Non-iterative method for the analysis of soil-structure interaction

Analyse non-itérative sur l'interaction du sol-structure

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SYPNOPSIS A procedure for the combined analysis of structure and soil, which consists in establishing compatibility between structure and soil displacements, is presented. Structure deformations are calculated as a function of soil reactions and settlements of soil are found as a function of loads applied to it (these loads are equal to the soil reactions, with opposite sign), employing the concept of influence values. Finally a simple example of application is included, to illustrate the method.

INTRODUCTION

The method for soil-structure interaction consists in the following procedure:

Suppose a reticular structure which rests over a soil with medium to high compressibility (Fig.1), in which for the foundation we can use either a continuous footings or a slab with reinforced concrete beams. Since we don't know the soil reaction diagram, let us sustitute the distributed loads of soil (contact pressure) by distributed loads under columns and under the middle points of spans, as is shown in Fig. 1. We do this in order to simplify calculations; if we want to get a greater number of reactions r_i, we can consider each foundation beam as two or more beams, for analysis purposes.

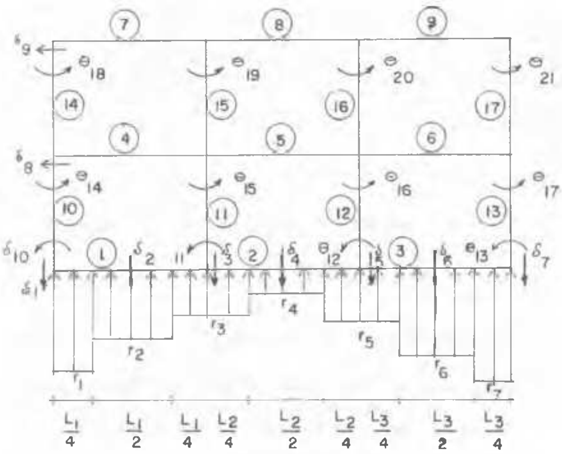

FIG. 1 MEMBERS, REACTIONS, LINEAR DISPLACEMENTS AND ROTATIONS NUMERATION

In order to make the combined analysis of structure and soil it is desirable to use stiffness method for structure analysis.

When we use this procedure we find the stiffness matrix and we know the loads vector, but we don't know the displacement vector. Nevertheless, as we see in Fig. 1, in this case we do not know the complete load vector, and the reactions of soil r_i also unknown. Neverthesless, when we make the settlement analysis of soil we can find soil displacements δ_i as a function of reactions r_i, taking these as unknown. Sustituting these equations in the expressions derived of the initial application of stiffness method, we find an equation system in which the unknown are only the rotations in the joints θ_i and the reactions of soil r_i. Solving this system of equations we find the rotations in the joints and the reactions of soil. As we have vertical displacements as a function of reactions, we can also find them.

STRUCTURAL ANALYSIS

Structural analysis is made using stiffness method, in which each joint must be in equilibruium, e.g., that the sum of the fixed moments and the moments due to displacements must be zero in each joint. Moreover, members (beams and columns) must be in equilibrium under effect of the sum of the fixed shear forces produced by displacements. In matrix form:

$$[K]\begin{Bmatrix}\delta_i\\ \theta_i\end{Bmatrix}+\begin{Bmatrix}V_i^e\\ M_i^e\end{Bmatrix}=0$$

or

$$[K]\begin{Bmatrix}\delta_i\\ \theta_i\end{Bmatrix}=\begin{Bmatrix}-V_i^e\\ -M_i^e\end{Bmatrix} \qquad (1)$$

where K is the stiffness matrix of structure, δ_i and θ_i linear and rotational displacements and V_i^e and M_i^e are fixed force shears and flexural moments. In equation systems 1 there are three types of unknown elements: displacements δ_i, rotations θ_i and reactions r_i (these can appear in V_i^e or M_i^e).

