Seepage losses from small irrigation reservoirs

Pertes par infiltration pour les réservoirs de petite capacité

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SYNOPSIS

A large number of small water reservoirs have been, and are currently being built in Israel. One of the most difficult aspects of the design of such reservoirs is the estimation of the expected water loss due to seepage. In fact this consideration is frequently the decisive factor with respect to the feasibility and profitability of a proposed reservoir.

Frequently, these reservoirs have the following characteristics:
a) The permeability of the foundation soil is higher or equal to that of the embankment.
b) Water table is deep or non-existant.
c) The operational policy of the reservoir (i.e., the function h = h(t) where t is time and h denoting the height of water in the reservoir) is periodic with period of one year.

Features (a) and (b) imply that the water losses are mainly due to vertical seepage. Features (b) and (c) imply that steady state is never realized.

In the present paper a procedure for the evaluation of water losses is presented. The procedure is based on the approximation of the actual flow regime by the vertical movement of horizontal wetting front. The procedure can handle any operational policy h(t). Solutions for simple operational policies are presented in graphical form.

1. INTRODUCTION

One dimensional vertical flow from the bottom of a reservoir can be analyzed using the classical infiltration theory (Raudkivi and Callander, 1976). Such an approach suffers from two practical limitations: i) The specification of an infiltration problem requires experimental determination of two constitutive functions describing the dependence of permeability and suction on water content. The small scale of most irrigation reservoirs makes the determination of these functions for each project prohibitively expensive. ii) The infiltration equation requires numerical solution for each operation policy h(t). Such numerical solutions are cumbersome to apply on a regular basis.

There exist two observations suggesting that an alternative simplified procedure may be feasible: i) The solution of the infiltration equation is not too sensitive to the exact form of the constitutive functions (Neuman, Feddes and Bresler, 1974). ii) The resulting distribution (with depth) of water content obtained from the solution of the infiltration problem can frequently be approximated by a sharp wetting front (Ibrahim and Brutsaet, 1968).

It appears therefore that the process of water losses from the bottom of a reservoir can be approximatively modeled by studying the advance of a sharp wetting front separating an upper wet (saturated) zone from a lower unsaturated one. Similar approximation has been used by Bear (1972).

2. FORMULATION OF THE PROBLEM

Consider a simplified flow regime as shown in Fig. 1.

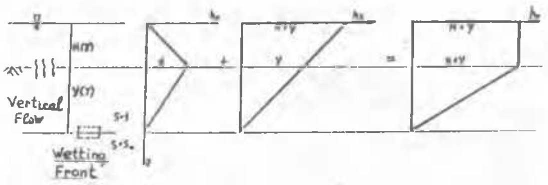

Figure 1 - Simplified Flow Regime.

On the basis of this figure it is possible to write:

$$q = k\,\frac{h + y}{y} \qquad (1)$$

where q is the specific discharge,
k - coefficient of permeability
h - height of water in the reservoir
(h = h(t) is the given operational policy of the reservoir) and y = y(t) is the depth of the wetting front at time t.

Neglecting volume change due to wetting and writing an equation for water balance at the wetting front, the following relation is obtained

$$q\,dt = n(S_f - S_o)\,dy \qquad (2)$$

where n is the porosity and S_f, S_o are degrees of saturation above and below the wetting front. Combining equations (1) and (2) yields

$$\frac{dy}{dt} = \frac{k}{n(S_f - S_o)}\left(1 + \frac{h}{y}\right) \qquad (3)$$

It is convenient to introduce the following non-dimensional variables

$$Y = y/h_m \qquad (4.1)$$

$$H = h/h_m \qquad (4.2)$$

$$T = kt/n(S_f - S_o)h_m \qquad (4.3)$$

where h_m is the maximum value of h, so that the function H = H(T) varies between zero and one. In terms of Y, H and T, eqn. (3) becomes:

$$dY/dT = 1 + H/Y \qquad (5)$$

Equation (5) is a non-linear ordinary differential equation which controls the motion of the wetting front. The term (1 + H/Y) is the hydraulic gradient i; hence the knowledge of its solution (the function Y = Y(T) makes it possible to calculate the rate of losses from the relation $q(T) = k(1 + H(t)/Y(T))$. Moreover, since all water lost from the reservoir are stored behind the wetting front the cumulative losses D(T) up to the time T are given by:

$$D(T) = n(S_f - S_o)h_m\,Y(T) \qquad (6)$$

The boundary conditions for the solution of eqn. (5) are H(T = 0) = Y(T = 0) = 0. Notice that T = 0 is a singular point of eqn. (5) since at that time the term H/Y is not well defined (it is of the form 0/0).

For convenience, it is possible to approximate any arbitrary operational policy as a combination of linear segments as shown in Fig. (2).

