Analysis of foundation behaviour using finite layer methods

Analyse de la conduite des fondations par les méthodes des couches finies

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SYNOPSIS Traditional methods of numerical analysis, such as finite element and finite difference techniques, require large amounts of computer storage, and for problems involving three dimensions, storage as well as computation time and data preparation time become excessive. For many problems in soil mechanics, where the soil is horizontally layered, finite layer methods may be used. Examples of the application of the method to a number of different two and three dimensional problems are given to illustrate its simplicity and efficiency.

INTRODUCTION

The behaviour of layered elastic materials has been of great interest to engineers in the past; this interest being mainly due to the fact that many earthworks, such as fills or pavements, consist of horizontal layers of materials of different types. Quite often natural deposits are horizontally layered also, and it is of interest to be able to predict the behaviour of structures built on such deposits.

Because of the interest in pavement design where wheel loads are roughly applied over a circular area, many analytic solutions have been produced for layered materials subjected to a circular load. Burmister (1945) and Fo. (1948) have presented results for two layered systems where the underlying layer was infinitely deep. Solutions have also been obtained for three layered systems by Jones (1962) and Ueshita and Meyerhof (1968), where again the underlying layer was infinitely deep.

Gerrard (1967) has presented solutions for a strip loading on a layered material. He analysed a two layered system where the layers were of finite thickness. He also investigated the effects of anisotropy.

Numerical methods of solution have in recent years proved very attractive as they may be used to analyse problems where each layer has a different thickness or different material properties. Layers may also be anisotropic. Straightforward application of the finite element method (see Zienkiewicz 1976) may be used, but this is an inefficient method of solution.

Methods using Fourier or Hankel transforms (Rowe and Booker 1981a) or Fourier series (Cheung and Fan 1979, Tham and Cheung 1981) are a much more efficient way to solve such problems and require only a fraction of the computer storage of conventional finite element methods.

FINITE LAYER METHODS

The finite layer method depends on three simple observations. These will be introduced by considering the elastic deformation of the horizontally layered deposit shown in Fig. 1 under conditions of plane strain.

The first observation is that if the deposit is subjected to a periodic load having a period L then the response will also be periodic.* This is illustrated in Fig. 1.

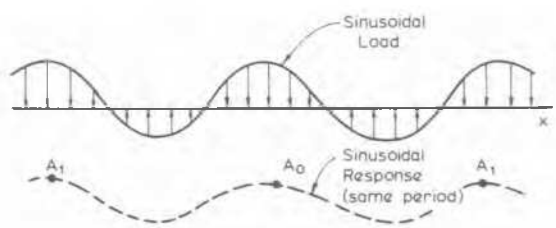

Fig. 1 Spatially Periodic Loading.

The second observation is that if the applied load happens to be sinusoidal as shown in Fig. 2, then the response will also be sinusoidal having the same period as the applied load. In particular, if the surface loads have the form

$$\sigma_{xz} = R \sin \alpha x \quad (1a)$$

$$\sigma_{zz} = P \cos \alpha x \quad (1b)$$

then the deflections throughout the layered system will have the form

* The assumption of periodic loading places no restriction on the method since an isolated load may be simulated by taking the period L to be large.

$$u_x = U(z)\ \sin\ \alpha x \qquad (2a)$$

$$u_z = W(z)\ \cos\ \alpha x \qquad (2b)$$

Thus for the sample form of loading given by equation (1) it is only necessary to determine the variation of the displacement amplitudes U, W with depth since the form of variation in any horizontal plane is known. It follows that for this case the two dimensional problem has been reduced to a one dimensional problem.

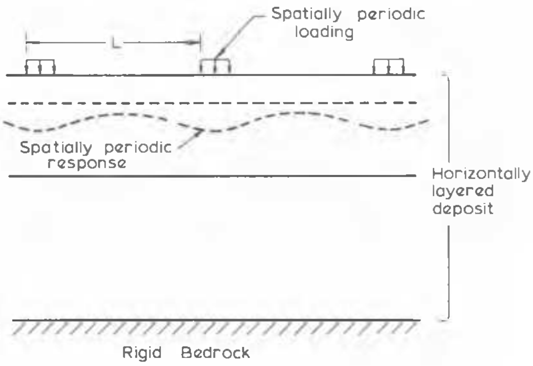

Fig. 2 Response of a Layered Elastic Soil to Sinusoidal Loading.

The third observation is that any prescribed loading can be broken into a number of Fourier components, so that

$$\sigma_{xz} = r(x) = \sum_{n=0}^{\infty} R_n\ \sin\ \alpha_n\ x \qquad (3a)$$

$$\sigma_{zz} = p(x) = \sum_{n=0}^{\infty} P_n\ \cos\ \alpha_n\ x \qquad (3b)$$

where

$$R_n = \frac{2\varepsilon_n}{L} \int_0^L r(x)\ \sin\ \alpha_n\ x\ dx$$

$$P_n = \frac{2\varepsilon_n}{L} \int_0^L p(x)\ \cos\ \alpha_n\ x\ dx$$

$$\alpha_n = 2n\pi/L$$

$$\varepsilon_n = 1/2\ (n = 0)$$

$$\varepsilon_n = 1\ (n \neq 0)$$

The decomposition of a uniform strip loading is shown in Fig. 3.