Stiffness matrix determination, and fixed shears and moments determination, can be made for each member and then they must be integrated over the complete structure (Beaufait et.al., 1970). The procedure outlined in the preceding paragraphs is general, but in order to simplifly, beams with continuous supports are presented only. Reaction blocks are included at the middle of spans, so auxiliary equations are needed for these reactions blocks; these expressions can be found by the use of the conjugate beam theorems.

Stiffness matrix of member j with continuous (Fig.2) is:

$$K_J = \begin{array}{c} \begin{array}{cccc} \theta_p & \theta_q & \delta_r & \delta_s \end{array} \\ \left[\begin{array}{cccc} \frac{4\,EI}{L} & \frac{2\,EI}{L} & -\frac{6\,EI}{L^2} & \frac{6\,EI}{L^2} \\ \frac{2\,EI}{L} & \frac{4\,EI}{L} & -\frac{6\,EI}{L^2} & \frac{6\,EI}{L^2} \\ -\frac{6\,EI}{L^2} & -\frac{6\,EI}{L^2} & \frac{12\,EI}{L^3} & -\frac{12\,EI}{L^3} \\ \frac{6\,EI}{L^2} & \frac{6\,EI}{L^2} & -\frac{12\,EI}{L^3} & \frac{12\,EI}{L^3} \end{array}\right] \end{array} \begin{array}{c} \\ \theta_p \\ \theta_q \\ \delta_r \\ \delta_s \end{array} \qquad (2)$$

in which: L = lenght of member j
E = material elasticity modulus of member j
I = moment of inertia of member j
θ_p = rotation at joint p
θ_q = rotation at joint q
δ_r = displacement at joint r
δ_s = displacement at joint s

Rotations are considered positive in a counterclockwise sense and down displacements are positive. Positive flexural moments (member over joint) have clockwise sense and positive shear forces (member over joint) are upward.

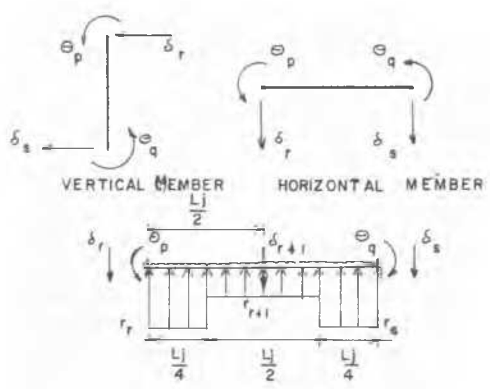

FIG. 2 FOUNDATION MEMBER WITH CONTINUOUS SUPPORTS AND DISTRIBUTED LOAD

Fixed moments and shears (member over joint) due to a uniform load w_j are (Fig. 2):

$$M_p^e = \frac{w_j L_j^2}{12} \quad (3) \qquad M_q^e = -\frac{w_j L_j^2}{12} \quad (4)$$

$$V_r^e = \frac{-w_j L_j}{2} \quad (5) \qquad V_s^e = -\frac{w_j L_j}{2} \quad (6)$$

For the uniform loads at foundation (Fig.2):

$$M_p^e = -\frac{67}{3072} L_j^2 r_r - \frac{11}{192} L_j^2 r_{r+1} - \frac{13}{3072} L_j^2 r_s \qquad (7)$$

$$M_q^e = \frac{13}{3072} L_j^2 r_r + \frac{11}{192} L_j^2 r_{r+1} + \frac{67}{3072} L_j^2 r_s \qquad (8)$$

$$V_r^e = \frac{121}{512} L_j^2 r_r + \frac{1}{4} L_j r_{r+1} + \frac{7}{512} L_j r_s \qquad (9)$$

$$V_s^e = \frac{7}{512} L_j r_r + \frac{1}{4} L_j r_{r+1} + \frac{121}{512} L_j r_s \qquad (10)$$

Vertical displacements δ_{r+1} at foundation level, in the middle of the span (Fig.2), for a member with continuous supports, is found using conjugate beam theorems:

$$\frac{EI}{L}\theta_p - \frac{EI}{L}\theta_q - 8\frac{EI}{L^2}\delta_r + 16\frac{EI}{L^2}\delta_{r+1} - 8\frac{EI}{L^2}\delta_s$$
$$+ \frac{13}{384} L^2 r_{r+1} + \frac{1}{256} L^2 r_s = \frac{1}{24} w_j L^2 \qquad (11)$$

Concluding, structural analysis is made with system equations 1 and auxiliary equation 11.

