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# Wave induced soil motion and inverted in-situ shear modulus

## Mouvement du sol dû aux ondes de gravité et module de cisaillement résultant local

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**ABSTRACT** In-situ complex shear moduli at large shear strains are determined by inverting the measured soil particle motion in a soil-wave tank. The elasto-plastic propagator matrix theory for wave-soil interaction developed by Yamamoto (1983) was used in the inversion. The inverted shear modulus (real part) is found to be in good agreement with the torsional shear data in the literature. However, the inverted damping (imaginary part) is found to be much larger than the torsional shear data. Observed mixing and mass transport processes in clay bed under wave action are considered to be responsible for the large damping by clays.

### INTRODUCTION

The dynamic behavior and energy dissipation (damping) mechanism of soils have been broadly studied in recent years (5, 7, 11). This interest stems from the fact that a better understanding of soil interaction and soil-structure interaction is needed for the anti-seismic design of nuclear power plants, earth dams and structures in general. The knowledge of the dynamic properties and behavior of submerged sediments is no less important in the design of offshore structures such as oil drilling platforms and offshore gravity oil tanks. Such sediments may be subjected to cyclic loading by ocean waves as well as seismic sources. In addition, accurate modeling of acoustic and seismic propagation in the oceans for underwater surveillance and explorations, demands information of the elastic properties of bottom sediments.

Soils behave elastically only under small strain amplitudes less than  $10^{-5}$ . Under intermediate strain amplitudes from  $10^{-5}$  to  $10^{-2}$ , soils behave like something in between an elastic material and a plastic material -- an elasto-plastic material. Under large strain amplitudes larger than  $10^{-2}$ , soils are fluidized and can no longer be considered to be solid materials. Because the strains caused by ocean acoustics and ocean seismic explorations are usually less than  $10^{-5}$ , one may treat the marine sediments as elastic media. On the other hand, earthquakes and the ocean waves can cause large strain amplitudes in marine sediments which fall in the last two classifications. Nonetheless, Seed and Idriss (1969) have demonstrated that the dynamic soil behavior under intermediate strain amplitude excitations caused by earthquakes may be adequately modeled by the equivalent linear system. The equivalent linear stress-strain model is defined by

$$\tau = G^* \gamma \quad (1)$$

Here,  $\tau$  and  $\gamma$  are shear stress and shear-strain respectively and are assumed to be time-harmonic.  $G^*$  is the complex shear modulus given by

$$G^* = G_r + i G_i = G_r(1 + i \delta) \quad (2)$$

The real part  $G_r$  is the shear modulus and the imaginary part  $G_i$  the loss modulus. The ratio of the two  $\delta$  is the loss coefficient.  $G_r, G_i$  may be dependent

on the shear strain amplitude but are considered to be nearly independent of excitation frequency. This work stimulated the determination of equivalent linear soil moduli in torsion triaxial cells by many investigators.

Seed and Idriss (1970), Anderson and Woods (1975), Richart and Wylie (1977) and Ishihara (1975) have summarized previous experiences on the variation of the shear modulus with the shear strain for different types of clays. Figures 1 and 2 show the envelopes of these torsional shear data of the normalized shear modulus and loss coefficient (damping) vary with the shear strain for a maximum value of the shear strain of about 1 percent. No values of the shear modulus and damping are found in the literature for strain amplitudes larger than 1 percent.

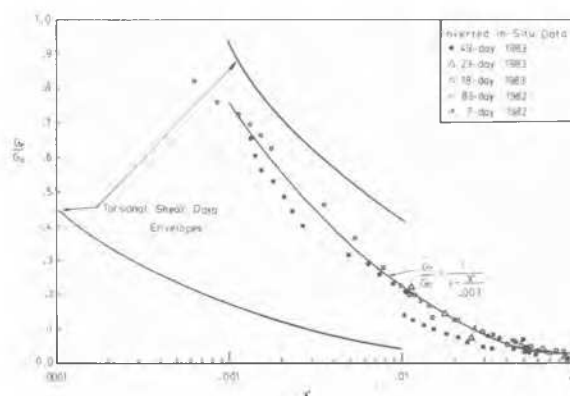


Fig. 1 Inverted In-Situ Data of Shear Modulus vs. Strain Amplitude

Wave tank tests performed by the writers (6 and 9) have indicated that submerged soils may be subjected to strain amplitudes larger than one percent. The analysis of offshore structures founded on marine sediments and subjected to wave action thus requires the determination of shear moduli values for large strain amplitude.

