End bearing capacity of pile foundations by means of characteristics

Force portante à la base des pieux avec la méthode des caractéristiques

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SYNOPSIS Following an approximate approach suggested by Vesic (1963) and Berezantzev et al. (1963), the punching failure mechanism occurring below the base of a pile is simulated by classical plasticity theory through a reduced extent of the slip volume.
Values of the bearing capacity coefficient Nq are obtained by the method of characteristics in axial symmetry; the reduction of slip volume is obtained by considering a variation between 0 and $\pi/2$ of the angle ω formed by the limit surface and the horizontal plane through pile base.
In addition, the effect of base roughness, self weight and distribution and inclination of the overburden pressure is also analysed.
The results obtained are compared with previous findings, and some conclusions are drawn.

INTRODUCTION

As shown by Vesic (1964) the failure mechanism below the base of a pile cannot be but a punching failure, even in very dense soils. Accordingly, the end bearing capacity of piles should be evaluated by means of an elastic-plastic model, accounting for soil deformations before failure.
In order to avoid the difficulties involved by this model, two simplified approaches have been suggested.
The first one (Vesic, 1963; Berezantzev et al., 1963) makes use of the rigid-plastic model of classical plasticity, but reducing the extent of the slip volume. The second one (Skempton et al., 1953; Vesic, 1977) resorts to the theory of the expansion of a spherical cavity in an elastic-plastic medium.
In this paper the end bearing capacity of a pile is evaluated by means of characteristics following the first approach, and a reduction of the extent of the slip volume is obtained by considering a variation between 0 and $\pi/2$ of the angle ω formed by the limit surface and the horizontal plane through pile base (fig. 1).
In addition, the effect of: i) base roughness; ii) self weight and iii) distribution and inclination of the pressure q acting on the limit surface, is also analysed.
The results obtained are compared with previous findings and some conclusions are drawn.

$$\frac{\partial \sigma_r}{\partial r} + \frac{\partial \tau_{rz}}{\partial z} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \quad (1a)$$

$$\frac{\partial \tau_{zr}}{\partial r} + \frac{\partial \sigma_z}{\partial z} + \frac{\tau_{zr}}{r} = \gamma \quad (1b)$$

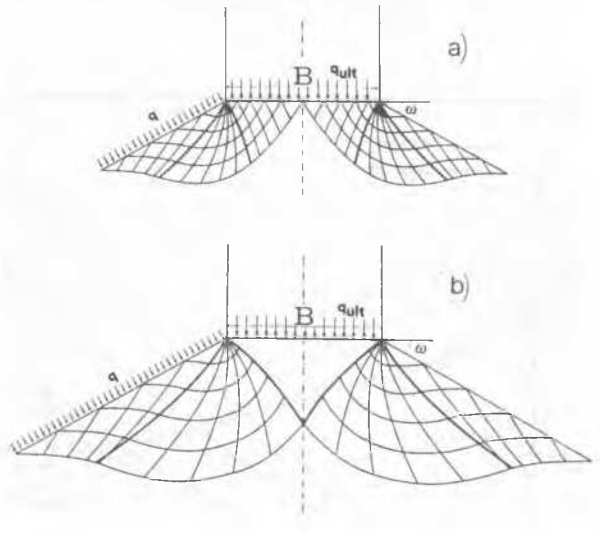

Fig. 1. Failure volume under pile point: a) smooth base; b) rough base.

GOVERNING EQUATIONS

The axially symmetric problem of a Mohr-Coulomb medium is governed by the equilibrium equations:

and the yield condition:

$$\sqrt{\left(\frac{\sigma_r - \sigma_z}{2}\right)^2 + \tau_{rz}^2} = \frac{\sigma_r + \sigma_z}{2}\sin\varphi + c\cos\varphi \quad (2)$$

