Undrained bearing capacity of strain-softening clay

Capacité portante non-drainée d'argile ramollie

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SYNOPSIS This paper shows how simple finite element models, in conjunction with elasto-plastic theory, can give excellent collapse load predictions for footings resting on either an elastic-perfectly plastic or strain-softening clay. The model employed is based on the Tresca yield criterion which can be considered sufficiently appropriate for a total stress analysis. A new approach called "displacement control method" has been proposed to solve the elasto-plastic problem.

INTRODUCTION

Most soils display a decrease in strength with increasing strain from peak values to the ultimate or residual value. This occurs under both drained and undrained conditions and constitutes an important departure from most inelastic idealizations. It is now generally acknowledged that progressive failure arising from strain-softening has a significant effect on many problems such as slope stability, earth pressures, bearing capacity, even on the strength measured in a test like the shear vane test.

It was emphasized by Peck (1967) that a complete understanding of the progressive failure would require a finite element solution for the strain-softening soil. Höeg (1972) pioneered the application of incremental elasto-plastic finite element models to simulate the strain-softening behaviour of a clay foundation. The theoretical justification for using the classical theories of platicity to simulate strain-softening behaviour was later provided by the work of Prevost and Höeg (1975) who proved the existence of uniqueness and stability.

Attempts to incorporate strain-softening behaviour have been connected with the use of rather sophisticated models which render them unsatisfactory for the design office. This paper presents a simple plane strain finite element model simulating the strain-softening behaviour of clays using the theory of plasticity and an associated flow rule for the post-peak behaviour down to and including the residual state.

NUMERICAL SOLUTION PROCEDURE

Strip footing on elastic-perfectly plastic soil

The implementation and application of the "displacement control method" to the problem of bearing capacity is better described in two parts used sequentially as follows:

In the first part, the program developed calculates a linear elastic solution for the problem under the given loading. It then proceeds to calculate the load factor, λ, on the given loading which will just cause yield in one element. The internal stresses and displacements corresponding to this load factor are then calculated.

The vertical displacement under the centre of the footing (the controlled displacement) is conveniently taken to be the norm of the displacement vector. The specified value, C, of the controlled displacement increment in each loading step is based on that of the elastic solution. It is calculated as the above-mentioned norm of the elastic solution divided by a factor,m, which must be given as data. Finally, the starting approximate values of the displacement vector increment, {DSP}, and the load factor increment, $\Delta\lambda$, to be used in the next loading step are calculated as:

$$\{DSP\} = \{SP\}/m \qquad (1)$$

$$\Delta\lambda = \lambda/m \qquad (2)$$

where {SP} is the displacement vector corresponding to the load factor which will just cause yield in one element.

The second part of the program calculates the correct increase of load factor and the correct displacement vector increment for one loading step. Furthermore, the norm of the displacement vector increment (the controlled displacement) is maintained equal to the previously specified constant value, C, throughout the iterations. It starts by assuming that the displacement vector increment is equal to the converged displacement vector increment of the previous loading step. This is progressively improved to take into account the non-linear behaviour of the problem and a new laod factor increment is calculated with each improvement so as to limit the controlled displacement to the specified value. The procedure is repeated up to convergence when the load factor increment remains very nearly constant.

Basis for iterations within one loading step

If, for a given increment in load factor $\Delta\lambda$, the correct (converged) displacement vector increment {DSP} is calculated then the relationship given below will apply exactly:

$$[K_e]^{-1} (\Delta\lambda \{P\}+\{dR\}) = \{DSP\} \qquad (3)$$

where $[K_e]$ is the overall elastic stiffness matrix, {P} is the load vector and {dR} is the load vector due to "initial stresses". If for a given $\Delta\lambda$ an approximate value of {DSP} is known then the following recursive formula may be used to obtain improved values of {DSP}:

$$\{DSP\}_{(i+1)}=[K_e]^{-1} (\Delta\lambda \{P\}+\{dR\}_{(i)}) \qquad (4)$$

where suffix "i" denotes the iteration number.

This recursive formula is derived intuitively from equation (3) and no proof regarding convergence is given. In fact it can be proved that it will diverge when $\Delta\lambda$ brings the total loading above the collapse load. This difficulty has been commonly encountered by researchers in the field where they found it impossible to calculate the load displacement part of the curve around the collapse load. In the strip footing problem for example the last point in the curve of applied pressure versus settlement, and before divergence is indicated, is usually used to represent a good approximation of the lower bound of the collapse load.