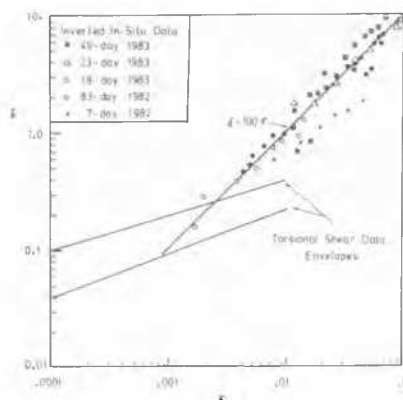


Fig. 2 Inverted In-Situ Data of Loss Coefficient vs. Strain Amplitude

Recently, Yamamoto (1983) developed the theory of wave-soil interaction based on the equivalent linear relation given by Eq. (1) and the propagator matrix.

An inversion method (3) of the propagator matrix theory was recently developed by the writers to back calculate the variation of the shear modulus and damping with depth within the soil bed from the measured soil particle motion under wave action. The inverted in-situ soil moduli are determined from the previous data (9) as well as the newly measured wave induced soil particle motions (6). Experimentation and the inversion procedure are discussed in this paper. The inverted soil moduli data are compared with the torsional shear data in Figs. 1 and 2.

#### WAVE TANK EXPERIMENTS

The experimental facility and procedures are essentially the same as previous tests (6, 9, 13, 14). The wave tank used in the experiments had the dimensions of 18 m long x 0.85 m wide x 1.2 m high. Wave absorbers were placed on both ends of the tank in order to reduce wave reflections. The soil tank (10 m long x 0.62 m deep x 0.85 m wide) is located at the mid-section of wave tank. Bentonite clay was mixed with sea water and allowed to settle for a minimum of 7 days before the beginning of the experiments. Shear strength profiles, soil particle motions, water wavelength, and water wave damping for several bed conditions were measured. The mud-line height (hs) decreased from 69 cm at the beginning of the experiments (7th day after mixing the clay), to 52 cm (79th day after mixing the clay) as consolidation proceeded. During the series of experiments, the wave height (H) varied between 1 and 20 cm, whereas the wave period (T) varied between 0.75 and 2.5 seconds.

#### Soil Property Measurements

Porosity and shear strength were measured during the investigation such that a record of their variation with depth and with time after mixing (time of consolidation) could be kept. Soil porosity decreased from 0.87 to 0.82 between the 7th and 79th days after mixing the clay.

Undrained shear strength ( $S_u$ ) consistently increased with depth below the mudline. Undrained shear strength also increases with consolidation time. From a value of 200  $N/m^2$  at the bottom of the clay bed on the 23-24th day after mixing,  $S_u$  increased to 400  $N/m^2$  on the 49th day after mixing, and further increased to 700  $N/m^2$  on the 79th day at the same locations. From vane shear tests ran before and after wave generation, it was found that

the clay bed suffered a temporary softening after which, there was gradual recovery and further increase in the shear strength of the soil.

The shear strength increases roughly linearly with depth in a clay bed under natural consolidation. The strength gradient of the bed also increases with time as a result of natural consolidation. The average vane shear strength gradients of the 24, 49 and 79 days old bed are 330, 830 and 1700  $Pa/m$ , respectively. Note: (1  $Pa = 1 N/m^2$ ). The measured shear strength gradients were used to estimate the maximum shear modulus  $G_o$  (9). Different values of the conversion factor  $C$  are recommended by different investigators. Actually  $C$  is not a constant but a function of  $S_u$ . Since no investigations cover the range of  $S_u = 0-4,000 N/m^2$  measured in the present study, an extrapolated value obtained from these data is used (9,14). A constant conversion factor of 100 is used. The physical properties of the clay beds which are used for the inversion analysis are tabulated in Table 1.

TABLE I

Physical Properties of Test Clay Beds

Bed No.	(1)	(2)	(3)	(4)	(5)
Bed Age (days)	49	23	18	83	7
Reference No.	6	6	6	9	9
Bed Thickness hs(m)	0.58	0.62	0.62	0.24	0.28
Average Porosity(%)	84	85	85	79	84
Shear Strength					
Gradient (kPa/m)	0.83	0.33	0.25	4.0	1.2
Max. Shear Modulus					
Gradient (kPa/m)	83.	33.	25.	400.	120.

