitu stress condition. The deformation has been considered to consist of volumetric and shear components. Any Stress-path is considered to be composed of a consolidation component and a shear component. The parameters should be determined from consolidation - rebound and pure shear tests conducted on samples consolidated following a stress ratio equal to the insitu stress condition. Eight parameters determined from three tests are used to predict the constitutive behaviour of sand along any stress-path in the p-q stress space. A procedure of referring the anisotropic consolidation line has been adopted to consider the insitu stress condition. It has been shown that the model is capable of predicting the stress-strain-volume change behaviour of dense and along various stress-paths covering the entire stress space satisfactorily. The model however is restricted to axi-symmetric stress condition only.

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