The underestimation of the true collapse load and the poor representation of the load-displacement curve in the vicinity of collapse arise because $\Delta\lambda$ is usually assumed fixed. It may be overcome by varying $\Delta\lambda$, (which is the same as varying the load increment) in such a way as to keep constant a norm of {DSP}. In this work the increment of the settlement at the centre of the footing, denoted by $\{DSP\}_c$, is maintained constant throughout. In applying the displacement control with elastic-perfectly plastic soils the above recursive formula (3) is adjusted so as to keep $\{DSP\}_c$ constant and equal to C at each stage. This is achieved by assuming that {DSP} is proportional to $\Delta\lambda$ and hence a factor h is calculated so that

$$h=\frac{C}{\{DSP\}_c}$$

which insures that

$$\text{new } \Delta\lambda = h\ \Delta\lambda \qquad (5)$$

$$\text{new } \{DSP\}_{i+1} = h\ \{DSP\}_{i+1} \qquad (6)$$

This assumes a linear relationship between $\Delta\lambda$ and {DSP}. This assumption is not absolutely correct because some elements transit from elastic to plastic mode but the process being iterative, the small errors involved are automatically corrected in the next iteration. On convergence, the internal stresses, displacement vector and load factor are updated. The loading steps can be repeated as many times as necessary.

There are always out of balance forces which remain at the end of each loading step. If the program does not account for such forces they continue to accumulate and built up a certain amount of non-equilibrating load. The effect of the out of balance forces was minimized by calculating them at the end of each loading step and applying them together with the increment of loading vector in the next loading step. The treatment of the out of balance forces with displacement control needs, in the case of a flexible footing, the introduction of a second displacement vector increment (Kalteziotis, 1981).

Strip footing on strain-softening soil

The idealized stress-strain relationship assumed in this work is shown in fig. 1 and it is composed of three regions (OKMN). The first

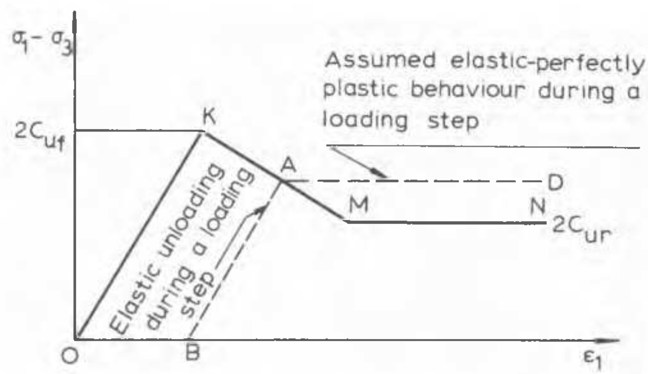

Fig.1 Representation of the Stress-Strain Curve and Procedure for the Treatment of the Strain-Softening Case

ascending portion, OK, represents initial elastic behaviour of the soil (clay), in which the load rises linearly with a slope of E up to the peak strength. The second region, KM, is a descending straight line in which the strength is reduced from peak to residual linearly with increased plastic deformation. The third region, MN, is a constant strength, c_{ur}, region where the stress remains constant with further plastic deformation. It is assumed further that the yield condition is described by the Tresca criterion. In order to characterize the brittleness of a soil, both the total reduction in strength and the rate of reduction in strength between the peak and the residual state must be considered. The extent of strain-softening is described by the brittleness index, I_B, given by:

$$I_B = \frac{c_{uf} - c_{ur}}{c_{uf}} \qquad (7)$$

where c_{uf} and c_{ur} are the peak and residual shear strength respectively. The rate of strain-softening is described by the ratio H'/E, H' being the slope of the axial stress-axial plastic strain curve in an uniaxial test (H'<0), and E is the Young's modulus of the soil (clay).

For the treatment of the strain-softening case and at the start of a loading step, the strength of each element, which is known from the previous deformation history, is assumed to remain constant throughout that step. Under these conditions the load-deformations characteristics of an element which has previously undergone plastic deformation is as shown with the dashed lines of Fig. 1. Line AB represents elastic unloading of the element whereas line AD represents constant strength elasto-perfectly plastic loading. This way the problem was reduced to the elastic-perfectly plastic problem described before. Further details can be found elsewhere (Kalteziotis, 1981).

