# Dynamic properties and behavior of copper tailings

## Propriétés dynamiques et fonctionnement de déchets de cuivre

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SYNOPSIS The shear modulus and damping ratio of three samples of dry and saturated copper tailings with different gradation characteristics were determined with the use of a Hardin Resonant Column Apparatus. The general dynamic behavior of tailings with respect to content of fines, initial void ratio, confining pressure, and percent shear strain is discussed. The results are compared with averages published for sand, and average curves and useful relationships are presented for tailings.

## INTRODUCTION

During the mining and refining of copper, huge quantities of waste in the form of copper tailings are generated as a by-product. As a consequence, the safe storing of copper tailings is an item of major concern. Today, one of the most widely used processes, called cycloning, involves the separation of a significant portion of the fine grained material from the coarse, and the use of the latter for the construction of a dam to retain the former. In order to be able to analyse and design such impoundments safely in areas where seismic activity is common, one must make use of the dynamic properties of tailings. However, the dynamic characteristics of these materials are not well understood. The work covered in this paper deals with the evaluation of two dynamic parameters, shear modulus and damping ratio, under low shear strain levels using the Hardin Resonant Column device, and general findings of the dynamic behavior of copper tailings.

In the past several years, significant advances in the analysis of problems involving the dynamic loading of soils have been made and the study of soil dynamics has attained remarkable growth and importance. Some investigators such as Whitman and Richart (1967) have addressed problems related to foundation vibrations whereas others such as Seed (1967) have concentrated on the effects of earthquakes and the elements of protective construction. However, as Richart, Hall and Woods (1970) have pointed out, "the process of obtaining representative values for the critical soil properties is probably the most difficult part of the design study," and this is still true today. In the design of earthen impoundments subject to dynamic forces under symmetrical cyclic loading conditions but involving no residual soil displacements, the shear modulus and the damping ratio are the most important soil properties. The shear modulus, G, is the slope of the line joining the extremities of the cyclic stress-strain loop in Figure 1, and $G_{max}$ is defined as the shear modulus at zero percent shear strain or for all practical purposes at very low percent strain amplitudes ($\gamma \approx 3x10^{-4}\%$). The damping ratio, D, is the area of the above loop divided by 4π times the triangular area shown in Figure 1.

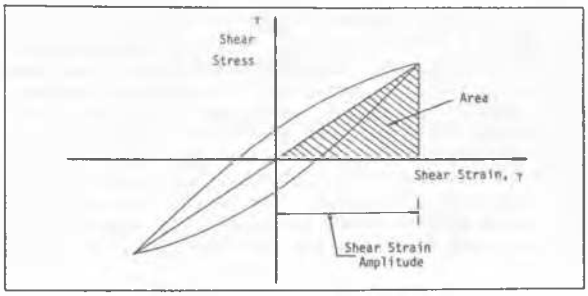


Figure 1. Definition of Shear Modulus and Damping Ratio.

Efforts to experimentally evaluate the dynamic properties of soil using laboratory equipment and establish empirical relationships date back to the work of Casagrande and Shannon (1948) on the effect of bomb explosions on the stability of the Panama Canal. Since that time, various dynamic testing devices such as the cyclic triaxial apparatus, oscillatory simple shear test apparatus, M.I.T. apparatus for rapidly loaded triaxial tests, and resonant column apparatus, have been used to determine the dynamic characteristics of soil. In addition, investigators such as Hardin and Black (1968), Hardin and Drnevich (1970), Roesler (1979), and Yu and Richart (1984) have presented useful empirical relationships. Hardin and Black have expressed the shear modulus at zero shear strain as a function of void ratio and mean principal effective stress, whereas Roesler has incorporated the individual components of the confining pressures, and Yu and Richart have accounted for the effects of stress ratio. The following general form of these expressions as discussed by Seed and Idriss (1970) may also be used for the shear modulus:

$$G = 1000\ K_2\ (\bar{\sigma}_o)^{0.5} \qquad (1)$$

in which $\bar{\sigma}_o$ is the mean principal effective stress and $K_2$ is a parameter which accounts for the influence of void ratio and shear strain amplitude. On the other hand, Hardin and Drnevich defined a hyperbolic strain function, $\gamma_H$, as follows:

$$\gamma_H = (\gamma/\gamma_r)\left[1 + a\ \exp\{-b(\gamma/\gamma_r)\}\right] \qquad (2)$$

in which

$$\gamma_r(\%) = (\tau_{max}/G_{max})100 \qquad (3)$$

and

$$\tau_{max} = \{(\frac{1+K_o}{2}\,\bar{\sigma}_v \sin\bar{\phi} + \bar{c}\cos\bar{\phi})^2 - (\frac{1-K_o}{2}\,\bar{\sigma}_v)^2\}^{0.5} \qquad (4)$$

where $K_o$ = coefficient of earth pressure at rest.