#### Soil Particle Movements

Soil particle movements were measured from 8 mm movie pictures after following the movement, through the glass panels of the wave tank, of specially prepared devices. Lightweight hollow plastic bars (2 mm in diameter), cut to the wave tank width, were filled with a bentonite clay-seawater slurry (to ensure a density close to that of the clay bed) and sealed at both ends. These bars were inserted horizontally into the clay bed at different depths with one of the ends clearly seen through the glass panels of the wave tank.

Soil particle movements at four given depths below the mud-line are depicted in Fig. 3. Soil particles rotate in an approximate elliptical pattern and in the same direction as water particles due to wave action for this case. The soil particle moves slightly in advance in phase relative to the water particle for this case. The amplitudes of horizontal and vertical displacements may be determined from the elliptical patterns of soil particle motions and may be used as input for the inverse theory as shown in Fig. 4.

Similar to the water particle motion in the wave field, the wave-induced soil particle motion has a component of steady horizontal mass transport. An example of the wave-induced mass transport in the clay bed is shown in Fig. 5. The soil mass transport is limited to the upper portion of the bed where the soil is experiencing a large shear deformation. The soil mass transport as depicted in Fig. 5 is caused by large wave energy dissipation in this layer of clay bed. The mass transport is always associated with the mixing of soil mass similar to the turbulence mixing of fluid. This mixing may be responsible for the unusually large damping determined from the inversion of the soil motion in this layer.

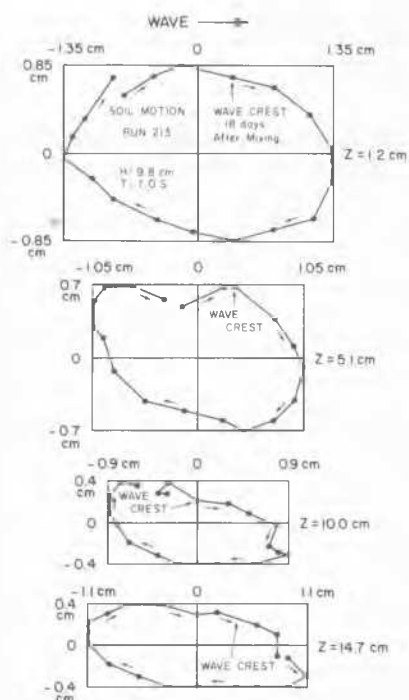


Fig. 3 Wave-Induced Soil Particle Motion

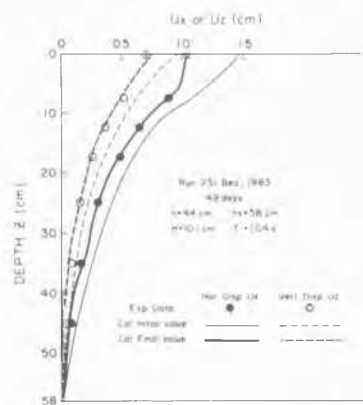


Fig. 4 Measured and Calculated Amplitudes of Horizontal and Vertical Displacements

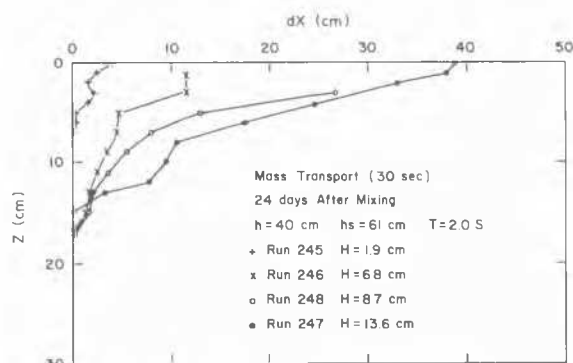


Fig. 5 Wave-Induced Soil Transport Data

## INVERSE THEORY AND SHEAR MODULUS DETERMINATION

The inverse problem for a linearized system as suggested by Aki and Richards (1980), in combination with the propagator matrix method for seabed response to water waves developed by Yamamoto (1983), has been used to determine shear moduli values with depth within a soil profile subjected to wave action. The combination of these two concepts allowed the determination of shear moduli for large strains, based on soil particle movement measurements obtained during the wave tank experiments previously described.

