Numerical and experimental methods in analysis of hydraulic structure stability

Méthodes numériques et expérimentales d'analyse de stabilité des constructions hydrotechniques

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SYNOPSIS The paper concerns numerical simulation of real behaviour of system: construction-soil-water. Equilibrium equations of saturated soil are discussed. Differential scheme stability condition in finite element method is also discussed. In the course of model tests the experiments with the deformation of subsoil were made and their results were compared to those obtained with numerical calculations. The good compatibility of both methods was found.

INTRODUCTION

All the phenomena concerning the water flow in the subsoil are very important for the stability of construction. Close relationship exists between mechanical and hydraulical phenomena occurring in soil equlibrium state which determines hydraulical characteristics of the medium. Analysis emloyed so far were made on basis of superposition of three independent states: bearing capacity state, limited state of deformation and water flow state, as result of complexity of the phenomena occurring in the soil stability of construction. Recent progress in numerical methods of construction analysis allows accurate testing by numerical simulation of all states of real behaviour of system: construction -soil-water. The equation of continuous medium equilibrium state, rules of water flow in porous medium, rule of water flow continuity and Terzaghi´s assumption of effective stress are used in the description of phenomena occurring in two phase medium.

SOIL MEDIUM EQUILIBRIUM EQUATIONS

The following assumptions are made in the discussed problem of soil medium stability:

/i/ soil material is homogeneous and isotropic,

/ii/ strains are infinitesimal,

/iii/ total strain is the sum of elastic and plastic strains

$$\varepsilon_{ij} = \varepsilon_{ij}^{e} + \varepsilon_{ij}^{p} \qquad /1/$$

/iv/ constitutive rule describes relationship between strain and stress tensor components in elasto-plastic domain

$$\sigma_{ij} = D_{ijkl}^{ep}\, \varepsilon_{kl} \qquad /2/$$

/v/ water flow in the subsoil is discribed by Darcy´s law.

Navier´s equilibrium state equations according to Terzaghi´s effective strain law can be expressed as follows:

$$(\sigma_{ij} + \gamma_w \Psi \delta_{ij})_{,j} + F_i = 0 \qquad /3/$$

Water existing in the continuous medium exerts body force on it and this force is the sum of two components: hydrostatic pressure and filtration force /Louis, 1977/. The basic equation describing ground water movement is equation of mass conservation, which in the course of assumption /v/ for fully saturated medium can be expressed as follows:

$$[k_i(\Psi + z)_{,i}]_{,i} = \frac{\partial}{\partial t} u_{i,i} \qquad /4/$$

where: k_i - component of the hydraulic conductivity tensor of the porous medium,
Ψ - hydraulic potential,
z - position head relatively to the reference level,
u_i - component of the displacement vector function.

Equations /3/ and /4/ make complete partial differential equation system of eliptic-parabolic type.
During the action of the load on semispaces surface,the isotropic pressure is decomposed in the time t=0, partially to water and soil skeleton. Taking into account that water is non compressible, it takes over total isotropic pressure in the saturated soil in the time t=0, then:

$$\Psi(x_j,t) = \Psi(x_j,0), \quad u_i(x_j,t) = u_i(x_j,0) \quad x_j \in D \qquad /5/$$

with boundary conditions of Dirichlet

$$u_i(x_j,t) = \bar{u}_i(x_j,t), \quad \Psi_i(x_j,t) = \bar{\Psi}_i(x_j,t) \quad x_j \in \partial D \times D_t \qquad /6/$$

and Neumann´s

$$\sigma_{ki}(x_j,t)\,l_i(x_j) = T(x_j,t) \qquad /7/$$

$$k_i\,(\Psi+z)_{,i}\;l_i(x_j) = -Q(x_j,t) \qquad x_j \in \partial D \times D_t$$

The equations system /3/ - /4/ with above conditions is the proper stated nonstationary boundary problem.