$\bar{\sigma}_v$ = effective vertical stress.

$\bar{c}$, $\bar{\phi}$ = static effective strength parameters.

and were able to derive the following unique expressions for both sand and clay:

$$G/G_{max} = 1/(1+\gamma_H) \qquad (5)$$

$$D/D_{max} = \gamma_H/(1+\gamma_H) \qquad (6)$$

## LABORATORY TESTING PROGRAM

During the testing program, three different samples were tested using a "free-fixed end" Hardin Resonant Column Apparatus. One sample, designated tailings-A, was made of copper tailings as they came from the mine, another sample, labeled tailings-C, comprised of tailings that had undergone cycloning, and the third sample, called tailings-W, was manufactured in the laboratory by washing away all of the fines (particles smaller than 0.074 mm) from a sample of tailings-C. The general characteristics of the three samples which geotechnically range from clean fine sand to fine silty sand are given in Figure 2.

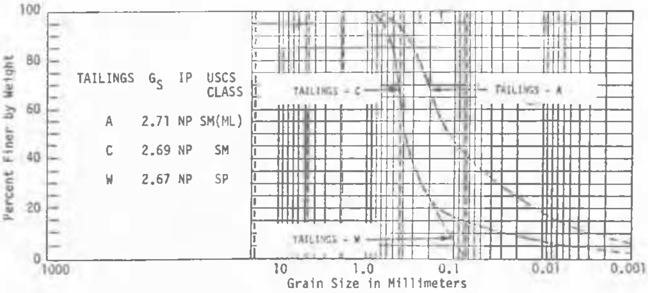


Figure 2. Gradation and Basic Physical Properties of Tailings.

In total, 51 dynamic tests were performed on samples of the material at void ratios representing approximately 50%, 70% and 90% relative densities. Twenty seven tests were performed on air dried samples using effective confining pressures of 147.11, 294.21 and 490.35 kPa, and 24 tests were conducted on saturated samples using effective confining pressures of 147.11, 294.21 and 441.32 kPa. After the dynamic tests were completed, triaxial compression tests were performed on the samples. In the former tests, the number of cycles ranged from 5,000 to over 100,000 cycles and the shear strain ranged from approximately $5 \times 10^{-4}$ to $3 \times 10^{-2}$%, whereas in the latter, the maximum deviator stress occurred on the average at about 8% axial strain. In the case of the dynamic tests, the percent shear strain levels were so low that no appreciable volume change or development of excess pore pressure took place.

