# A test for poro-elastic properties of sediments

## Propriétés poro-élastiques des sédiments marins

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SYNOPSIS. For the analysis of wave-induced transient pore-water pressures and displacements, the soil properties which control the behavior can be combined, in one-dimensional strain condition, into a single parameter termed "composite poro-elasticity factor, $\alpha'$". Composite poro-elasticity factor includes all the pertinent parameters, i.e., shear modulus, Poisson's ratio, and coefficient of permeability of the soil and the compressibility and unit weight of the fluid along with the frequency of wave loading and a characteristic dimension (such as the length of the maximum drainage path). A new laboratory test for the determination of $\alpha'$is described. The theoretical basis of the test and the method of analysis of the results are briefly presented based on a consideration of the coupled response arising from the porous and elastic nature of marine soils (under typical transient strain levels). The results of tests performed on three different soils at varying densities indicate a close correlation between the measured composite poro-elasticity factors and the theoretical values computed by extrapolating on the basis of frequency and stress variations. The test provides a more efficient means of determining the poro-elasticity of marine soils and the test results reveal interesting aspects of the poro-elasticity of marine soils.

## INTRODUCTION

Recent advances in offshore construction have created a need for improved methods of property characterization of marine soils. One of the important sources of loading is due to sea waves which induce pore-water pressures directly through the mudline as well as through the forces transmitted by the offshore structure. Both transient and residual pore pressures may result from the coupled instantaneous response of the soil skeleton and pore water to wave-induced loads. The transient pore pressures have attracted attention from a large number of investigators and for which there is some laboratory and field evidence (Sleath, 1970; Yamamoto, et al., 1978; Madsen, 1978; Mei and Foda, 1981). In the coupled analysis of the transient pore pressure, the response of the sea bed is determined by taking into account the elastic deformation of the porous medium, compressibility of the pore fluid and Darcy flow. The coupled analyses are largely based on the three-dimensional consolidation theory of Biot (1941) and are referred to herein as the "poro-elastic" methods. The linear poro-elastic approach provides the fundamental characteristics regarding the wave-induced response of offshore sediments and structures and it is justified for small cyclic shear strain amplitudes which appear to be the case even under very severe storm conditions (Mizikos and Hicher, 1982). For the poro-elastic analysis, the soil properties needed are shear modulus, Poisson's ratio, hydraulic conductivity, porosity, and degree of saturation, and the pertinent fluid properties are density and compressibility. These properties are to be determined individually from separate tests. The individual tests are not simple and the properties measured are often affected by the test boundary conditions and procedures as well as certain environmental factors such as levels of confining and cyclic stresses, frequency of loading, etc. Based on these considerations, a single parameter which combines the soil and fluid properties and a new laboratory test of measuring this parameter in a single test are presented.

## THEORETICAL BACKGROUND

For a homogeneous, isotropic seabed, the governing equations in one-dimensional vertical strain may be written as (Toha, 1983)

$$\frac{2(1-\nu)}{1-2\nu}\kappa i\frac{d^3P}{dz'^3}+\{1+\frac{2\xi(1-\nu)}{1-2\nu}\}\frac{dP}{dz'}=0 \qquad (1)$$

where $i = \sqrt{-1}$, $\kappa$ is the poro-elasticity factor $(kG/\gamma_f\omega a^2)$, $\xi$ is the fluid compressibility factor $(nG\beta)$, $\nu$ is the Poisson's ratio, P is the non-dimensionalized axial variation of pore pressure, and z' is the non-dimensionalized axial coordinate. The non-dimensionalization parameters are **a** (a characteristic dimension) and $P_o$ (the amplitude of pressure at the surface). The other parameters are k = coefficient of permeability, G = shear modulus, $\gamma_f$ = fluid unit weight, $\omega$ = angular frequency, n = porosity, and $\beta$ = fluid compressibility. Eq. 1 combines the conservation of mass principle and Darcy's law with the elastic equilibrium utilizing the effective stress principle. The characteristic roots are $A_1 = 0$ and $A_{2,3} = \pm\alpha$ where the modulus of $\alpha$ is given by