The basic idea of the finite layer method is to use the principle of superposition and to break the applied load into the sum of Fourier components, to find the solution for each Fourier component (as remarked earlier this reduces to a one dimensional problem) and then to obtain the complete solution by superimposing the component solutions.

The solution for each Fourier component is found by a procedure which is analogous to the well known finite element method. Consider a typical loading of the form (1) ($\alpha = \alpha_n$, $R = R_n$, $P = P_n$), then the response will have

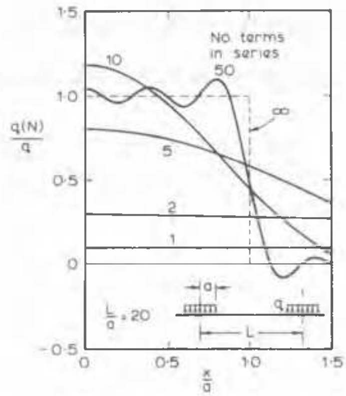

Fig. 3 Approximation of Uniform Strip Loading by Fourier Series.

the form given by equation (2). Now consider the typical layer bounded by node planes $z = z_m$, $z = z_p$ and shown in Fig. (4). This will be acted upon by tractions

$$\begin{aligned}
\sigma_{xz} &= R_m\ \sin\ \alpha x \quad (z = z_m) \\
\sigma_{zz} &= P_m\ \cos\ \alpha x \quad (z = z_m) \\
&\text{and} \\
\sigma_{xz} &= R_p\ \sin\ \alpha x \quad (z = z_p) \\
\sigma_{zz} &= P_p\ \cos\ \alpha x \quad (z = z_p)
\end{aligned} \qquad (4)$$

and will undergo node plane deflections

$$\begin{aligned}
u_x &= U_m\ \sin\ \alpha x \quad (z = z_m) \\
u_z &= W_m\ \cos\ \alpha x \quad (z = z_m) \\
u_x &= U_p\ \sin\ \alpha x \quad (z = z_p) \\
u_z &= W_p\ \cos\ \alpha x \quad (z = z_p)
\end{aligned} \qquad (5)$$

It is possible to determine the relationship between the node plane deflection amplitudes and the node plane tractions, either analytically (Booker and Small, 1984) or approximately, using an energy principle Cheung (1976). This relationship has the form

$$\begin{bmatrix} R_m \\ P_m \\ -R_p \\ -P_p \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix} \begin{bmatrix} V_p \\ W_p \\ V_m \\ W_m \end{bmatrix} \qquad (6)$$

where the 4x4 coefficient matrix k is called the layer stiffness matrix for the harmonic α.

The layer stiffness matrices (6) can be assembled using the conditions of equilibrium of stress and compatibility of displacements to lead to the total stiffness equations for the harmonic under consideration. This equation has the form

$$\underset{\sim}{K}\ \underset{\sim}{A} = \underset{\sim}{F} \qquad (7)$$

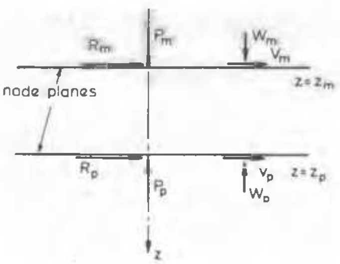

Fig. 4 Typical Layer.

where

K = the total stiffness matrix for the harmonic α.

$\tilde{A}$ = $(V_1, W_1, \ldots\quad)^T$ is the vector of unknown displacement amplitudes

$\tilde{F}$ = $(R_1, P_1, 0, 0, \ldots)^T$ is the vector of applied traction ampitudes.

The assembly procedure for a three layer system is shown in Fig. 5.

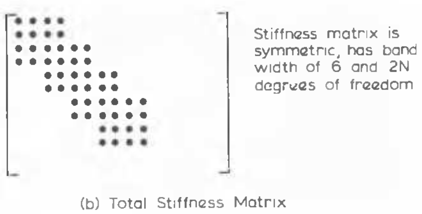

Fig. 5 Total Layer Stiffness Matrix.

Equations (7) can be solved for the unknown displacement amplitudes for each harmonic $\alpha = \alpha_n$ and the displacements at any position may then be found by adding the component solutions (2).

Application of the Finite Layer Method

The finite layer theory developed in the previous section was strictly applicable to the plane strain of an isotropic elastic material under conditions of plane strain. The method can however be extended to solve three dimensional problems for layered soils consisting of a number of horizontal anisotropic elastic layers. To illustrate this consider the problem of a strip, circular or rectangular loading applied to the surface of a layered soil, as shown in the inset to Fig. 6(b). The soil consists of an upper layer A and a lower layer B, which are anisotropic and have different properties to each other. (See Booker & Small (1982) for properties). Results for stresses in the vertical (σ_{zz}) and horizontal directions (σ_{rr} or σ_{xx}) along the centreline beneath each loaded area are shown in Figs 6a,b. Stresses and displacements may, of course, be calculated anywhere within the layer.