Consideration of the Haar-Von Karman (1909) hypothesis:

$$\sigma_\vartheta = \sigma_3 \qquad (3)$$

makes the problem a statically determined one. It can be solved by the method of characteristics, since it is of the hyperbolic type (Cox et al., 1961).
As it is well known, the cohesion may be accounted by Caquot's corresponding states theorem, leading to the expression:

$$N_c = (N_q - 1)\cot\varphi \qquad (4)$$

Accordingly, only a cohesionless medium ($c = 0$; $\varphi \neq 0$; $\gamma \neq 0$) needs be considered.
The stress state at a point may be defined by means of the angle ϑ formed by the direction of the major principal stress σ_1 and the horizontal r-axis, and of the Sokolowskii (1965) stress parameter χ defined as.

$$\chi = \frac{\cot\varphi}{2} \ln\frac{p}{p_r} \qquad (5)$$

in which p is the mean stress and p_r a reference stress. The governing equations (1) to (3), rewritten in terms of χ and ϑ, with respect to the characteristic curves α and β:

$$\frac{dy}{dx} = \tan(-\vartheta \mp \varepsilon) \qquad (6)$$

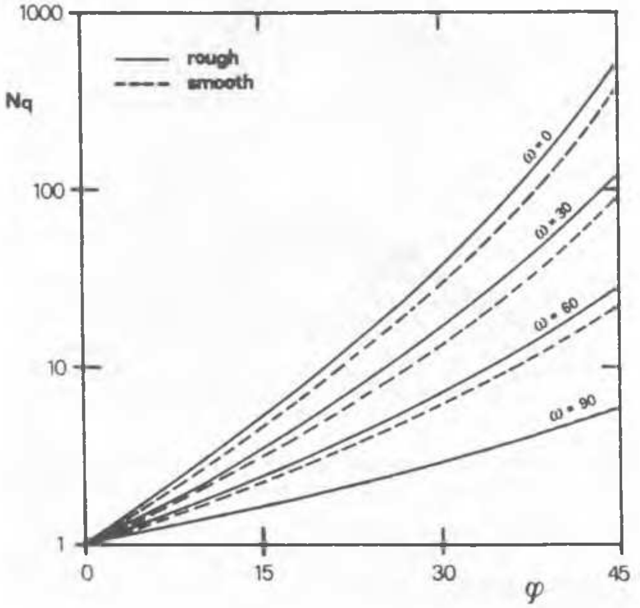

Fig. 2. Bearing capacity factor N_q for rough and smooth base.

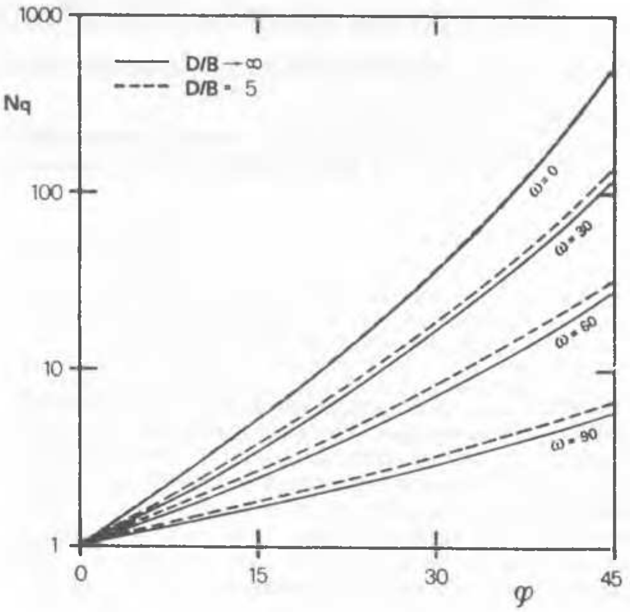

Fig. 3. Bearing capacity factor Nq for D/B→∞ and D/B = 5

become:

$$\frac{\partial\chi}{\partial s_\alpha} + \frac{\partial\vartheta}{\partial s_\alpha} = -\frac{\sin\varepsilon\cos\vartheta}{r} - \frac{\gamma\cos(-\vartheta+\varepsilon)}{2p_r\sin\varphi}\exp(-2\chi\tan\varphi) \qquad (7a)$$

$$\frac{\partial\chi}{\partial s_\beta} - \frac{\partial\vartheta}{\partial s_\beta} = -\frac{\sin\varepsilon\cos\vartheta}{r} + \frac{\gamma\cos(\vartheta+\varepsilon)}{2p_r\sin\varphi}\exp(-2\chi\tan\varphi) \qquad (7b)$$

where s_α and s_β are curvilinear abscissae along the characteristic curves α and β, and $\varepsilon = \pi/4 - \varphi/2$.