The proposed method involves a trial and error procedure in which displacements calculated using the propagator matrix method (shear moduli values with depth are needed as input parameters) developed by Yamamoto (1983) are compared with the displacements obtained during the experiments. If the calculated and measured displacements are appreciably different, new displacements need to be calculated through the numerical integration technique. How many cycles are needed to match the calculated to the measured displacements depends on how accurate the initial estimate of the shear modulus is with depth. The mathematical formulation and the procedures required for this inverse method are presented in a recent paper by the writers (3). An example set of the initial guess of the shear modulus profile, and the final estimates of the shear modulus profile are shown in Fig. 6. The corresponding set of the measured values, the initial and the final calculated values of horizontal and vertical displacements are shown in Fig. 4.

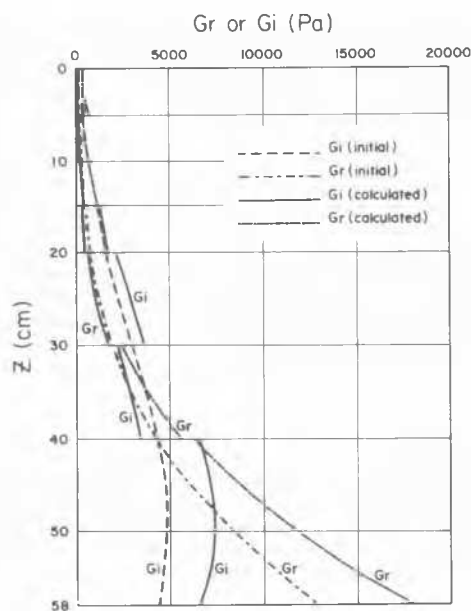


Fig. 6 Initial and Inverted Final Profiles of Complex Shear Modulus

## RESULTS AND THOUGHTS

Once the final values of the shear modulus were obtained, the numerical integration technique (12) allowed the determination of shear strains with depth. It was then possible to plot modulus reduction and damping curves. Figures 1 and 2 show comparisons of modulus reduction and damping curves respectively obtained by the inversion technique with similar curves found in the literature. It should be indicated that there is better agreement in the modulus reduction curve than in the damping curve with the values generally found in the literature. The

modulus reduction curve found by the inverse method falls within the general range of values and follows a similar trend to those curves for the torsional shear data. For example, the inverted in situ modulus reduction curve is closely approximated by a hyperbolic relation (4) as

$$\frac{G_r}{G_0} = \frac{1}{1 + \gamma/0.003} \quad (3)$$

The damping curve, however, lies above the outermost limit of the torsional shear data and has a steeper slope than these curves as shown in Fig. 2. The inverted in-situ damping data are approximately given by a linear relation as

$$\delta = 100 \gamma \quad (4)$$

According to Eqs. (3) and (4), the damping modulus  $G_i$  approaches a constant value as  $\gamma$  becomes larger, or

$$G_i = G_r \delta + 0.3 G_0 \text{ as } \gamma \rightarrow \infty \quad (5)$$

This limiting relation suggests that the soil in the mixing layer behaves as a "viscous fluid". In the turbulent fluid flow, energy is quickly dissipated compared to the laminar flow. Similarly the larger damping value obtained from the mass transport layer of clay bed may be due to the turbulent mixing of the soil particles in comparison to the laminar motion of soil under torsional shear tests.

## CONCLUSIONS

The inverse method based on the elasto-plastic propagator matrix theory is successfully used to determine the complex shear modulus of clays at a large shear strain level from the measured motion of soil particle under surface water wave action.

The inverted dynamic shear modulus (the real part) as a function of the shear strain amplitude is found in close agreement with the torsional shear data in the literature

The inverted damping (the imaginary part) is, however, found to be much larger than the torsional shear data. This large damping of clays is attributed to the observed mixing and the forward mass transport in the clay bed which are induced by wave action. In the mixing layer, the clay behaved like a viscous fluid.

## ACKNOWLEDGEMENTS

This research was mainly sponsored by the National Science Foundation with a Grant No. CEE-8117454. A portion of the inverse analysis was sponsored by the Office of Naval Research with a Contract #N00014-83-C-0165. Mr. Toshihiko Nagai assisted in the experiments.

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