PRESENTATION AND DISCUSSION OF RESULTS

In order to illustrate some applications of the proposed model, a plane strain problem is solved herein using the finite element method and the Tresca criterion. It consists of a 3.10m wide strip footing resting on the surface of a weightless linearly elastic-perfectly plastic or strain-softening soil (clay). The soil was assumed to have a Young's modulus of 206,850 kPa and to be isotropic and homogeneous. The finite element mesh used to analyse the strip footing problem consisted of constant strain triangular elements. Boundary conditions in the finite element mesh were identical to those used by Höeg et al. (1968).

The load-settlement curves for a flexible surface, strip footing resting on a nearly incompressible clay (ν = 0.48) which exhibits a strain-softening behaviour are illustrated in Fig. 2. The curves of this figure have been obtained for a brittleness index of 40% and each corresponds to a different rate of softening. The strain-softening clay is assumed to possess an undrained peak shear of 120.66 kPa and a residual shear strength of 72.40 kPa. The load-settlement curves for the elastic-perfectly plastic clay having constant shear strengths of 120.66 kPa and 72.40 kPa respectively are also shown in Fig. 2. After a common initial elastic portion, the load-settlement curves for the strain-softening clay rise to a peak value (peak ultimate load) which is less than the collapse load for the elastic-perfectly plastic clay having a constant shear strength equal to the peak shear strength of the strain-softening clay. After rising to a peak, the load-settlement curves exhibit a descending portion and finally approach the ultimate residual load at larger settlements. The peak ultimate load clearly depends on the rate of strain-softening, H'/E, when the other parameters are held constant.

The value of the residual ultimate bearing capacity of a footing on a strain-softening clay, is on the other hand, practically independent of H'/E as it is illustrated in Fig. 2. The load-settlement curves, after passing the peak load, approach the ultimate failure load of a footing on an elastic-perfectly plastic clay with a strength equal to the residual shear strength of the softening clay. This way the footing is liable to catastrophic failure if the actual loading reaches or exceeds the peak ultimate