SOIL SETTLEMENTS

Vertical displacements δ_i are found as a function of soil reactions r_i, taking the reactions r_i as unknowns, for the stratigraphy and properties shown in Fig. 3. Settlement of stratum j under section i, due to load r_k located at k is:

$$\delta_{ijk} = M_{vij}\, H_j\, (\Delta\sigma)_{ijk}$$

in which: δ_{ijk} = deformation of rectangle ij, due to a reaction located at $k(r_k)$
M_{vij} = strain modulus of soil in rectangle ij
H_j = thickness of stratum j
$(\Delta\sigma)_{ijk}$ = normal vertical stress increment in the rectangle ij, due to a pressure r_k/b_k in the contact area between soil and foundation
b_k = foundation width, corresponding to the load r_k

but $(\Delta\sigma)_{ijk} = I_{ijk}\,(r_k/b_K)$

where I_{ijk} = influence value in the rectangle ij, due to a unit pressure in k

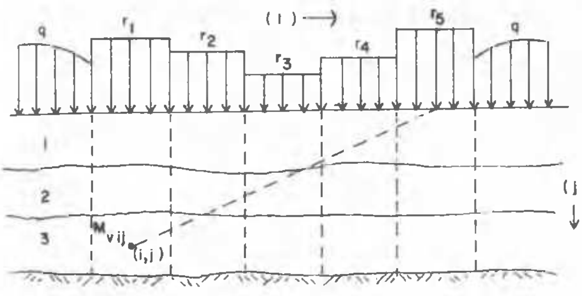

FIG. 3 SOIL SETTLEMENTS DETERMINATION

Values of I_{ijk} can be obtained by computing the stress caused by a unit pressure in b_k over the rectangle ij (Zeevaert, 1973). Settlement in the rectangle ij due to all loads r_k and to external load q (Fig.3) is

$$\delta_{ij} = M_{vij} H_j (q_{ij} + \sum_{k=1}^{m} I_{ijk}\, r_k / b_k)$$

in which m is the total number of reactions applied at surface and q_{ij} is the normal vertical stress due to pressure q at the external surface (Fig. 3).

Now, the settlements under i will be:

$$\delta_i = \sum_{j=1}^{p} \delta_{ij}$$

so

$$\delta_i = \sum_{j=1}^{p} M_{vij} H_j\, q_{ij} + \sum_{j=1}^{p} M_{vij} H_j \left(\sum_{k=1}^{m} I_{ijk}\, r_k / b_k \right) \quad (12)$$

in which p = number of subsoil strata

In this equation we see the soil displacements as a function of the loads r_k.

DISPLACEMENTS COMPATIBILITY

Once structure analysis and soil settlements determinations are made, displacements compatibility between them must be established (Deméneghi, 1979): soil settlements obtained by Eq. 12 are sustituted in the resulting system equation from Eqs 1 and 11 of the structural analysis. In this way, vertical displacements are no more unknown and the unknown are only the rotations and the reactions r_i. It is easy to see that the number of equations is the same as the number of unknown elements; the equation system can be solved and the values of the rotations and the reactions can be determined. Using Eq. 12 soil settlement can be computed, as values of r_i are now known

EXAMPLE

Find the reaction and settlement diagrams for the structure, stratigraphy and properties of example shown in Fig.4. Foundation width is 8 m. E= 1.5811×10^6 ton/m^2.