$$\alpha' = \frac{\alpha}{\sqrt{i}} = \sqrt{\frac{1}{\kappa}\left[\xi+\frac{1-2\nu}{2(1-\nu)}\right]} = \sqrt{\frac{\gamma_f\omega a^2}{k}\left[n\beta+\frac{1-2\nu}{2G(1-\nu)}\right]} \qquad (2)$$

where $i = \sqrt{-1}$. The parameter α' is seen to incorporate the pertinent soil and fluid characteritics as well as the frequency into one composite parameter that controls the harmonic response of a one-dimensional poro-elastic medium and it is termed as the "composite poro-elasticity factor" herein. A close examination of α' suggests that it is related to the coefficient of consolidation, $c_v$ and to the thickness of the boundary layer in Mei and Foda's theory (1981). This one-dimensional approach is most appropriate to conditions found beneath shallow water waves (long waves and near the bed surface).

## TEST CONDITIONS

In the development of a suitable procedure for the determination of α', a general solution of Eq. 1 is obtained. The constants of the solution are determined using the test boundary conditions. Based on considerations presented elsewhere (Toha, 1983), two different sets of test boundary conditions were found to be suitable for coarse and fine-grained soils, respectively. In either case, however, the test is performed in a rigid cylindrical chamber under one-dimensional strain condition. For coarse-grained soils, the best test configuration is achieved by applying a cyclic stress to the soil skeleton through a rigid and porous disc at the top of the specimen while maintaining an undrained condition at the bottom. For this condition, the non-dimensional axial variation of pore pressures is given for a fully saturated soil as

$$P = 1 - \frac{\cosh(\alpha + \alpha z')}{\cosh(\alpha)} \qquad (3)$$

It is clear that P is a function of α alone under saturated conditions. Therefore, if the pore pressure distribution with depth in the specimen is available, α can be obtained by curve matching with the theoretical solutions. Pore pressure ratio (the modulus of the non-dimensionalized complex P) distribution, according to Eq. 3 is shown in Fig. 1 for various values of α' as a function of non-dimensionalized elevation, z' (elevation divided by the specimen height). As the magnitude of α'increases beyond 2, the pore pressure ratio distribution becomes less sensitive to α'. Therefore, the boundary conditions described should not be used to determine α'when it is estimated to exceed 2.

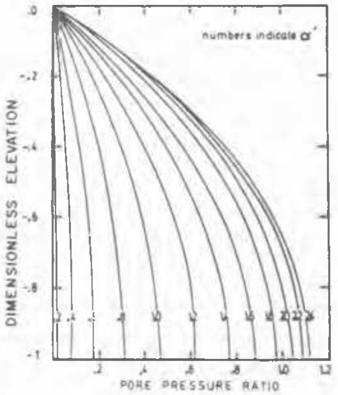

Fig.1 Pore Pressure Ratio Distribution

## TEST APPARATUS AND PROCEDURE

A cylindrical cell to accommodate a soil specimen of 150 mm in diameter and 300 mm in height and to suit the boundary conditions abov was built. A sketch of the complete test setup is shown in Fig. 2. The pore-pressure measurement ports are installed at proper elevations and a recorder is used in collecting the data. In order to guarantee a one-dimensional strain condition, three or four steel straps are fitted around the plexiglass. Two pore pressure ports are provided at the base, one of which is placed in the center. This provides an additional check on one-dimensional strain condition since the ports at the same elevation should register the same reading. The center port is also used in measuring permeability by circulating water through the specimen and measuring the outflow rate.

Before placing the sample in the test cell, a latex membrane with the proper diameter is fitted in the cell after lubricating the inner wall of the cell in order to eliminate side friction on the specimen. Saturating the specimen is a difficult but a critical step in the procedure. A back pressure of about 140 kPa is often sufficient to improve saturation, but in the tests 250 kPa was ordinarily applied.

Subsequent to saturation, the static vertical stress of the desired level is applied to simulate the overburden stress and finally the cyclic loads at various frequencies and magnitudes are applied while recording the pore pressures at various depths. The recorded and normalized pore pressure data are plotted on Fig. 1 and the composite poro-elasticity factor, α'is determined by curve matching.