NUMERICAL SOLUTION

Following Cox et al. (1961), eqs.(6) and (7)have been solved numerically by finite differences.
A parametric study has been carried out, considering the influence of: i) the angle ω varying between 0 and $\pi/2$; ii) the base roughness; iii) the self weight and iv) the distribution and inclination of the pressure q acting on the limit surface.

In the case of smooth base (fig. 1a) all the three failure zones occur, namely the passive or Cauchy zone, the intermediate one or Prandtl fan, and the active or mixed one, and the failure mechanism is of the Hill type. For the rough base (fig. 1b) only the first two zones occur; a rigid wedge forms below the base and the failure mechanism is of the Prandtl type.

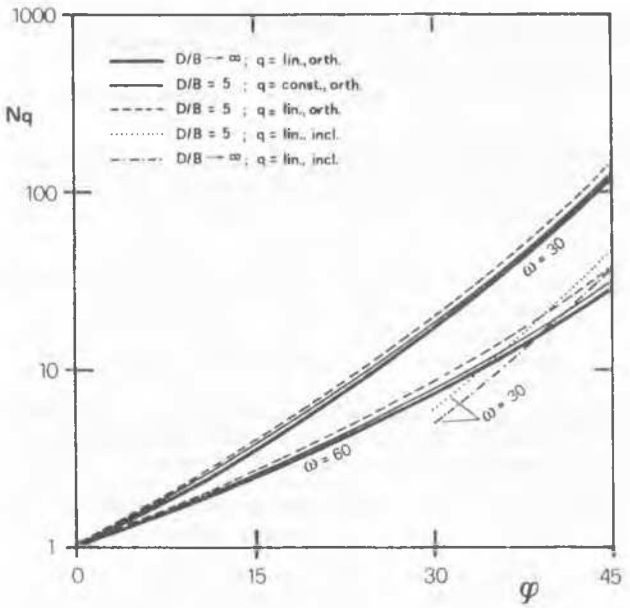

Fig. 4. Bearing capacity factor N_q for different overburden pressure conditions.

The bearing capacity factor Nq is defined by the relation:

$$q_{ult} = N_q \cdot q \tag{8}$$

where q_{ult} is the end bearing capacity of the pile.
Values of Nq for rough and smooth pile base are plotted in fig. 2 against φ, for $0 \leq \omega \leq \pi/2$.
The effect of the self weight, generally neglected in piles end bearing capacity calculations, is shown in fig. 3 (rough base) where Nq is plotted for $D/B \rightarrow \infty$ (weightless medium) and D/B = 5, being D and B respectively depth and diameter of the pile base.
Finally, the influence of different hypotheses about the inclination and distribution of q is depicted in fig. 4. The pressure acting on the limit surface has been considered either constant or linearly increasing with depth, and either orthogonal to the limit surface or vertical, and hence inclined to the normal of an angle ω . In the latter instance, of course, only the cases with $\varphi > \omega$ are possible.

ANALYSIS OF THE RESULTS

Figs. 2 and 3 shown that Nq decreases considerably as the angle ω increases from 0 to $\pi/2$.
This had to be expected, since the increase of stress from the value at the limit surface (q) to the value at the pile base (q_{ult}) depends on the rotation of the principal stress within the Prandtl fan, and hence on the opening of such fan.
The same effect occurs in plane strain, where the coefficient Nq has the expression:

$$N_q = \frac{1+\sin\varphi}{1-\sin\varphi}\exp(2\psi\tan\varphi) \tag{9}$$

the angle ψ being the opening of Prandtl fan.
Consideration of base roughness implies a significant increase of Nq (fig. 2), while it is well known that for the plane strain case it affects only N_γ . The influence of roughness increases with increasing φ and decreasing ω ; for $\omega = \pi/2$; the influence vanishes. It is believed that the case of smooth base has no practical interest; accordingly, further results are shown for the only case of rough base.
The effect of self weight (fig. 3) seems unimportant, thus substantiating the usual practice of disregarding it.
Finally, the effect of the distribution of the overburden pressure q (fig. 4) seems to be unimportant when q is normal to the limit surface, and significant when q is vertical, i.e. inclined of ω to the normal. The decrease of Nq with increasing inclination of q may be explained with the above considerations on the opening of Prandtl fan; the inclination of q, indeed, makes the angle ϑ on the limit surface increase, thus reducing the width of the Prandtl fan.

COMPARISON WITH PREVIOUS FINDINGS

In order to compare the present results with those obtained by other Authors, the values of Nq by Vesic (1963, 1977)and by Berezantzev et al. (1961, 1963) are plotted in fig. 5 together with

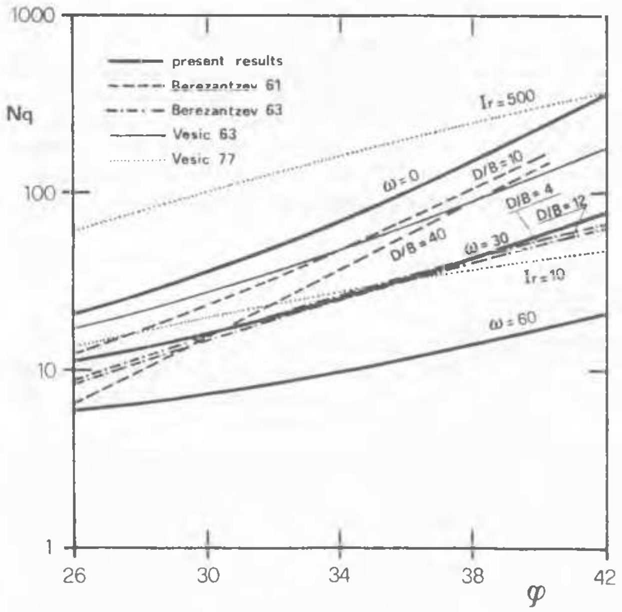

Fig. 5. Comparison between present Nq values and previous ones.

those calculated by the writers for rough base, q normal to the limit surface and $D/B \to \infty$ (self weight = 0).
It is to remember that Vesic (1963) and Berezantzev et al. (1963) adopt a reduced opening of Prandtl fan, equal respectively to 1.9φ and to $(\pi/4+\varphi)$. Berezantzev et al. (1961) assume $\omega = 0$, and Vesic (1977) makes use of the spherical cavity expansion theory. It is also to note that Vesic (1977) introduces a coefficient N_σ that is related to Nq by the expression:

$$N_\sigma = N_q \frac{3}{1+2K_0} \qquad (10)$$

To the writers' knowledge, the results by Berezantzev et al. (1961) are widely used in practice and believed to provide the best fit with experimental evidence for medium diameter piles (B=40÷60cm). The results by Berezantzev et al. (1963) were proposed for caissons and are sometimes adopted for large diameter bored piles, since they account for the reduction in end bearing capacity with increasing pile diameter.
Fig. 5 shows that the values of Nq obtained by the writers with $\omega = 30°$ fit rather well Berezantzev et al. (1963) ones. To fit Berezantzev et al. (1961) values, values of ω decreasing from 30° to 10° with increasing φ should be considered. This trend could be interpreted as a reduction of the local failure effect with the increase of φ.

CONCLUSIONS

The procedure proposed in the present paper is to account for the punching character of pile base failure through a reduced extent of the slip volume; this approach, first suggested by Vesic (1963), allows a unified treatment of both medium and large diameter piles.
The results obtained fit those by Berezantzev et al., provided suitable values are assigned to the angle ω. For large diameter bored piles a value $\omega = 30°$ seems to apply; for medium diameter piles a value of ω decreasing from 30° to 10° with increasing φ should be used.

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