EXAMPLE

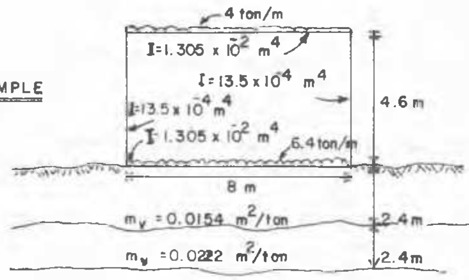

FIG. 4 STRUCTURE, STRATIGRAPHY AND PROPERTIES (EXAMPLE)

Rotations and displacements of each member are (Fig. 5)

Member number	θ_p	θ_q	δ_r	δ_s
1	θ_5	θ_6	δ_1	δ_3
2	θ_7	θ_8	δ_1	δ_3
3	θ_7	θ_5	-	-
4	θ_8	θ_6	-	-

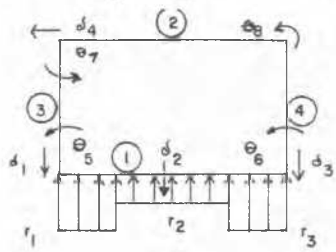

FIG. 5 MEMBERS, REACTIONS AND DISPLACEMENTS NUMERATION (EXAMPLE)

Stiffness matrix determination for each member:

Member 1

E= 1.5811×10^6 ton/m^2 ; I= 1.305×10^{-2} m^4;

L= 8 m

Using Eq.2:

$$\begin{array}{cc} & \begin{array}{cccc} \theta_5 & \theta_6 & \delta_1 & \delta_3 \end{array} \\ K_1 = & \left[\begin{array}{cccc} 10\,316.67 & 5\,158.34 & -1\,934.38 & 1\,934.38 \\ 5\,158.34 & 10\,316.67 & -1\,934.38 & 1\,934.38 \\ -1\,934.38 & -1\,934.38 & 483.59 & -483.59 \\ 1\,934.38 & 1\,934.38 & -483.59 & 483.59 \end{array}\right] \begin{array}{c} \theta_5 \\ \theta_6 \\ \delta_1 \\ \delta_3 \end{array} \end{array}$$

Using Eqs. 3 and 4: $M_5^e = \dfrac{6.4 \times 8^2}{12} = 34.1333$ ton-n

$M_6^e = \dfrac{-6.4 \times 8^2}{12} = -34.1333$ ton-m

Using Eqs. 7 and 8:

$M_5^e = -1.3958\, r_1 - 3.6667\, r_2 - 0.27083\, r_3$; $M = 0.27083\, r_1 + 3.6667\, r_2 + 1.3958\, r_3$

Using Eqs. 5 and 6:

$V_1^e = \dfrac{-6.4 \times 8}{2} = -25.6$ ton; $V_3^e = \dfrac{-6.4 \times 8}{2} = 25.6$ ton

Using Eqs. 9 and 10:

$$V_1^e = 1.8906r_1 + 2r_2 + 0.1094r_3; \quad V_3^e = 0.1094r_1 + 2r_2 + 1.806r_3$$

In a similar way, stiffness matrices and fixed moments and shears for members 2, 3 and 4 are obtained:

$$K_2 = \begin{array}{cc} \begin{array}{cccc} \theta_7 & \theta_8 & \delta_1 & \delta_3 \end{array} & \\ \begin{bmatrix} 10\,316.67 & 5\,158.34 & -1\,934.38 & 1\,934.38 \\ 5\,158.34 & 10\,316.67 & -1\,934.38 & 1\,934.38 \\ -1\,934.38 & -1\,934.38 & 483.59 & -483.59 \\ 1\,934.38 & 1\,934.38 & -483.59 & 483.59 \end{bmatrix} & \begin{array}{c} \theta_7 \\ \theta_8 \\ \delta_1 \\ \delta_3 \end{array} \end{array}$$

