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# Sand liquefaction under rotation of principal stress axes

## Liquéfaction des sables avec la rotation des axes de contrainte principaux

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**SYNOPSIS** Using a hollow cylindrical torsion shear test apparatus, three types of cyclic loading tests were performed on samples of the Japanese standard sand prepared with different densities. The types of tests were the cyclic triaxial shear test, the cyclic torsion shear test and the test in which the triaxial mode and torsional mode of shear stress were cyclically applied with a phase difference of  $90^\circ$  whereby producing a continuous rotation of principal stress direction in the test sample. Comparison of the results of these three types of tests disclosed the fact that the resistance of sand to cyclic loading is significantly reduced if the rotation of principal stress direction is involved in the cyclic application of shear stresses.

### INTRODUCTION

The laboratory tests on sand liquefaction simulating cyclic loading conditions during earthquakes have been performed using the triaxial shear, torsional shear and simple shear test apparatus. The stress conditions in these tests are characterized by a sudden change in the principal stress direction which takes place when the direction of shear stress application is reversed. However, there are several other cases in the geotechnical engineering in which the mode of cyclic loading is different from that induced by ground motions during earthquakes. Typical of these cases are the cyclic stresses induced in the seabed deposit during the propagation of ocean waves. As pointed out by Ishihara (1983) and Ishihara and Towhata (1983), the mode of cyclic stress application due to ocean waves is characterized by the continuous rotation of principal stress direction. The resistance of sand to liquefaction under conditions simulating such a mode of cyclic stress alteration was investigated by Ishihara and Towhata (1983) using a triaxial torsion shear test apparatus. The cyclic behavior of sand undergoing the continuous rotation of principal stress direction was also studied by Symes et al. (1984) by means of a hollow cylindrical torsion shear test device. However, these studies have not been comprehensive enough to provide test data on sand with different densities that can directly be used for practical purposes. The objectives of the present study is to fill this gap and to offer a more complete set of test data on the sand response.

cylindrical test specimen, 10 cm in outer diameter, 6 cm in inner diameter and 10.4 cm in length, is encased in rubber membranes and placed in the triaxial chamber. In the present test scheme, the chamber pressures both inside and outside were held equal at a constant level during the cyclic loading test. Therefore, the two components of stress, i.e., vertical stress and torsional stress were varied to produce various types of cyclic excursions of shear stress in the test specimen. Cyclic loads in vertical and torsional modes were applied through belofram cylinders operated by air pressure. Pore water pressures were measured by an electric transducer installed at the base of the triaxial chamber.

The sand used in the present test is the Japanese standard sand called Toyoura sand. The mean particle size,  $D_{50}$ , and uniformity coefficient,  $U_c$ , are 0.17 mm and 2.0, respectively. The maximum and minimum void ratios of this sand are 0.98 and 0.60, respectively, and the specific gravity is 2.65. The specimen was prepared by the method of air pluviation in which air-dried sand was rained into the mold from different height. The heights of fall of about 12 cm, 35 cm and 180 cm were required to prepare specimens with relative densities of about 45 %, 77 %, and 92 %, respectively. After forming the specimen, it was saturated under a back pressure of  $98 \text{ kN/m}^2$  with the aid of  $\text{CO}_2$  gas. The B-value exceeding 0.95 was obtained with this procedure. The sinusoidal cyclic stresses were applied undrained with a period of 12 sec.

### TEST APPARATUS AND MATERIAL

The test apparatus used was a triaxial torsion shear apparatus which permits torsional shear stress to be applied to a specimen independently of vertical and horizontal stresses. A hollow

### LOADING SCHEME

If stresses are assumed to be uniformly distributed in the radial direction across the wall of the hollow cylindrical sample, it can be proved that the stress in the circumferential direction,  $\sigma_h$ , is equal to the cell pressure when equal

pressures are applied both in the inner and outer chambers. When the vertical stress,  $\sigma_v$ , and the torsional stress,  $\tau_{vh}$ , are varied cyclically while keeping the chamber pressure, hence, the horizontal circumferential stress,  $\sigma_h$ , at a constant level, the principal stresses,  $\sigma_1$ , and  $\sigma_3$ , in the plane of the cylindrical wall are given by

$$\left(\frac{\sigma_1}{\sigma_3}\right) = \frac{\sigma_v + \sigma_h}{2} \pm \sqrt{\left(\frac{\sigma_v - \sigma_h}{2}\right)^2 + \tau_{vh}^2} \quad (1)$$