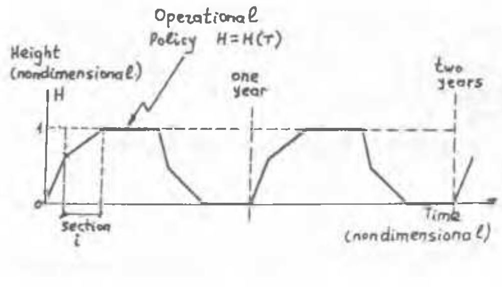

Figure 2 - Operational Policy

Hence, it is sufficient to solve eqn. (5) for the linear operational policy H(T) = aT + b using the boundary conditions $H(T = T_1) = H_1$ and $Y(T=T_1) = Y_1$. The end values of Y of the solution in one section is then taken as the initial value in the next section and the procedure is advanced section after section.

3. MATHEMATICAL ANALYSIS

Define a new variable $\eta(T)$ as

$$\eta(T) = \frac{H(T)}{Y(T)} > 0 \qquad (7)$$

In terms of η, eqn. (5) becomes

$$H\frac{d\eta}{dT} = -\eta\Phi(T) \qquad (8.1)$$

where

$$\Phi(T) = \eta^2 + \eta - a \qquad (8.2)$$

In solving eqn. (8.1) one has to distinguish between the case $\Phi = \Phi(T) \neq 0$ and the case $\Phi = 0 \neq f(T)$. The first case corresponds to a regular solution of eqn. (8.1) while the second one represents a singular solution of this equation.

<u>3a. The regular solution</u> - Here too, it is necessary to deal separately with the case a=0 (constant water level in the reservoir) and the case $a \neq 0$.

(i) The case $a \neq 0$:
In that case dT = dH/a and eqn. (8.1) becomes

$$\frac{d\eta}{\eta\Phi} = -\frac{1}{a}\frac{dH}{H} \qquad (9)$$

The solution of this equation is obtained using standard tables of integrals. This solution can be represented in the following form:

$$Y = \exp[(C - \ln|\Phi| + I(\eta))/2] \qquad (10.1)$$

with

$$I(\eta) = \begin{cases} \dfrac{U}{|U|}\dfrac{1}{\sqrt{4a+1}}\ln\dfrac{|U|}{\sqrt{4a+1}\,(2\eta+1)} & \text{for } a > -1/4 \\ -2/(2\eta+1) & \text{for } a = -1/4 \\ \dfrac{1}{\sqrt{4|a|-1}}\ \text{are tan}\ \dfrac{2\eta+1}{\sqrt{2|a|+1}} & \text{for } a < -1/4 \end{cases} \qquad (10.2)$$

$$U(\eta) = (2\eta+1) - \sqrt{aY} + 1 \qquad (10.3)$$

and C is an integration constant. Equations (10) are implicit representations of the solution since η and Φ depend on Y, but this does not cause any serious difficulties in application. The segmented form of eqn. (10.2) may cause the impression that a = -1/4 corresponds to some change in the character of flow. A careful examination shows, however, that the solution is continuous at a = - 1/4 only changing its formal mathematical representation. At T=0, H=Y=0 and η, Φ are not defined, hence it is not possible to use the information at T=0 (H=Y=0) in order to evaluate the integration constant C in eqn. (10.1). Consequently, this solution cannot be utilized in the vicinity of the singular point T=0.

(ii) The case a = 0:
In that case $H = b \neq f(T)$ and eqn. (8.1) can be written as

$$dT = -\frac{b\,d\eta}{\eta^2(1+\eta)} \qquad (11)$$

The solution of this equation is

$$T = Y + b\ln(1+\eta) + C \qquad (12)$$

with C an integration constant.

Obviously, this solution cannot be applied at T=0 when H(0) = 0 since in that case a=0 implies that the reservoir remains empty.

3b. The singular solution - The solutions presented above were based on the assumption that $\Phi \neq 0$. It was shown that these solutions do not apply in the vicinity of the point T=0. Since the boundary conditions of the problem are specified at that point, it is necessary to find a solution which is valid at T=0. Let us investigate if such a solution may be obtained on the basis of the assumption

$$\Phi = \eta^2 + \eta - 1 = 0 \quad (13)$$

Using eqn.(13) and H=aT in eqn.(8.1) one get $aT(d\eta/dT) = 0$. Hence, eqn.(13) implies that $\eta=\eta_o$=const. is a solution of the differential equation (8.1). The value of the constant η_o can be obtained from eqn.(13)

$$\eta_o = \frac{\sqrt{1+4a} - 1}{2} = \frac{H(T)}{Y(T)} \quad (14)$$

Since η_o is constant. eqn.(14) implies that a must also be a constant, or in other words, this singular solution is valid only for a linear variation of H with time. Substituting H(T)=aT into eqn.(14) and solving for Y(T) yields

$$Y(T) = \beta T \quad (15.1)$$

$$\beta = \frac{2a}{\sqrt{4a+1} - 1} \quad (15.2)$$

Eqn. (15) shows that the wetting front advances at a constant rate. Notice that in this solution the hydraulic gradient $i = 1+H/Y = 1+a/\beta$ is well defined at the singular point T=0, H=0, Y=0.

