Scale effect in 1g-model tests on horizontally loaded piles

Pieux en modèle sous charges horizontales – Effet d'échelle

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SYNOPSIS In spite of some expectations arisen in the last years it has revealed that 1g-model tests have results which are scale dependent. It is shown in this paper that the scale effects are due to the influence of the elasticity and the crushing strength of the sand grains. A series of model tests on horizontally loaded vertical piles with varying diameters was performed to determine an analytical description of the scale effects which can be used for the transposition of the model test results to prototypes.

1 PROBLEM

Centrifuge model tests are very expensive and often not available. That is why conventional model tests performed in the natural gravity 1g-field are still of great importance. But soil behaviour is dependent on stress level, stress path and strain rate. Therefore scale effects occur when model tests are used to predict prototype behaviour. In this paper it is demonstrated that scale effects have to be taken into account and can be assessed by a series of conventional model tests if model test results are used to predict prototype performance. This restriction disappears only if sand can be considered as an assembly of rigid, unbreakable grains which is called "psammic material" according to DIETRICH (1977) and if the influence of shear bands (narrow rupture zones) can be neglected. DIETRICH (1977) has shown that real sand behaves like psammic material at low stress intensity. Until now it is still under discussion up to which mean stress level sand can be taken as psammic material such that no scale effects are exhibited. GUDEHUS (1980) proposed that for mean stress levels smaller than

$$\sigma_m = \frac{\sigma_x + \sigma_y + \sigma_z}{3} \leq 1 \text{ MPa} \qquad (1)$$

sand behaves rigid. The test results presented in this paper show that such a limitation is not valid and that particularly the results obtained from small scale model tests with very low stress level indicate scale effects. Insofar older publications e.g. that of LUNDGREN (1957) and DE BEER (1963) were approved.

2 TASK AND DIMENSIONAL ANALYSIS OF THE PROBLEM

By order of the German Federal Railway (Deutsche Bundesbahn) small scale model tests on supporting piles for sound absorbing walls have been performed. The results of the model tests had to be transposed on the prototype. In order to obtain the scaling law relationships a dimensional analysis of the problem acc. to fig. 1 was carried out. The dimensional analysis gives the complete set of similarity requirements that must be fulfilled both by the model and by the prototype. In fig. 1 the lengths d, l, h, the flexibility of the pile EI and the displacements U and F are defined. The test sand is characterized by the unit weight γ_0 at test begin, the average grain size d_G, the modulus of elasticity E_G and the crushing strength C_G of

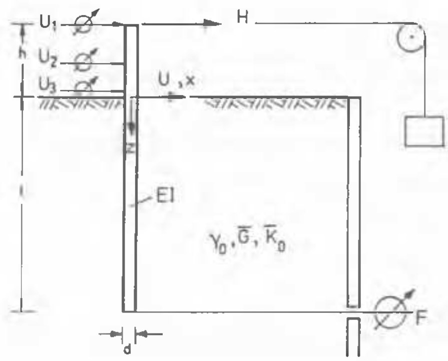

Fig. 1 Cross section of the testing device

the grains (the last two are infinite for psammic material) and by two more dimensionless parameters $\bar{G}$ and $\bar{K}_0$ (DIETRICH 1982). $\bar{G}$ is the "granulometry", summarizing the influences of grain shapes, grain size distribution and normalized grain sizes. $\bar{K}_0$ is the "grain configuration" at test begin, representing the fabric of the sand and insofar exceeding the information given by the relative density. It should be mentioned, that $\bar{G}$ and $\bar{K}_0$ are only needed as symbols and that no knowledge of their magnitude is required. The parameters involved in the problem are

$$U = f(H, d, l, h, EI, \gamma_0, d_G, E_G, C_G, \bar{G}, \bar{K}_0) \qquad (2)$$