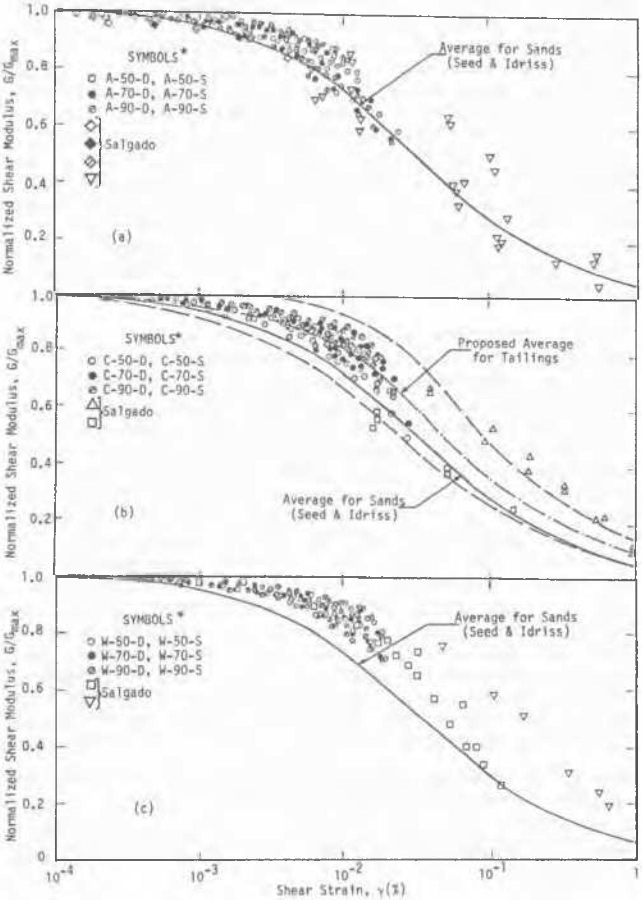
## LABORATORY TESTS RESULTS

The results given below represent data obtained by Rojas-González (1982) and Ben-Khayal (1983), as well as that compiled by Salgado (1983). Where the results for dry and saturated samples are close, they are represented by the same symbol in the figures. In order to generate comparisons with published results, parameters and forms of equations presented by previous investigators have been used where possible. Since the range of the number of cycles used did not appreciably influence the tests' results, it was not considered as a factor in the laboratory testing program.

### Shear Modulus

The variation of the normalized shear modulus, $G/G_{max}$, with the average percent shear strain is shown in Figure 3.

As expected, $G/G_{max}$ for the copper tailings cited decreases in a non-linear fashion as percent shear strain increases and exhibits the general pattern of behavior presented by Seed and Idriss (1970). However, most of the data plots above the graph which represents the average for sands. As shown in Figure 3, the results for tailings-A yield the best agreement with the average for sands, whereas those for tailings-W give the worst.



* A-50-D = Tailings A, 50% relative density, dry.

Figure 3. Variation of Normalized Shear Modulus with Percent Shear Strain.

In other words, the data presented tend to approach the average given for sands as the percentage of silty fines increases. Also, the saturated samples gave values which were generally lower than those for the dry samples. At this time, no explanation has been found for the above behavior of tailings with silty fines but it is believed to be a function of gradation, grain characteristics and structure of the soil. In any case, the proposed average curve presented in Figure 3(b) appears to be a better representation for copper tailings than the average curve given for sands.

Again as expected, $G_{max}$ increases with increasing effective confining pressure and decreasing void ratio. However, if the data is presented as shown in Figure 4, in a form similar to that discussed by Seed and Idriss (1970), the average values for $\bar{K}$ (which is really a function of $\bar{\sigma}_o$) are shown to be for the most part below those for clean sands. Further comparison between the data presented and that for clean sands indicates fair agreement for clean tailings but generally poor agreement for tailings with silty fines. Based on the data presented, the proposed curve given in Figure 4 is thought to be more representative for cycloned tailings.

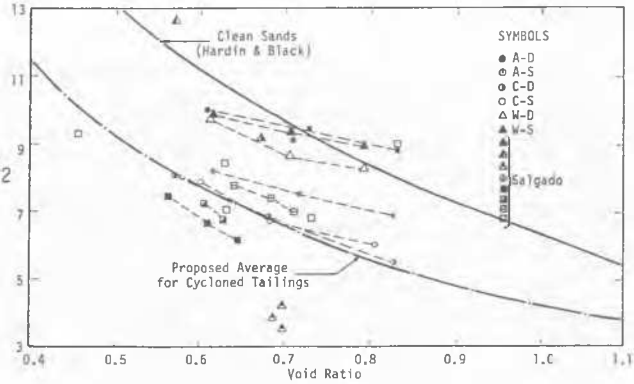


Figure 4. Variation of $K_2$ with Void Ratio

On the other hand, if the data is replotted according to Hardin and Drnevich (1970), values for parameters a and b may be obtained for dry and saturated samples, and their relation with void ratio and mean effective principal stress determined as shown in Table I.

TABLE I. Expressions for Parameters Related to $G/G_{max}$

| Samples | a | b |
|---|---|---|
| A-DRY | $0.7+(3.8e-1.2)\exp(-3.7/\bar{\sigma}_o)$ | $12.4\exp(-0.8/\bar{\sigma}_o)$ |
| A-SAT | $0.5+2.9\exp(-0.6/\bar{\sigma}_o)$ | $13.5\exp(-2.3/\bar{\sigma}_o)$ |
| C-DRY | $(8.7e-4.5)\exp(-1/\bar{\sigma}_o)$ | $11.5\exp(-1.5/\bar{\sigma}_o)$ |
| C-SAT | $0.5+(7e-3)\exp(-0.75/\bar{\sigma}_o)$ | $14.9\exp(-3/\bar{\sigma}_o)$ |
| W-DRY | $-0.5+(10.5e-2.6)\exp(-5/\bar{\sigma}_o)$ | $0.14+67\exp(-4.5/\bar{\sigma}_o)$ |
| W-SAT | $0.5+(11.8-13.2e)\exp(-4/\bar{\sigma}_o)$ | $0.14+16\exp(-4/\bar{\sigma}_o)$ |