## TEST RESULTS

In order to evaluate this new test, three soils ranging from coarse sand to silt were used as summarized in Table I. The testing program included 156 measurements on 9 specimens

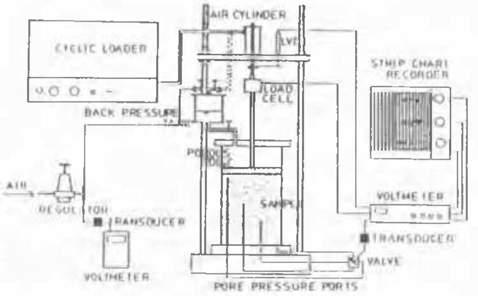

Fig.2 Sketch of the Test Setup

Table I

Properties of Soils Tested

| Sample | $G_s$ | Grain Size (mm) | $e_{max}$ | $e_{min}$ | $C_u$ | k(mm/sec) at $D_r$=70-80% |
|---|---|---|---|---|---|---|
| Madison Silt | 2.83 | 0.005-0.074 | 1.20 | 0.66 | 1.8 | 0.10 |
| Ottawa 70-140 | 2.55 | 0.105-0.210 | 0.90 | 0.52 | 1.5 | 0.79 |
| Ottawa 50-80 | 2.66 | 0.177-0.297 | 0.87 | 0.52 | 1.5 | 0.89 |

reconstituted from the three soils samples at different relative densities ranging from 10 to 100%. The range of static and cylic stresses were, respectively, 15 to 232 kPa and 7 to 345 kPa. The cylic stress frequencies varied over a range of 0.1 to 2 Hz.

Several frequencies of cyclic loads were applied at each stress condition on each specimen during the experimental program. If all parameters, except frequency, remain constant during the measurement of α', then α' can be expressed as

$$\alpha'_i = C\sqrt{\omega_i} \qquad (4)$$

where C is a constant depending on soil type, density, and stress state. All of the test results involving different soils, densities, and stress levels were analyzed using Eq. 4. A comparison of the theoretically estimated values of α' as a function of frequency and the measured values is shown in Fig. 3. The estimates of α' were obtained by extrapolation over a frequency range as high as fifteen times the reference frequency. At large frequency changes, the values of α' are overestimated; however, if extrapolation is limited to a range twice the reference frequency, the discrepancy becomes less than 10% and acceptable. Assuming that the power law is correct, it is apparent that the coefficient C in Eq. 4 itself varies with large changes in frequency. This is due to the fact that certain elastic properties depend on the frequency of loading as will be shown later in the case of Poisson's ratio. Based on these observations, it is suggested that the tests to be conducted within a frequency range of 0.15 to 0.25 Hz. This will allow safe extrapolation to the reported wave frequencies which are mostly between 0.07 and 0.50 Hz.

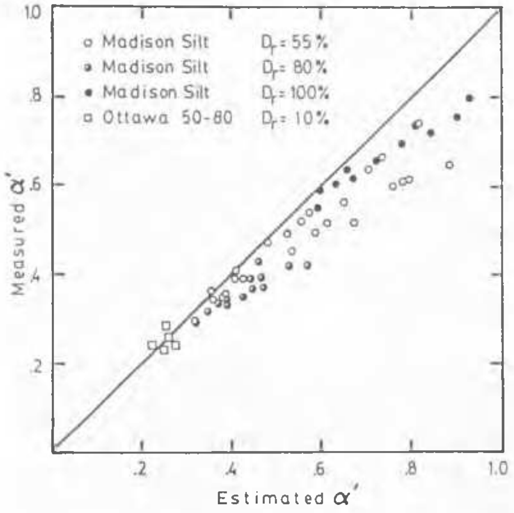

Fig.3 Measured vs Estimated α' (For Frequency)

The effect of static stress level, i.e., the confining pressure which is proportional to the vertical stress $\sigma_v$ for the at-rest condition, is characterized for the incompressible fluid by

$$\frac{\alpha'_1}{\alpha'_2} = \left(\frac{G_2}{G_1}\right)^{1/2} = \left(\frac{\sigma_{v2}}{\sigma_{v1}}\right)^{1/4} \qquad (5)$$

since shear modulus is approximately proportional to the square root of the confining pressure (Hardin and Drnevich, 1972). This effect can be studied by examining the data having the same cyclic stress amplitude to static vertical stress ratio and the same frequency of loading. Eq. 5 was used in extrapolating values measured at a reference vertical stress to other stress levels. A comparison of the measured and extrapolated values is given in Fig. 4 and once again the results are satisfactory. As shown by these checks (frequency and stress level), the values of α' determined from the tests basically follow the trends expected from the theory.

