Limit equilibrium analysis of reinforced soil walls

Analyse des murs en terre renforcée par équilibre limité

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SYNOPSIS Innovations are proposed for the limit equilibrium analysis of reinforced soil. The forces required for equilibrium in the soil are separated from the forces made available by reinforcement layers. A triangular distribution for required horizontal stress across potential slip mechanisms is suggested. This allows analysis where the mobilised soil shear strength is chosen and the required forces distributed to the reinforcement layers. Back analysis of instrumented reinforced soil walls indicate agreement with measured force distributions in metallic strip reinforcements, and overall face movements in a case with polymer reinforcement. A curve representing overall compatibility between soil and reinforcement strains indicates how design assumptions affect the expected maximum strains at equilibrium. One intriguing result is that the locus linking points of maximum reinforcement force depends on the reinforcement length, bond characteristics and the soil properties and appears not to be the consequence of a single worst potential failure mechanism as currently thought.

INTRODUCTION

Fresh studies on limit equilibrium analysis have been carried out stimulated by wider applications of reinforced soil, including the reinforcement of clay slopes with polymer materials. A method of analysis giving a consistent pattern for steep slopes was found, and a design procedure with charts has been proposed for slopes up to $\beta = 80°$, Jewell, Paine and Woods (1984). The method is reviewed here as it might apply to the vertical case, and may assist where polymer reinforcements and lower quality fills are contemplated. The attraction is that simple, conventional parameters describe the soil and reinforcement. The method offers guidance on the selection of reinforcement spacings and lengths, and provides a basis for choosing safety factors and for the estimation of deformations.

METHOD OF ANALYSIS

The analysis separates the forces required to provide the desired state of equilibrium in the soil and the set of forces made available by the inclusion of reinforcement layers.

The required forces are independent of the reinforcement and depend on the soil properties, pore water pressures, slope geometry and surcharge loading. The gross horizontal force required to maintain equilibrium on a potential slip mechanism may be calculated by applying forces external to the slope or wall face to provide equilibrium. Analysis of a full set of mechanisms in the soil assuming a fixed magnitude of mobilised soil shear strength defines two surfaces, Fig 1. The locus of zero required force defines the zone in the soil where reinforcement forces are required to maintain equilibrium. The most critical mechanism passing through the toe separates a zone at the face where uniformly high reinforcement forces are required, from a zone of decreasing required forces. The magnitude of required reinforcement forces and the position of the two surfaces in Fig.1 depend on the value of mobilised soil shear strength.

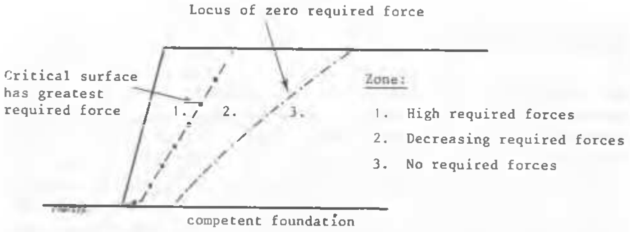


Fig.1. Zones of required force for equilibrium (Case 4:1 slope $\phi = 35°$)

The value of available force at a point along a reinforcement layer depends either on the mobilised bond stress or the reinforcement material properties, Fig. 2. Ultimately the limiting maximum force is governed by the reinforcement strength. For design, a factored long term strength or the force which would develop a limiting allowable strain would be selected for calculation.

Reinforcement distribution

Where there is sufficient bond, the force in extensible (polymer) reinforcement depends on the magnitude of extension which is directly controlled by strain in the adjacent soil. In contrast, the force in inextensible (metallic) reinforcement is governed by the soil stresses.

The spacing of reinforcement layers should ideally be chosen so that each can maintain the soil locally at or below the desired stress ratio. This concept, and

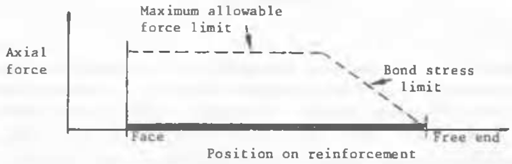


Fig.2. Envelope of maximum available force , for full load connection to face

empirical findings from limit equilibrium analyses, indicate that for slopes up to $\beta = 80°$ the gross horizontal force can be represented as a triangular distribution of required horizontal stress across a slip mechanism , Fig. 3, Jewell et al (1984). Thus equal strength reinforcement layers should ideally be spaced as an inverse function of depth so that each would be equally loaded, Fig. 3.