The angle between the major principal stress and the vertical direction,  $\beta$ , is given by

$$\tan 2\beta = \frac{2\tau_{vh}}{\sigma_v - \sigma_h} \quad (2)$$

and the deviator stress is given by

$$\frac{\sigma_1 - \sigma_3}{2} = \sqrt{\left(\frac{\sigma_v - \sigma_h}{2}\right)^2 + \tau_{vh}^2} \quad (3)$$

Corresponding to the stress analysis as above, the principal strain components,  $\epsilon_1$  and  $\epsilon_3$ , can be defined as

$$\left(\frac{\epsilon_1}{\epsilon_3}\right) = \frac{\epsilon_v + \epsilon_h}{2} \pm \sqrt{\left(\frac{\epsilon_v - \epsilon_h}{2}\right)^2 + \left(\frac{\gamma_{vh}}{2}\right)^2} \quad (4)$$

where  $\epsilon_v$  and  $\epsilon_h$  denote vertical and horizontal strains, respectively, and  $\gamma_{vh}$  is the shear strain in the torsional mode of deformation.

Three types of cyclic loading tests were conducted by changing the vertical stress and torsional stress either singly or in combination. The loading scheme of these three types of tests is illustrated in Fig. 1 in which the shear stress and the stress difference are represented in a rectangular coordinate system.

In the cyclic triaxial shear test, only the vertical stress was varied cyclically whereby producing cyclic change in the stress difference,  $(\sigma_v - \sigma_h)/2$ , with no variation in the torsional shear,  $\tau_{vh}$ . This type of test is identical to the conventional cyclic triaxial shear test. In the case of the cyclic torsion shear test, only the torsional stress,  $\gamma_{vh}$ , was applied cyclically to the specimen. This type of test is identical to the conventional type of cyclic torsion shear test. In the third type of test illustrated in Fig. 1(c), both the torsional stress and vertical stress were varied so that the deviator stress defined by Eq. (3) is maintained at a constant value throughout the

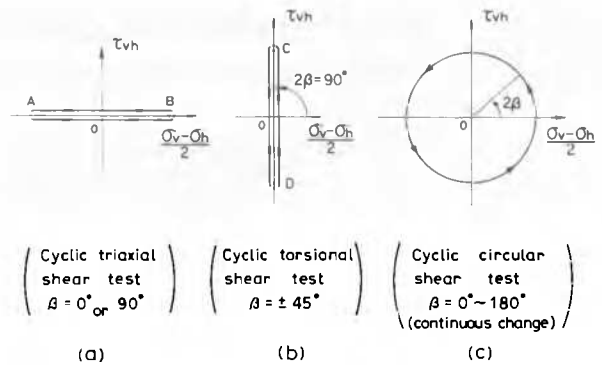


Fig. 1 Loading Schemes

cyclic loading test. In other words, the torsional stress and vertical stress were applied alternately so that a continuous rotation of principal stress direction can be executed in the specimen. This type of test will be referred to as the test with circular stress path.

TEST RESULTS

One of the direct records obtained in the test of circular stress path is presented in Fig. 2 in which several key variables monitored are displayed versus time. The test was performed on a loose sample with a relative density of 48 % by employing a cyclic stress ratio with an amplitude of  $(\sigma_1 - \sigma_3)/2\sigma_0 = 0.126$ , where  $\sigma_0'$  is the initial confining stress. It may be seen in Fig. 2 that the torsional stress,  $\tau_{vh}$ , and

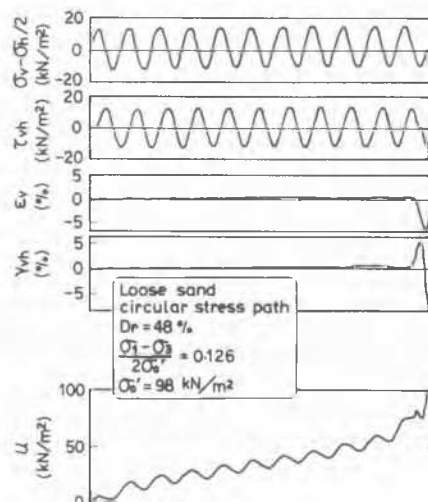


Fig. 2 Test Results with Circular Stress Path

the stress difference,  $(\sigma_v - \sigma_h)/2$ , with identical amplitude were applied cyclically with a phase difference of  $90^\circ$ . Like the conventional

cyclic triaxial test, the pore water pressure is seen building up steadily until it eventually became equal to the initial confining pressure, whereupon unlimitedly large strains developed both in the vertical and circumferential directions. Similar cyclic loading tests were carried out on loose samples with varying amplitudes and the results of this test series are presented in Fig. 3, where the cyclic stress