$\bar{\sigma}_o$ in kg/cm$^2$

Figure 5 indicates that the functional relationships developed for a and b yield results that are in good agreement with the hyperbolic curve presented by Hardin and Drnevich. This means that if the maximum shear stress can be obtained from Equation 4, the reference shear strain may be found from Equation 3 using a value of $G_{max}$ from Figure 4, the hyperbolic strain may be obtained from Equation 2, and the shear modulus determined from Equation 5. Figure 6 furnishes the relationship between $\bar{\Phi}$ and e for the tailings samples considered, and thus affords the evaluation of the maximum shear stress and ultimately the determination of G once the void ratio is known.

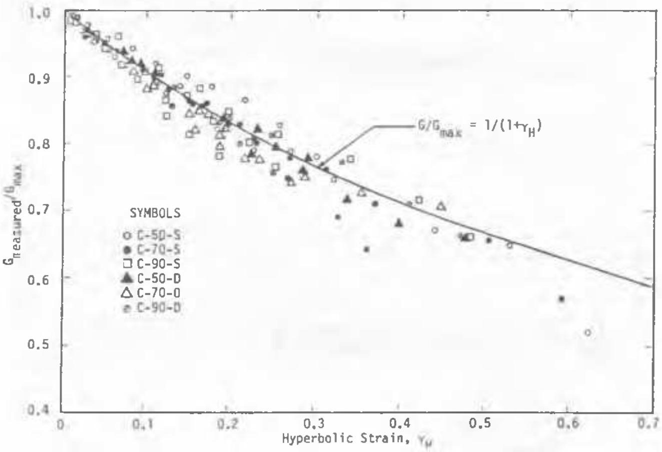


Figure 5. $G_{measured}/G_{max}$ vs. $G_{calculated}/G_{max}$.

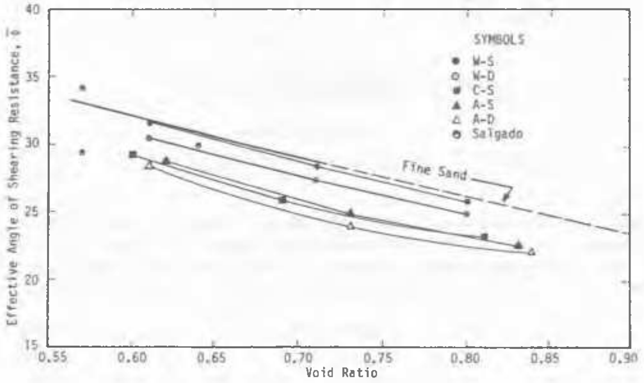


Figure 6. Effective Angle of Shearing Resistance vs. Void Ratio.

Damping Ratio

With regard to the damping ratio, the data also follows the general pattern of behavior previously established for natural soils. As shown in Figure 7, the damping ratio, D, increases as the average percent shear strain increases, but most of the data presented falls below the curve representing the average for sands and this is especially true at shear strains greater than approximately 0.001%. For all values of percent shear strain considered, the saturated samples gave slightly higher values of D. At high percent shear strain, the damping ratio is extrapolated to be in the relatively low range of approximately 18%. As with the shear modulus, the agreement with the average for sands is fairly good for tailings-A but relatively poor for tailings-C and W, and it is believed that the proposed curve in Figure 7(b) is a better representation for copper tailings than the average curve presented for sands.