SECOND ORDER DIFFERENTIAL SCHEME OF APPROXIMATION WITH THE INITIAL CONDITION

The approximation of the nonstationary problems is divided into two steps. One step introduces the approximation of the problem in relation to space variable. Using the finite element method in variation to restate equations /3/ and /4/ according to Galerkin--Bubnov method, one can obtain the differential equations with the time variable and difference equation with the space variable satisfied in Cartesian product of two sets: one of discretisation points of region D_h and second of the time variable D_t region. Difference equation according to /3/, /4/ as

$$[K]\{u\} + [C]\{\Psi\} = \{G\} \qquad /8/$$

$$[A]\{\Psi\} + [B]\,\frac{\partial}{\partial t}\{u\} = \{H\} \qquad in \quad D_h \times D_t$$

where: [K], [C], [A], [B] - are matrix of integrates resulting from approximation of space variable,
{G}, {H} - vectors of right sides of equation,
{u} - vector of displacement function.
Differential approximation of /8/ with aid of Crank-Nicholson scheme leads to:

$$\frac{\omega_{j+1}-\omega_j}{\tau} + [W]\,\frac{\omega_{j+1}+\omega_j}{2} = X \qquad /9/$$

$$\omega_0 = \begin{Bmatrix} u_0 \\ \Psi_0 \end{Bmatrix} = g_0$$

where

$$[W] = [B]^{-1}\begin{bmatrix} [K] & [C] \\ [0] & [A] \end{bmatrix}$$

$$\omega_j = \begin{Bmatrix} u \\ \Psi \end{Bmatrix} \qquad X = [B]^{-1}\begin{Bmatrix} G \\ H \end{Bmatrix}$$

Different scheme /9/ is the second order approximation in relation to time interval τ of /8/
Solution of /9/ could be described as

$$\omega_{j+1} = T^j\omega_j + \tau\left(E+\tfrac{1}{2}\tau\,[W]\right)^{-1}X \qquad /10/$$

$$T^j = \left(E+\tfrac{1}{2}\tau\,[W]\right)^{-1}\left(E-\tfrac{1}{2}\tau\,[W]\right)$$

where
T^j - is so called transition operator,
E - unit matrix.

Operator norm is estimated according to Kellog´s limit:

$$\|T^j\| \leq 1 \qquad /11/$$

so, from /10/ we can obtain

$$\|\omega_{j+1}\| \leq \|T^j\|\,\|\omega_j\| + \tau\,\|(E+\tfrac{1}{2}\tau\,[W])^{-1}\|\,\|X\| \qquad /12/$$

Taking into account that $[W] \geq 0$ is positively determined it is easy to show that:

$$\|(E-\tfrac{1}{2}\tau\,[W])^{-1}\| \leq 1 \qquad /13/$$

Applying /11/ and /13/ in inequality /12/ we obtain:

$$\|\omega_{j+1}\| \leq \|\omega_j\| + \tau\,\|X\| \qquad /14/$$

Assuming that $\|\omega_0\| = \|g_0\|$ and using recuration relation /14/ one can obtain

$$\|\omega_j\| \leq \|g_0\| + j\tau\,\|X\| \qquad /15/$$

Inequality /15/ is the stability of concerned difference scheme and additionally is the aprioric estimation of the solution.

MODEL TESTS

The model tests in the condition of plain strain state were made in Geotechnical Department of the Hydroengineering Faculty of Technical University in Gdańsk. The main goal of these experiments was to estimate bearing capacity and deformation of natural cohesionless soil medium /sand/ under rigid foundation loading. Strain-controlled tests in model experiments were performed. Loading value and deformation of respective characteristic points of subsoil followed. The main part of the test stand was the box /2,65 x 0,5 x 1,07 m/ with the front glass wall 35mm thick. The rigid foundation models of rectangle shaped plate 2 centimeters thick, 20 cm in width and 50 cm in length was axially and vertically or eccentrically loaded. Homogeneous subsoil was fine dry or saturated sand of various thickness restricted by underformable layer. Such structure of the subsoil is typical for southern Poland and must be taken into account in solving engineering problems concering a resting water plant /e.q.dams/. In this region there is underformable bed rock under the surface layer of the ground /e.g.sand/. The photogrammetric method was applied for the measurements of subsoil deformation.
400-800 special markers were placed /fig.1/ in certain points of the medium symbolizing nodal points of the finite elements mesh.