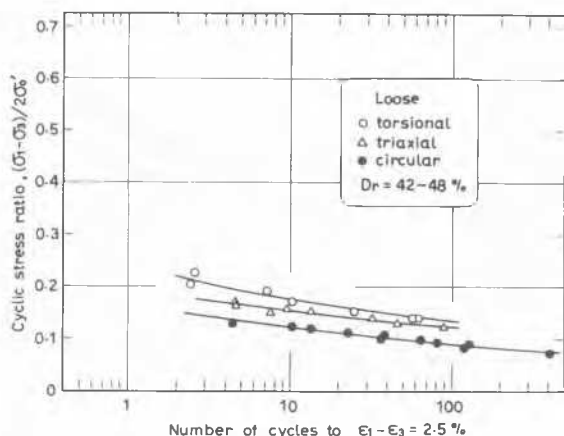


Fig. 3 Cyclic Strength of Loose Sand

ratio,  $(\sigma_1 - \sigma_3) / 2\sigma_0'$  causing 2.5 % deviator strain is plotted versus the number of cycles. Also plotted in Fig. 3 are the results of the cyclic triaxial shear test and the cyclic torsion shear test conducted on identically prepared specimens. It may be seen in Fig. 3 that the resistance of sand to cyclic stress application is reduced appreciably if the rotation of the principal stress direction is involved in the execution of cyclic loading.

Similar sets of cyclic tests were carried out on dense samples prepared with relative densities between 75 and 81 %. The results of the tests are presented in Fig. 4 where it may as well be

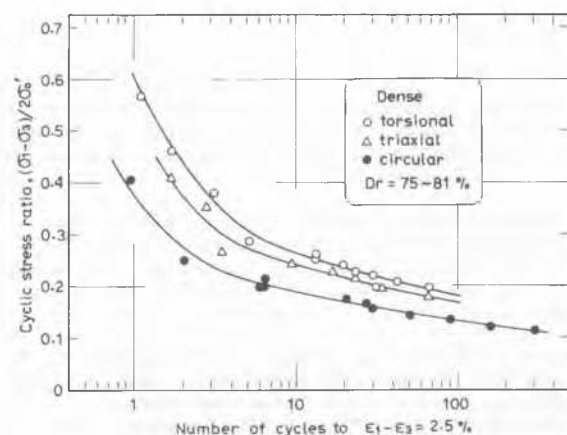


Fig. 4 Cyclic Strength of Dense Sand

seen that the cyclic stress ratio causing 2.5 % single-amplitude deviator strain in the test specimen is significantly lower for the tests involving the rotation of principal stress direction than for the conventional cyclic triaxial and torsion shear tests without the continuous rotation of the principal stress axis.

In the similar fashion, very dense samples of sand with relative densities of 89 to 95 % were tested by employing the three types of stress paths shown in Fig. 1. The results of these tests shown in Fig. 5 indicate also that the continuous rotation of the principal stress tends to decrease the resistance of sand to cyclic stress application.

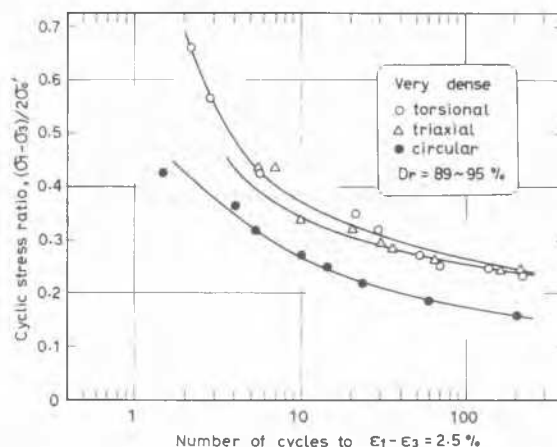


Fig. 5 Cyclic Strength of Very Dense Sand

To examine the effects of density, the cyclic stress ratio causing 1 %, 2.5 % and 4 % single-amplitude deviator strain in 5, 20 and 100 load cycles was read off from the data arrangements such as those shown in Figs. 3 to 5 and plotted versus the relative density of the test specimen. Such plots are presented in Fig. 6 where it may be observed that the cyclic stress ratio required to cause any level of deviator strain is always smaller for the cyclic loading condition involving the rotation of principal stress direction as compared to other two without such rotation. Fig. 6 also indicates that, as the required level of deviator strain increases, the cyclic stress ratio needed to attain that strain level increases sharply, when the sand is dense to very dense and when the number of cycles in question is small. Such characteristic behavior of sand resulting from cyclic mobility has been known to exist in the sample tested in the conventional cyclic triaxial and cyclic torsional loading conditions. It is apparent in Fig. 6 that the same cyclic mobility effect can be manifested as well in the cyclic loading involving the rotation of the principal stress direction.