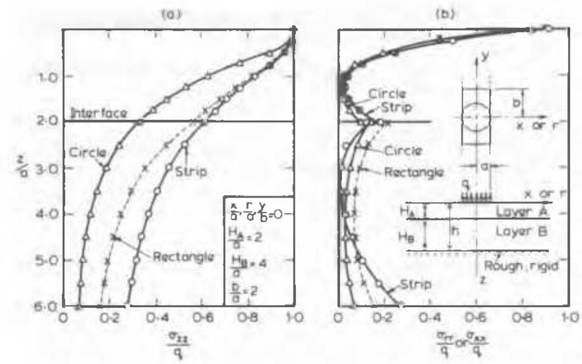

Fig. 6 Stresses Computed Beneath Loadings - Two Layers.

Often it is necessary to analyse soils which exhibit strong non-homogeneity, such as a Gibson soil having a modulus which varies linearly with depth and has the form $E = E_o + \rho z$.

One way of performing this analysis is to use a "stair case" approximation of the type shown in Fig. 7. It is difficult to obtain a good approximation using this approach; this is illustrated in Fig. 8, where a stair case approximation has been used to evaluate the behaviour of a strip footing acting on a layer of Gibson soil; Brown and Gibson (1979).

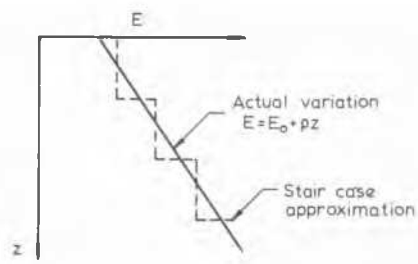

Fig. 7 Layer Approximation to Varying Modulus.

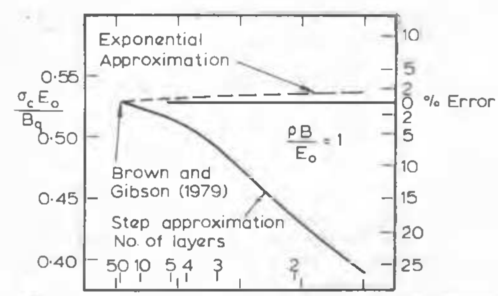

Fig. 8 Effect of Numerical Approximations upon Central Displacement of Circular Footing.

An alternative approach developed by Rowe and Booker (1982) approximates the variation of modulus in each finite layer by using an exponential variation as shown in Fig. 9. Referring to Fig. 8 it can be seen that it is possible to obtain an accurate solution with far fewer layers.

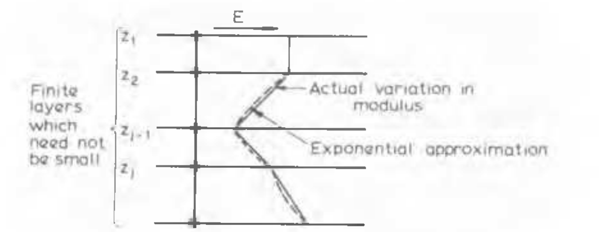

Fig. 9 Typical Non-Homogenous Soil Profile with Exponential Approximation.

This approach has been used to examine the behaviour of anchors in a Gibson soil (Rowe and Booker, 1981b) and the behaviour of surface loading on a soil having a crust (Rowe and Booker, 1981a).

The Finite Layer Method can also be used to analyse time dependent problems and a formulation which can analyse the consolidation of a layered soil has been developed by Booker and Small (1982) and to analyse soil creep (Small and Booker 1982b) as well as primary and secondary consolidation (Small and Booker 1982a).

To illustrate this approach the problem of a circular loading applied to a non-homogeneous soil as shown schematically in inset (i) to Fig. 10 was considered. Here two sublayers of soil A and B make up the overall layer. For this problem the lower layer B is four times as stiff but four times less permeable than the upper layer A. The solution for the settlement-time behaviour of the central point of the loading is shown in Fig. 10.

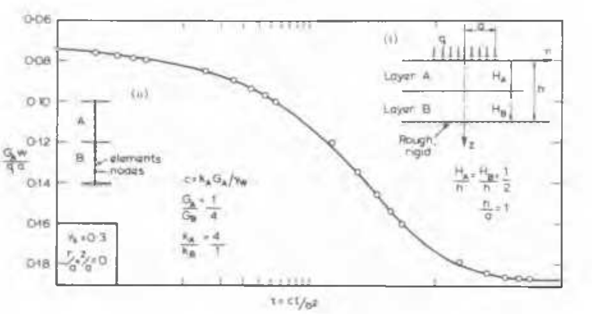

Fig. 10 Time-Settlement Behaviour of Two-Layer Soil System.

CONCLUSION

The finite layer method may be used for the analysis of a wide range of problems in soil mechanics, where the soil is horizontally layered. Because the method needs very little computer storage, two and three dimensional problems may be analysed using microcomputers, and savings may be made in data preparation and computational time.

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