$$K_3 = K_4 = \begin{array}{ccc} \theta_7 & \theta_5 & \\ 1\,856.07 & 928.04 & \theta_7 \\ 928.04 & 1\,856.07 & \theta_7 \end{array} \qquad M_7^e = 21.333 \text{ ton-m} \quad M^e = -21.333 \text{ ton-m}$$

$$V_1^e = -16 \text{ ton} \quad ; \quad V_3^e = -16 \text{ ton}$$

Stiffness matrix for the complete structure is determined by adding the stiffness matrices of each one of the members: $K = K_1 + K_2 + K_3 + K_4$; therefore:

$$K = \begin{array}{cc} \begin{array}{cccccc} \delta_1 & \delta_3 & \theta_5 & \theta_6 & \theta_7 & \theta_8 \end{array} & \\ \begin{bmatrix} 967.18 & -967.18 & -1\,934.38 & -1\,934.38 & -1\,934.38 & -1\,934.38 \\ -967.18 & 967.18 & 1\,934.38 & 1\,934.38 & 1\,934.38 & 1\,934.38 \\ -1\,934.38 & 1\,934.38 & 12\,172.74 & 5\,158.34 & 928.04 & 0 \\ -1\,934.38 & 1\,934.38 & 5\,158.34 & 12\,172.74 & 0 & 928.04 \\ -1\,934.38 & 1\,934.38 & 928.04 & 0 & 12\,172.74 & 5\,158.34 \\ -1\,934.38 & 1\,934.39 & 0 & 928.04 & 5\,158.34 & 12\,172.74 \end{bmatrix} & \begin{array}{c} \delta_1 \\ \delta_3 \\ \theta_5 \\ \theta_6 \\ \theta_7 \\ \theta_8 \end{array} \end{array}$$

Equilibrium of moments at the joints and of shears at the members (Eq.1)leads to the following expressions, considering that by symmetry of structure $\delta_3 = \delta_1$, $\theta_6 = -\theta_5$ and $\theta_8 = -\theta_7$:

Rotation θ_5(row 3 of stiffness matrix of structure):

$$-1\,934.38\,\delta_1 + 1\,934.38\,\delta_3 + 12\,172.74\theta_5 + 5\,158.34\theta_6 + 928.04\theta_7 + 0\theta_8 + 34.1333 - 1.3958r_1 - 3.667r_2 - 0.27083r_3 = 0 \tag{13}$$

Displacement δ_1 (row 1 of stiffness matrix of structure):

$$967.18\,\delta_1 + -967.18\,\delta_3 - 1934.38\,\theta_5 - 1934.38\,\theta_6 - 1934.38\,\theta_7 - 1934.38\,\theta_8 - 25.6 + 1.8906\,r_1 + 2r_2 + 0.1094\,r_3 - 16 = 0 \tag{14}$$

Rotation θ_7 (row 5 of stiffness matrix of structure):

$$-1934.38\,\delta_1 + 1934.38\,\delta_3 + 928.04\,\theta_5 + 0\,\theta_6 + 12172.74\,\theta_7 + 5158.34\,\theta_8 + 21.333 = 0$$

Using auxiliary equation 11 for member 1:

$$2579.2\,\theta_5 - 2579.2\,\theta_6 - 2579.2\,\delta_1 - 2579.2\,\delta_3 + 5158.3\,\delta_3 + 51858.3\,\delta_2 + 0.25r_1 + 0.25\,r_3 + 2.1667\,r_2 = 17.0667 \tag{16}$$

Soil settlements are found using Eq.12; influence values determination is as folows (Fig.6):

Influence values I_{ijk}

Unitary pressure located at area 1 (k=1) / Unitary pressure located at area 2 (k=2)

j \ i	1	2	3	1	2	3
1	0.4659	0.02502	0.000635	0.02793	0.9318	0.02793
2	0.2812	0.1063	0.00920	0.1362	0.5624	0.1362