where f is an unknown function. If we assume psammic behaviour, the modulus of elasticity E_G and the crushing strength C_G would not appear as relevant parameters. Using the π-theorem from the dimensional analysis, eq. 2 can be expressed in a dimensionless form, choosing γ_0

and d as the quantities with independent basic units

$$\frac{U}{d} = g\left(\frac{H}{\gamma_0 d^3}, \frac{l}{d}, \frac{h}{d}, \frac{EJ}{\gamma_0 d^5}, \frac{d_G}{d}, \frac{E_G}{\gamma_0 d}, \frac{C_G}{\gamma_0 d}, \bar{G}, \bar{K}_0\right) \quad (3)$$

g is another function which is to be determined by the model tests. In order to obtain similarity between the model and the prototype the model tests must be designed in such a way that the dimensionless products in eq. 3 attain the same values for the model and the prototype

3 SCALING LAW RELATIONSHIPS

Using the same sand in the model tests as it is assumed to exist in-situ and imitating an idealized sedimentation process by applying the 'rainfall analogy' (WALKER & WHITAKER 1967) for the preparation of the sand deposit in the model container one gets the same values of G, K and in the model tests and in-situ. This is an idealization as we cannot model the natural soil deposit in all details. Therefore model tests suffer from the same deficiency as a theoretical analysis does, an idealized material behaviour must be presumed.

The scale of length λ is defined as the ratio of corresponding lengths in the model test and in the prototype

$$\lambda = \frac{l_P}{l_M} \quad (4)$$

The index p denotes prototype and the index M denotes model. In the case of 1 g-gravity tests we have

$$\gamma_P = \gamma_M = \gamma_0 \quad (5)$$

and can derive the scale of forces $\varkappa$

$$\varkappa \cdot \lambda^{-3} = 1 \quad (6a)$$

respectively

$$\varkappa = \lambda^3 \quad (6b)$$

This shows that in a 1g-test the scale of length is the only independent scaling factor that can be chosen. The scale of stresses σ is obtained from the scale of length and the scale of forces by the following relationship

$$\sigma = \varkappa \cdot \lambda^{-2} = \lambda^3 \; \lambda^{-2} = \lambda \quad (7)$$

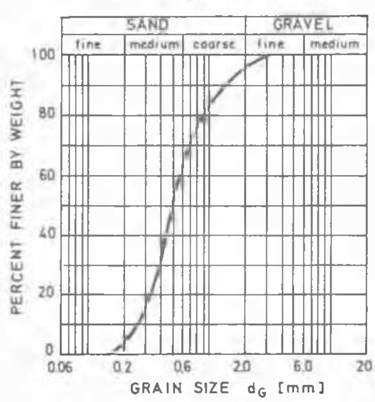

Fig. 2 Grain size distribution of the test sand

As the parameters E_G and C_G have the dimension of a stress they should have a value which is σ-times greater in-situ than in the model test. But this is impossible as usually the same sand in the model test and in case of the prototype is used. Therefore the dependence of the test results on E_G and C_G must be determined. Only for psammic material this dependence disappears because E_G and C_G are assumed to be infinite. The deviations from this restricting assumption in natural soil are responsible for the scale effects.

Acc. to eq. 3 the ratio d_G/d should be the same for model tests and for the prototype, this, however, is impossible. OVESEN (1980) has demonstrated by centrifugal tests on circular footings with diameter $\bar{d}$ on sand that the influence of the grain size can be neglected if $\bar{d}/d_G$ exceeds 30. In the model tests presented in this paper the average grain size of the sand was d_G = 0.5 mm (fig. 2) and the diameter of the smallest model pile was d = 18 mm, thus d/d_G = 36. Additionally the influence of shear bands on the load-displacement behaviour of the pile is vanishingly small. Shear bands arise only at the pile base and at the ground surface in front of the pile. Fig. 3 shows the field of discontinuities in front of the pile which might be of special interest, but this rupture zone is small in height compared with the length of the pile. Therefore the influence of the grain size can be omitted for this model tests.