As a final check, the coefficients of permeability determined on the same specimens after each test, the known values of shear modulus of these soils as given by Edil and Luh (1978), and a range of estimated Poisson's ratios were used in Eq. 2 in calculating α'. The calculated values were reasonable and compared well with the measured values within

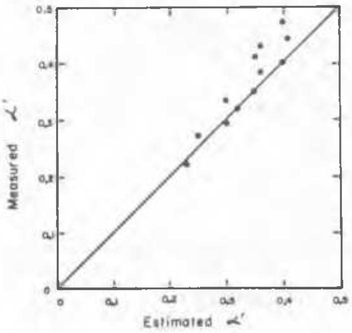

Fig.4 Measured vs Estimated α' (For Stress)

the range corresponding to these Poisson's ratios. Based on the premise that the measured $\alpha'$ is correct, Poisson's ratios were back-calculated using the measured values of $\alpha'$ and k and the shear moduli for these soils from

$$\nu = \frac{1 - [(2\alpha'^2 kG)/(\gamma_f \omega H_s^2)]}{2\,[(1 - \alpha'^2 kG)/(\gamma_f \omega H_s^2)]} \qquad (6)$$

where $H_s$ is the height of the specimen and the other terms are as defined previously ($n\beta = 0$ for fully saturated soil, i.e., incompressible fluid). Shear modulus does not vary significantly with frequency for granular soils as reported by Hardin and Drnevich (1972). The values of Poisson's ratio of Madison Silt as determined using Eq. 6, are shown in Fig. 5 as a function of frequency. Fig. 5 shows the influence of cyclic load frequency and amplitude on Poisson's ratio. The influence of frequency is greater than that of the amplitude within the range of variation of these parameters used in the tests. The dependency of Poisson's ratio on frequency can be explained by the time available for drainage and the associated volumetric strains at different frequencies.

The volumetric strain is lower at higher frequencies resulting in higher Poisson's ratios. While this explanation is plausible, possible effects caused by the experimental procedure cannot be ruled out. Poisson's ratio also depends on the static state of stress and density of the soil. Generally, Poisson's ratio increased with increasing relative density and decreasing confining pressure (vertical stress) in the tests.

## SUMMARY AND CONCLUSIONS

Based on the analysis of the test results of a range of non-plastic soils varying in size from silt to coarse sand, the new test system described herein is seen to offer a technique whereby the composite poro-elasticity factor of marine soils can be determined efficiently and accurately. Depending on the hydraulic conductivity of marine soil and practical limitations, the test can be conducted in different settings involving the drainage and mode of load application at the top and bottom surfaces of the specimen. Poisson's ratios back-calculated from the measured composite poro-elasticity factors indicate a dependency on frequency of loading, stress conditions (static and cyclic), and soil density.

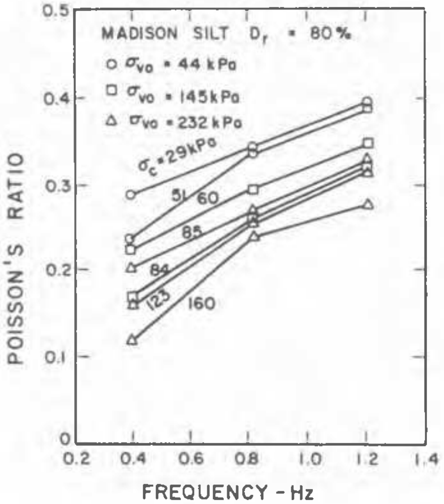

Fig.5 Influence of Frequency and Stress on Poisson's Ratio

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