Unitary pressure located at area 3 (k=3)

j \ i	1	2	3
1	0.000635	0.02502	0.4659
2	0.00920	0.1063	0.2812

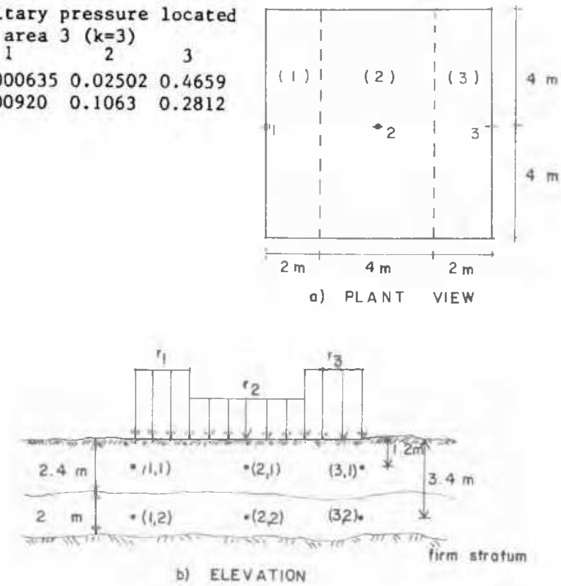

FIG. 6 INFLUENCE VALUES DETERMINATION

Making i =1 in Eq. 12:

$$\delta_1 = \sum_{j=1}^{2} M_{v1j} H_j \sum_{k=1}^{3} I_{1jk} r_k / b_k$$

$$\delta_1 = 0.0154\text{x}2.4\ (0.4659r_1/8 + 0.02793r_2/8 + 0.00065r_3/8)$$

$$\delta_1 = 0.003713r_1 + 0.0003849r_2 + 0.0007056r_3$$

Similarly:

$$\delta_2 = 0.0007056r_1 + 0.007426r_2 + 0.0007056r_3$$

but $r_1 = r_3$ by symmetry, hence:

$$\delta_1 = 0.003767r_1 + 0.008849r_2 \tag{17}$$

$$\delta_2 = 0.001411r_1 + 0.007426r_2 \tag{18}$$

By structure symmetry and sustituting Eqs 17 and 18 in Eq. 16:

$$5158.4\,\theta_5 - 5158.4\,(0.003767r_1 + 0.0008849r_2) + 5158\,4(0.001411r_1 + 0.007427r_2) + 0.5r_1 + 2.1667r_2 = 17\,0667$$

Equation system 13, 14, 15 and 16' is now:

$$7014.4\theta_5 + 928.04\theta_7 - 1\,6666r_1 - 3.667r_2 = -34.1333 \tag{13'}$$

$$r_1 + r_2 = 20.8 \tag{14'}$$

$$928.04\theta_5 + 7014\,4\theta_7 = -21.333 \tag{15'}$$

$$5158.4\,\theta_5 \quad -11.6533r_1 + 359076\,r_2 = 17.0667 \tag{16'}$$

Solving this system:

$\theta_5 = 0.002007$ $\qquad$ $\theta_7 = -0.003307$

$r_1 = 15.562$ ton/m $\qquad$ $r_2 = 5.238$ ton/m

Sustituting these values in Eqs. 17 and 18:

$\delta_1 = 0.0633$ m; $\delta_2 = 0.0609$ m these results are shown in Fig. 7

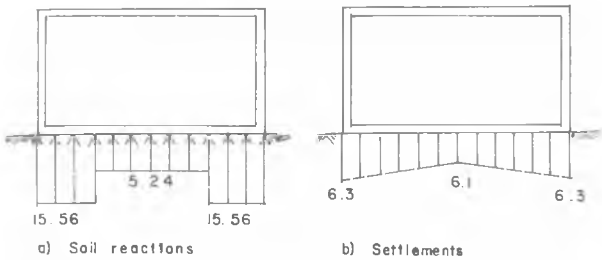

FIG.7 FINAL RESULTS (EXAMPLE)

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