Finally one has to establish the range of validity of the singular and regular solutions (eqns. (15) and (10). It was seen that the singular solution is valid only as long as a = const. For this constant value $\Phi_o = \Phi = 0$. When $\Phi = 0$ the regular solution (eqn.(10.1)) cannot be applied since ln(0)does not exists. It may be concluded therefore that the singular solution is valid from the time T=0 up to the first time when the function H(T) changes its slope. From that time on the regular solution applies.

4. RESULTS AND DISCUSSIONS

The above solution was implemented on a microcomputer and checked for number of conditions. In every case reasonable results have been obtained. Comparison with field observation is in progress at the present but no conclusive results have been obtained as yet.
An example of a solution is shown in Fig. 3. This solution is based on the following input data ($k = 10^{-6}$ cm/sec, $S_o = 0.8$, $S_f = 1.0$, n = 0.4 and h_m = 10 m) which is quite typical of conditions in Israel. For these conditions the non-dimensional time T for one year (one period) is 0.4. The solution was obtained for the periodic operational policy specified on the bottom of Fig. 3 and it is shown as the solid lines in the figure. For comparison purposes the solution by Bear (1972) which corresponds to the operational policy H=1 for T≥0 is shown as a dashed line on the same figure.

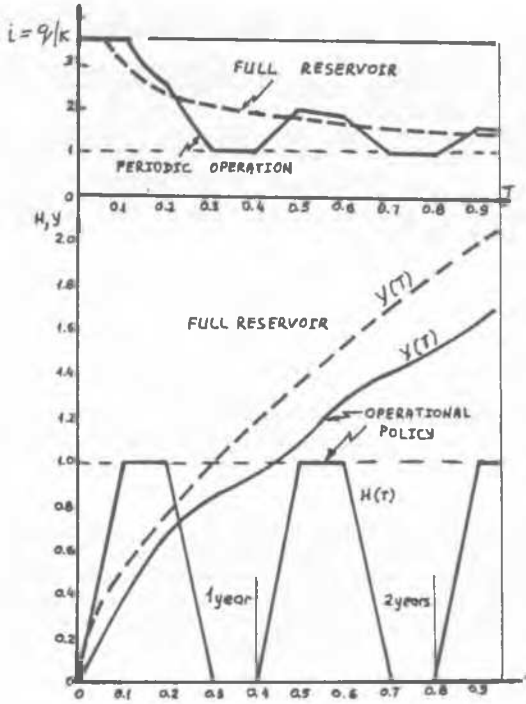

Figure 3 - Solution of an Example Problem

Following are number of observations (conclusions) which are based on the present analysis:

(a) On first filling the rate of advance of the wetting front (and hence also the rate of seepage losses) is constant (See Fig. 3). This result is fundamentally different from the one obtained by Bear (1972) which yields infinite losses at T=0.

(b) The rate of losses decreases with time and q approaches assymptotically the value of k. The decreases in the rate of seepage is rather slow and in the example shown in Fig. 3, q ≅ 1.5k after three years of operation.

(c) When the reservoir is empty (actually not empty but with minimum water level required to prevent drying and cracking) then i → 1 and q → k.

(d) If the wetting front meets a regional water table then the rate of seepage losses no longer decreases. It can be assumed that horizontal gradients in the regional aquifer adjust themselves to the inflow from the reservoir and a steady state is achieved. This observation shows clearly that the presence of high water table in the vicinity of a reservoir does not improve the situation from the stand point of seepage oss as is sometimes claimed.

REFERENCES

Bear, J. (1972). Dynamic of fluids in porous media. Elsevier Publ., Comp.

Ibrahim, H.A. and Brutsaet, W. (1968). Intermittent infiltration into soils with hysteresis. Jour. of the Hydraulic Div., ASCE, Vol. 94, No. HY1, pp. 113-137.

Neuman, S.P., Feddes, R.A. and Bresler, E. (1974). Finite element simulation of flow in saturated-unsaturated soils. Research Report No. ALD-SWC-77, Technion I.I.T., Haifa. Israel.

Raudkivi, A.J. and Callander, R.A. (1976). Analysis of ground water flow. John Wiley and Sons, N.Y.