Fig. 3 Narrow rupture zone at the sand surface

Acc. to eq. 3 the flexural stiffness of the model pile must satisfy the following condition

$$\frac{(EJ)_M}{\gamma_0 \, d_M^5} = \frac{(EJ)_P}{\gamma_0 \, d_P^5} \quad (8)$$

Using the scale of length this can be expressed as

$$\frac{(EJ)_P}{(EJ)_M} = \left(\frac{d_P}{d_M}\right)^5 = \lambda^5 \quad (9)$$

There are different possibilities in the design of a model pile to satisfy this requirement. First the same pile material for the model and for the prototype is considered, thus $E_P = E_M$. In this case pipe profiles can be used whereby the inner diameter d_i of the pipe can be chosen independently of the outer diameter d_o which must be determined acc. to the scale of length.

With $\eta = d_i/d_o$ and

$$I = \frac{\pi (d_o^4 - d_i^4)}{64} \tag{10}$$

we can rewrite eq. 9 in the following way

$$\eta^4 = \frac{\lambda - 1}{\lambda} \tag{11}$$

Obviously this term approaches unity with increasing values of λ. This means that the wall thickness must become smaller and smaller which is restricted by practical considerations. Therefore it is necessary to apply materials with a smaller modulus of elasticity for the model piles. In order to derive a criterion for the selection of suitable materials, the expression 9 has been transformed using the above mentioned abbreviations.

$$(d_o)_M = \frac{(d_o)_P^5}{(EJ)_P} \cdot \frac{\pi}{64} (1-\eta^4) \cdot E_M \tag{12}$$

This expression reveals the outer diameter of a model pile for a given prototype if a material with a specific modulus of elasticity E_M is chosen. In our special case the prototype was a bored pile with a diameter of 0,56 m cast-in-situ with concrete B 35 acc. to DIN 1045. Fig. 4 shows the model pile diameter as a function of the modulus of elasticity for this specific prototype acc. to eq. 12. It shows that for diameters smaller than 10 cm only synthetic materials are applicable. Unfortunately such materials tend to behave visco-elastic. In a series of preliminary tests it was found that polyvinylchlorid, polycarbonate and polypropylen complied with the requirements of the model tests.

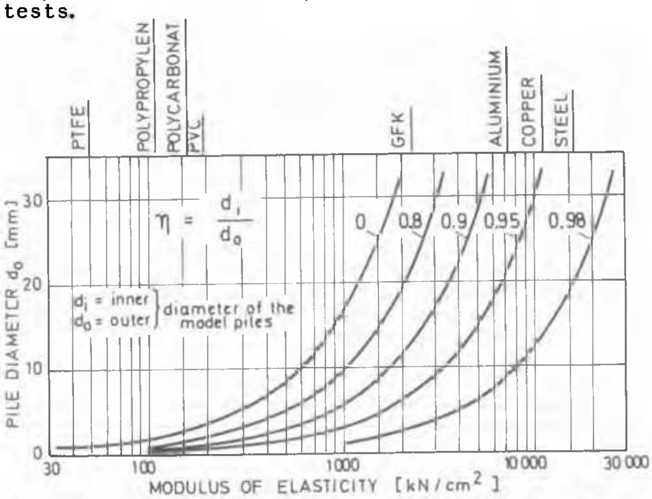

Fig. 4 Dependence of the modulus of elasticity on d_o and η

4 CONCEPT, PERFORMANCE AND EVALUATION OF THE MODEL TESTS

In order to determine the scale effect caused by the influence of the elasticity and the crushing strength of the grains expressing the deviation from psammic behaviour, a series of tests on a model-family was performed. The model-family consisted of a number of model tests on horizontally loaded piles with diameters ranging from 1.8 cm to 25.8 cm , all similar in accordance to eq. 3 except of $E_G/\gamma_0 d$ and $C_G/\gamma_0 d$. Differences in the load displacement behaviour must therefore be due to these influences.