As was the case for the normalized shear modulus, expressions given in Table II were developed for parameters a and b, and values of $D/D_{max}$ were plotted versus hyper-

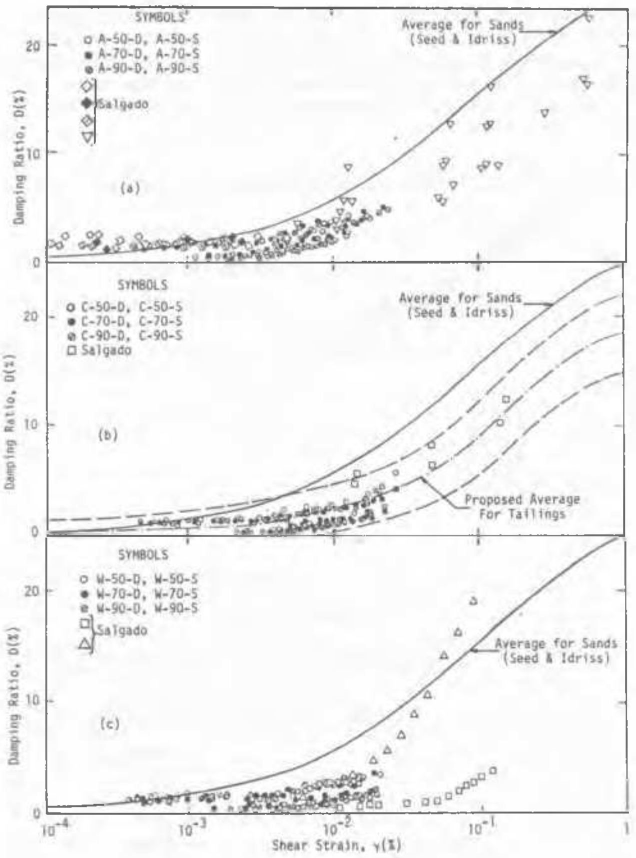


Figure 7. Damping Ratio vs. Shear Strain.

bolic strain for all three samples. However, only the results for cycloned tailings are shown in Figure 8. Once again, the data obtained is shown to fit the hyperbolic relationship presented by Hardin and Drnevich.

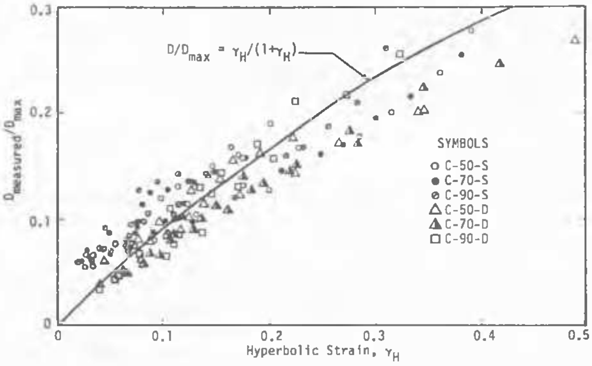


Figure 8. $D_{measured}/D_{max}$ vs. $D_{calculated}/D_{max}$.

TABLE II. Expressions for Parameters Related to $D/D_{max}$.

| Samples | a | b |
|---|---|---|
| A-DRY | $0.5+0.2\bar{\sigma}_o-0.4e$ | $9+6\bar{\sigma}_o-18e$ |
| A-SAT | $2+0.5\bar{\sigma}_o-0.4e$ | $18.5+8.7\bar{\sigma}_o-28e$ |
| C-DRY | $2.4+0.2\bar{\sigma}_o-2.7e$ | $32+2.6\bar{\sigma}_o-49e$ |
| C-SAT | $6+0.4\bar{\sigma}_o-3.8e$ | $12+11\bar{\sigma}_o-25.8e$ |
| W-DRY | $6.2+0.5\bar{\sigma}_o-8e$ | $81.5+4.3\bar{\sigma}_o-110e$ |
| W-SAT | $9+0.7\bar{\sigma}_o-7.7e$ | $10+14.9\bar{\sigma}_o-27e$ |

$\bar{\sigma}_o$ = kg/cm$^2$

## CONCLUSIONS

Based on the data presented, the following conclusions may be drawn regarding the shear modulus and damping ratio of copper tailings:

(1) Copper tailings follow the general pattern of behavior previously established for natural soils such as sand. However, when compared to sand, the tailings exhibit (on the average) lower $G_{max}$, higher normalized shear moduli and lower damping ratios for the same percent shear strain.

(2) Clean tailings do not always yield the best agreement with clean sands and the average for sands. Thus, it appears that gradation and possibly grain characteristics and soil structure do influence the shear modulus and damping ratio.

(3) There was not much difference in behavior between the dry and saturated samples. In general, the saturated samples gave lower values for the normalized shear modulus and higher values for the damping ratio.

(4) Curves and functional relationships have been presented for use in evaluating the shear modulus and damping ratio of tailings.

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