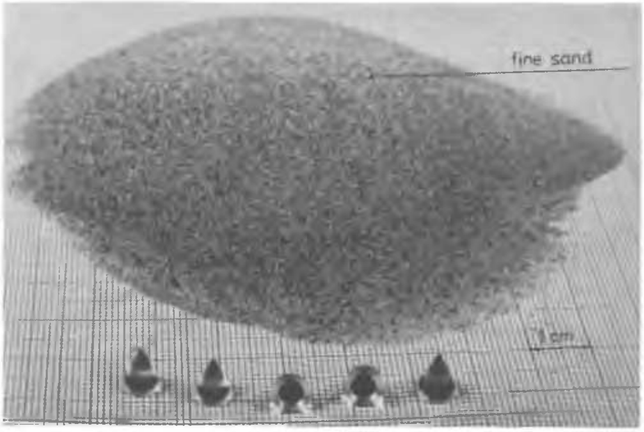

Fig.1 The markers used in model test

Those markers are made from plexiglass and are conical shaped with concave base. These markers were 7,5mm in height and 7,5mm base diameter. Because of shape employed these markers touch closely with their base to the glass wall of the box. During all time of the experiments movements of these markers resulting from the displacement of the model foundation were monitored by special photogrammetric camera placed 1,5m from the glass wall. Accurancy of measurments of position of respective markers was 0,3mm. Interpretation of results leads to the determination of trajectories and velocity of movements of the respective points. It is of particular importance when comparing test model results and results of theoretical calculations employing finite element method, discussed previously.

COMPARISION OF CALCULATIONS AND MODEL TEST RESULTS

The trajectories of the movements resulting from the model test were compared with these calculated for the cohesionless dry subsoil. The model tests for saturated subsoil are in progress and their results will be published soon. Figure 2 ilustrates an example of subsoil discretisation using triangular elements which are applied in calculation. The trajectories for vertical and eccentrical loading e/B=1/6 and cavity of underformable layer h/B= ∞ and h/B=1 of the movement in elastic phase resulting from tests and calculations are presented in fig. 3.
The comparison of values of model foundation displacements resulting from model test $/S_t/$ and calculation $/S_c/$ using finite element,