The tests were performed using an air-dried sand acc. to fig. 2. The sand was poured by applying the "rainfall analogy" such that the relative density was almost constant D = 0.575 ± 2% and γ_0 = 17 kN/m³. All test data were recorded automatically. Fig. 5 shows typical load displacement curves for the pile with 7.6 cm diameter as an example. Measuring the displacements at three different points at the pile head (s. fig. 1) enabled to determine the bending deformation of the pile. It was found that the bending deformation was small compared with the total displacement. Thus it can be assumed that the piles behaved rigidly. This is also confirmed by the following considerations:

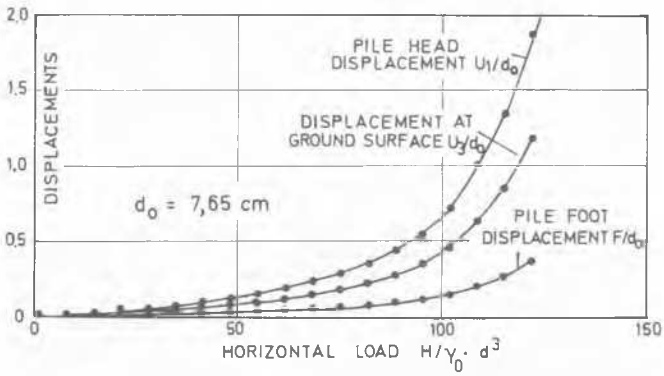

Fig. 5 Test result as an example

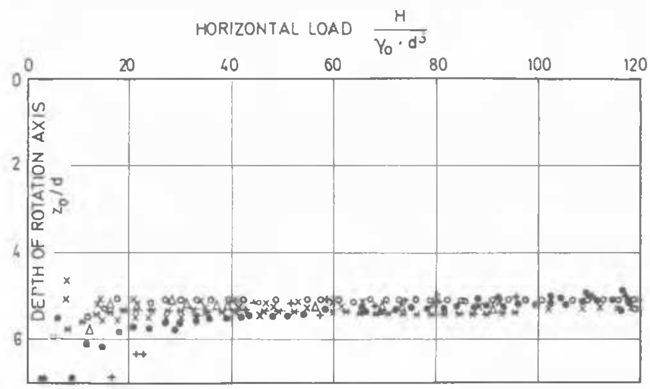

Fig. 6 Observed depths of rotation axis

DIETRICH (1982) has shown that for a horizontally loaded rigid pile in a psammic half space the axis of rotation remains at a constant depth during the loading process. Fig. 6 shows that the depth of the axis of rotation, evaluated from U_3 and F, remains constant with increasing load for all tests. The axis of rotation is located at a depth of z_0 = 5.3 d. Considering in addition that the pile is nearly rigid, only the determination of an analytical load displacement description for the pile head is needed to obtain the complete displacement field of the pile.