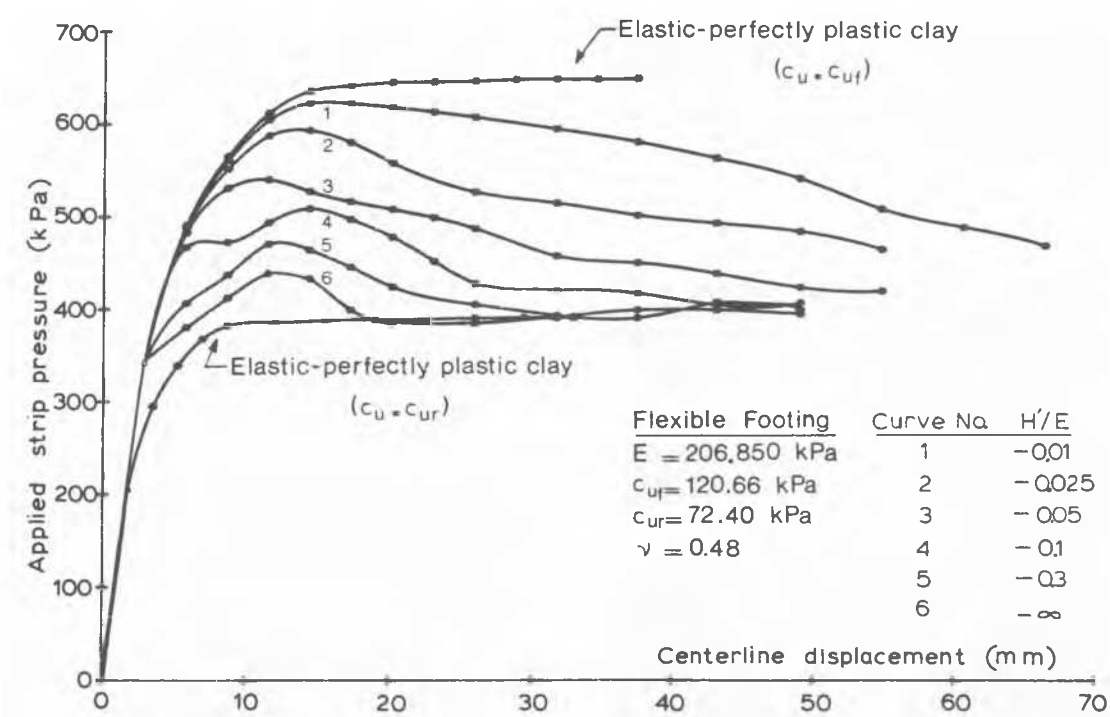

Fig. 2 Effect of Strain-Softening on Load-Settlement Curve

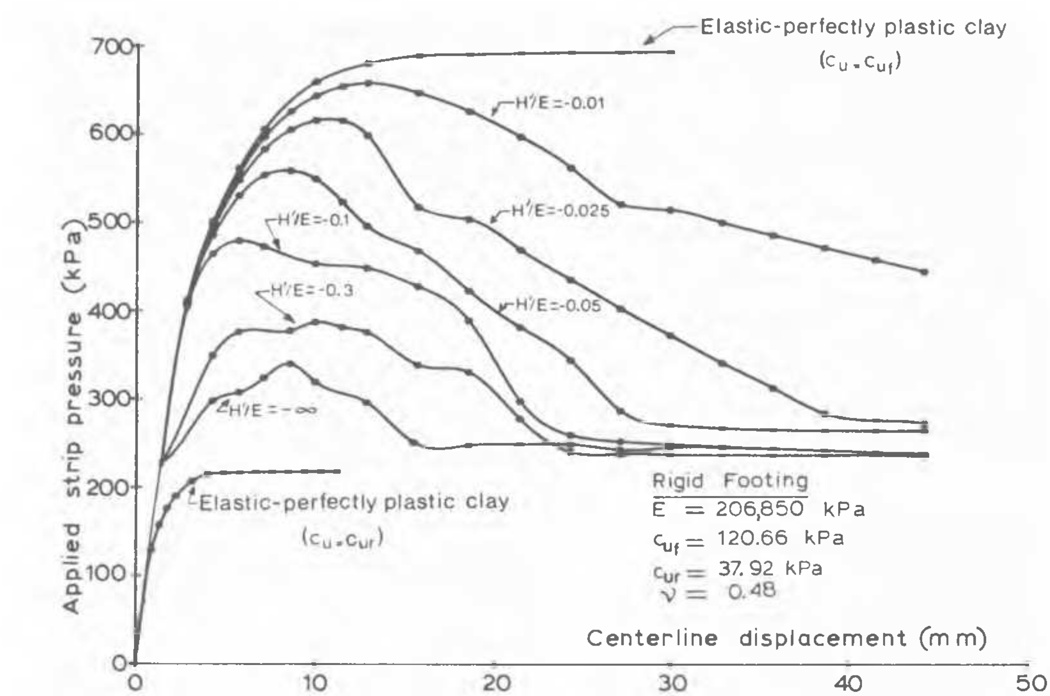

Fig. 3 Effect of Strain-Softening on Load-Settlement Curve

load. However, the actual settlement at which the residual ultimate load is obtained does in fact vary with H'/E. Referring to the same Fig. 2, curve No.1 is incomplete but it does however indicate that the correct residual load would have been obtained at larger settlement that the one at which the curve was truncated.

Similar conclusions can be drawn from the results of the analyses shown in Fig. 3 dealing with a rigid surface strip footing bearing on a nearly incompressible strain-softening clay ($\nu = 0.48$) which is assumed to possess the same undrained peak strength of 120.66 kPa but a higher brittleness index ($I_B = 68.6\%$).

CONCLUSIONS

The "displacement control method" was found to be capable of giving the complete picture of the load-displacement behaviour and a rather accurate determination of the collapse load of a strip footing bearing on either an elastic-perfectly plastic or strain-softening clay.

In general, the load-settlement curve of a footing on a strain-softening clay first rises to a peak and subsequently drops to a residual value. The residual ultimate load is nearly equal to the ultimate collapse load for an elastic-perfectly plastic clay with shear strength equal to the residual shear strength of the strain-softening clay. For the purpose of design, failure is normally assumed to occur at the peak ultimate load. This peak load is definitely less than the collapse load for an elastic-perfectly plastic clay with shear strength equal to the peak shear strength of the softening clay. Its value, and thus the effect of strain-softening, is a function of the rate of strain-softening and decreases as the absolute value of H'/E increases (i.e. as H'/E decreases towards - ∞).

The peak and residual ultimate loads for a strain-softening clay are reached at settlements which seem to depend on the rate of strain-softening.

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