#### CONCLUSIONS

Using a triaxial torsion shear test apparatus, three types of cyclic loading tests were conducted, that is, the cyclic triaxial shear test,

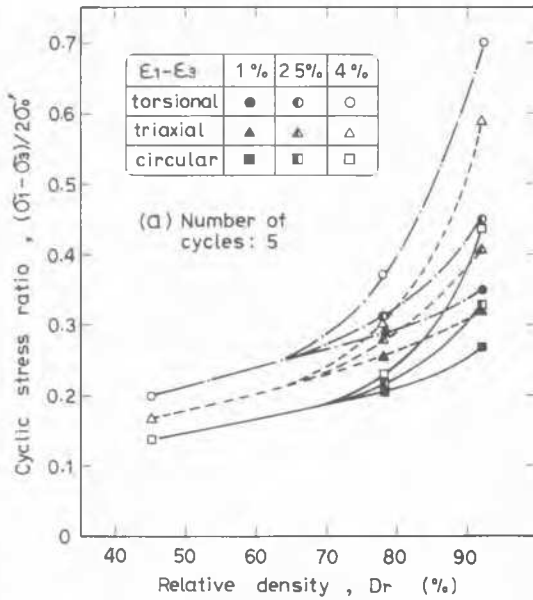


Fig. 6(a) Cyclic Strength at 5 Cycles

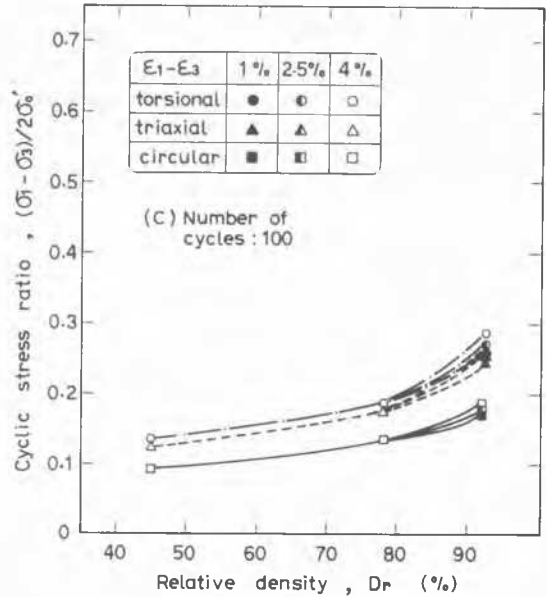


Fig. 6(c) Cyclic Strength at 100 Cycles

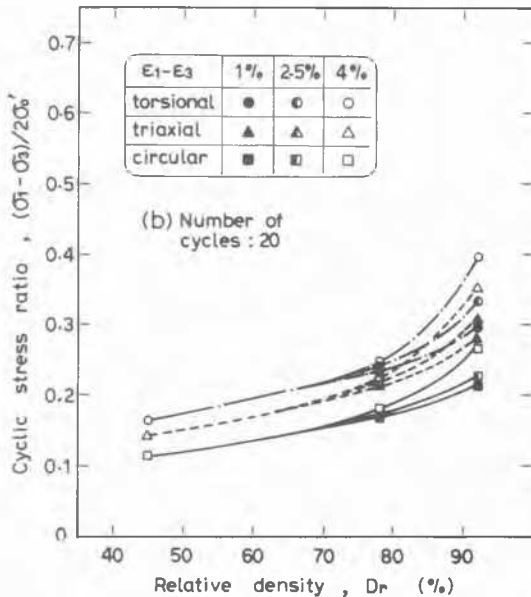


Fig. 6(b) Cyclic Strength at 20 Cycles

the cyclic torsion shear test, and the test in which the triaxial mode and the torsional mode of shear stress were applied cyclically with a phase difference of 90°. The latter type of test creates a continuous rotation of principal stress direction in the hollow cylindrical sample used in the present test. The results of the tests indicated that the cyclic stress ratio required to cause a certain level of strain is

significantly reduced for all densities of the samples tested, if the rotation of principal stress direction is involved in the cyclic loading. It was also shown that, for all three types of cyclic loading tests, the cyclic stress ratio needed to cause a certain magnitude of strain increases sharply with increasing level of required strain, particularly when the sand is dense and when the number of cycles in question is small.

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