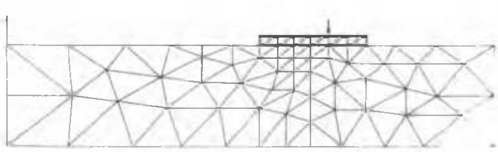

Fig.2 Discretisation of the subsoil using triangular elements

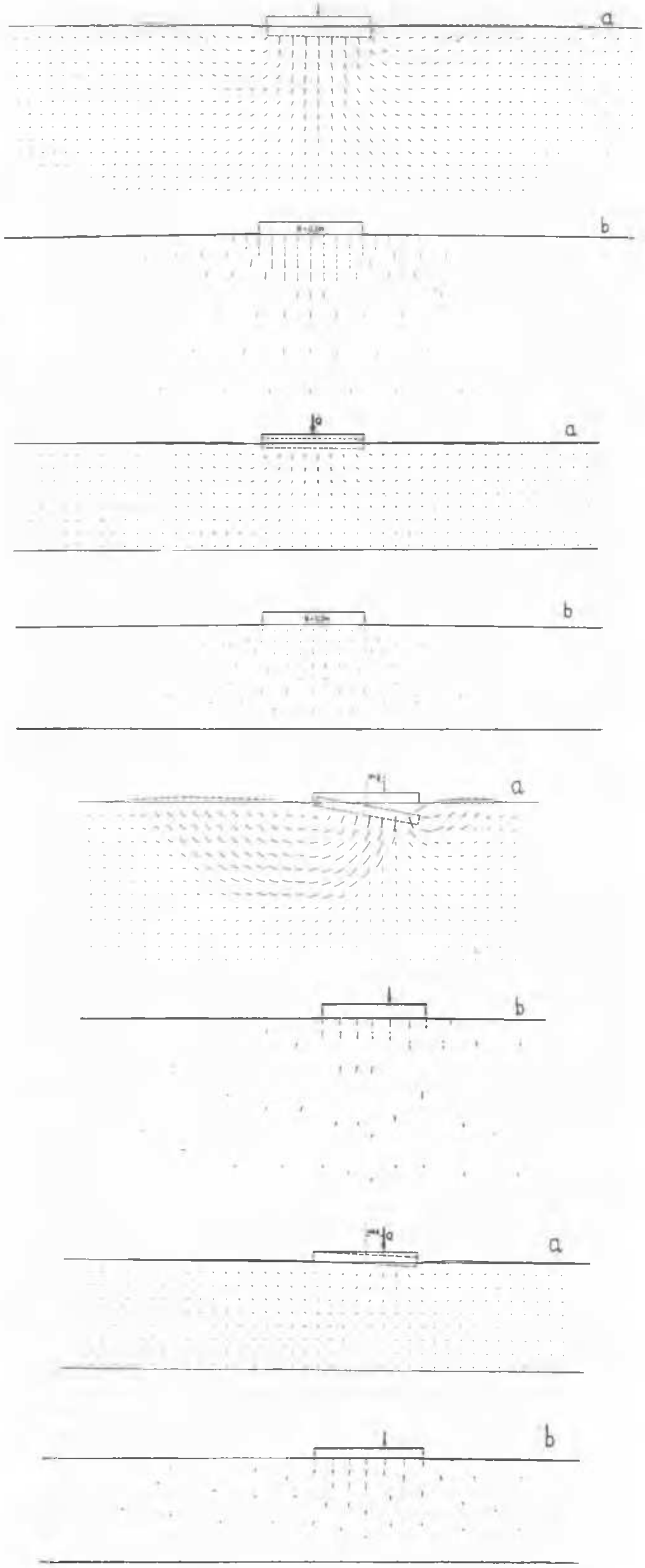

Fig.3 Trajectories of displacement in elastic phase a - model test b - calculations using FEM

method Polish Code /1981/ and Giroud tables /1972/ is shown in table 1. Coeficient $n = S_t/S_c$ is also presented.

TABLE 1
Comparison of the deformation value

Geometrical value			Foundation displacement /mm/				
B /cm/	$\frac{e}{B}$	$\frac{h}{B}$	model test	founda-tion calcu-lation in FEM	n	calcu-lation acc.to Polish Code	n
20	0	0,5	7,3	6,65	1,10	6,16	1,19
20	0	1,0	5,0	4,95	1,01	4,42	1,13
20	0	1,5	6,5	6,80	0,95	5,97	1,09
20	0	∞	8,0	8,60	0,93	7,34	1,09
20	1/12	0,5	5,0	5,34	0,94	3,24	1,54
20	1/12	1,0	3,6	3,32	1,08	2,69	1,33
20	1/12	1,5	4,0	4,08	0,98	3,32	1,20
20	1/12	∞	5,2	5,20	1,00	4,47	1,16
20	1/6	0,5	1,8	1,92	0,94	1,40	1,28
20	1/6	1,0	3,0	3,16	0,95	2,73	1,10
20	1/6	1,5	3,5	4,10	0,85	3,47	1,10
20	1/6	∞	4,0	4,44	0,90	3,96	1,01

From the fig.3 and table 1 one can see that calculations of displacement and results of model test are comparable and of good compatibility both in qualitative and quantitative terms.

CONCLUSIONS

The model of numerical simulation of real behaviour of the system construction-soil-water discussed here theoretically enable to make a more accurate analysis of the stability of the hydro-engineering constructions resting on the soil foundation. Comparison of theoretical calculation with the model test results /first for dry subsoil/ indicates qualitatively and quantitatively good compatibility and proves the accurancy of the proposed solution.

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