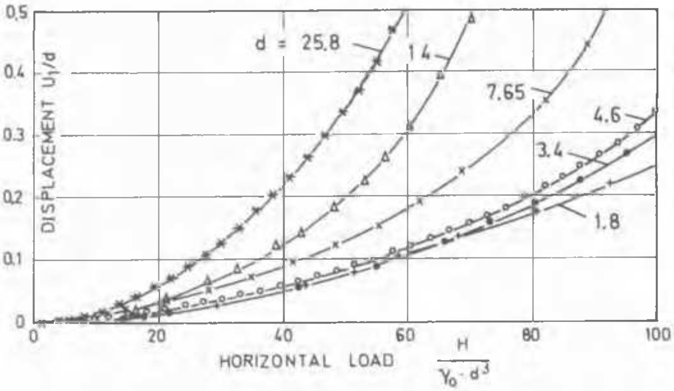

Fig. 7 Test results indicating scale effects

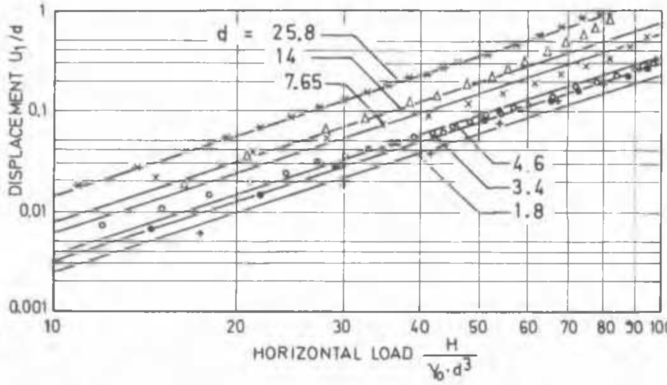

Fig. 8 Test results from fig. 7

Fig. 7 illustrates the observed scale effects due to the influence of E_G and C_G which can now be expressed in dependence on d. As E_G, C_G and γ_0 have had the same values in all tests $E_G/\gamma_0 d$ and $C_G/\gamma_0 d$ varies with the pile diameter d. It can be seen in fig. 7 that with increasing pile diameter the rate of horizontal displacements increases. For the approximation of the load displacement curves the following power law is assumed

$$\frac{U_1}{d} = \left(\frac{H}{\gamma_0 d^3}\right)^{1/\nu} \cdot d^{\alpha} f\left(\frac{l}{d}, \frac{h}{d}, \bar{G}, \bar{K}_0, \frac{E_G}{\gamma_0}, \frac{C_G}{\gamma_0}\right) \qquad (13)$$

where ν denotes the minor hardening exponent according to DIETRICH (1982) and f is constant for all tests of the model family. Fig. 8 shows the load displacement curves in a log-log scale as parallel lines with an average slope of $1/\nu$ =1.97 but different abscissa. In fig. 9 the mean values of $(U_1/d)/(H/\gamma_0 d^3)^{1.97}$ are plotted against the pile diameter for all model tests, yielding a straight line with a slope of α = 0.9 and an abscissa of f = 9.1 10^{-6}. Therefore eq. 13 can be rewritten

$$\frac{U_1}{d} = \left(\frac{H}{\gamma_0 d^3}\right)^{1.97} \cdot d^{0.9} \cdot 9.1 \cdot 10^{-6} \qquad (14)$$

(Diameter of the pile must be taken in cm)
This expression describes the load displacement behaviour of the investigated horizontally loaded pile taking into account the scale effects due to the stress level dependent behaviour of the soil. In a subsequent series of small scale model tests parameter studies with varying geometry and loading cases can be undertaken using eq. 14 to predict the prototype behaviour.

5 CONCLUSIONS

Using conventional small scale 1g-model tests an actual prototype behaviour can be predicted accounting for scale effects due to the influence of the elasticity and the crushing strength of the grains. It is demonstrated in this paper that the scale effects can be determined by a series of model tests all similar except of the influence of the elasticity and the crushing strength of the sand grains. An analytical description of the scale effect is derived in terms of the pile diameter. The knowledge of this description enables to study the influence of various parameters in subsequent model tests.

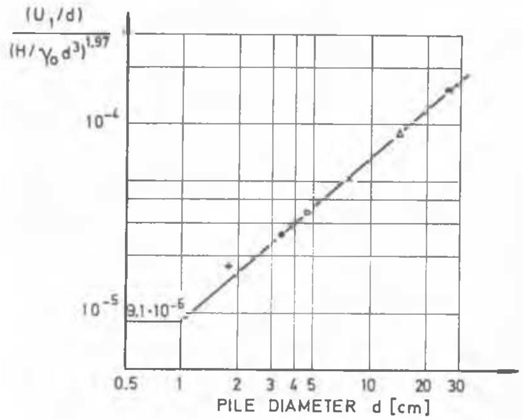

Fig. 9 Abscissa sections from fig. 8

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