This spacing arrangement would be appropriate for both extensible and inextensible reinforcement, and should provide equal loading of the reinforcement layers and a balanced improvement through the soil.

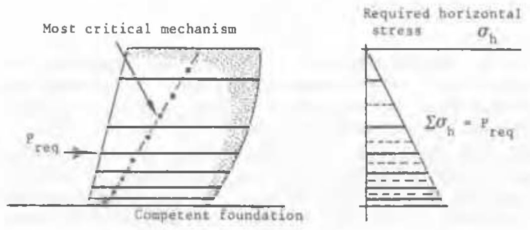


Fig.3. Triangular distribution of required force, and reinforcement spaced to carry equal loads (Case 4:1 slope $\phi = 35°$)

Slip mechanisms

As discussed by Jewell et al (1984) two-part wedge mechanisms with zero interslice roughness are considered both convenient and suitable for the analysis. They have the merit of simplicity, do not require complex assumptions about interslice forces (about which little is currently known for reinforced soil) and adequately model the basic mechanics of reinforced soil. Mechanisms emerging at the toe and at points up the front face are examined.

Analytical procedure I

This first procedure mirrors the convention for unreinforced analyses, and progresses by trial and error. The soil geometry and reinforcement layout are chosen. Design strength parameters for the soil(s) and maximum available force envelopes for the reinforcement layers are selected. A safety factor on any slip mechanism may then be calculated by iteration, reducing the soil strength and the available force on intersected reinforcement layers by the same (lumped) safety factor. The reinforcement length and spacing may be adjusted on the basis of the results until the required safety factor is achieved on all mechanisms.

Analytical procedure II

The second procedure is more cumbersome, but more useful for back analysis. In this case a fixed magnitude of mobilised soil shear strength is assumed, and the gross force required for equilibrium on any slip mechanism is distributed to the reinforcement layers intersected.

The distribution method adopted is to divide the gross force between the reinforcement layers in proportion to the area of the stress triangle supported by each layer, Fig. 3. The force at a point in any layer is limited to that available through bond, and should this value be exceeded the remainder is redistributed to the other layers in proportion as before.

An inadequate reinforcement arrangement which would cause collapse is indicated in one of two ways. A slip mechanism may be found for which excess required force cannot be allocated to the reinforcement layers because of bond limitations. Alternatively, the force at a point on a reinforcement may exceed the maximum allowable limit. (Forces causing overstress should not be redistributed to adjacent reinforcements).

LOCUS OF MAXIMUM FORCES FOR WALLS

Analysis using procedure II for a frictional soil wall reinforced by layers at an ideal spacing extending back to the locus of zero force gives the same pattern of force in each reinforcement layer, Fig. 4a. Each reinforcement supports the same uniform force in the front zone of the wall, and the force decreases gradually to zero at the locus of zero force. Varying the spacing from the ideal alters the magnitude of force but does not change the pattern of the force profile in the reinforcement layers.

Truncation of the reinforcement to give a practical arrangement of (say) equal lengths alters the force profile and destroys the uniformity, Fig. 4b. The resulting force profile shapes then depend on reinforcement bond characteristics, truncation length and the soil properties. The trend is for a peak force to occur in a reinforcement layer and at a point closer to the face than the most critical mechanism through the toe. In the lower layers the reinforcement carries additional forces shed due to lack of bond from the upper layers. The point of peak force moves markedly towards the wall face in the upper reinforcement layers, particularly for metallic strip reinforcement, directly as a result of bond stress limits, Fig. 4c.

The resulting locus linking the points of maximum force no longer represents a single slip mechanism, as assumed in the design rules for reinforced earth walls Schlosser et al (1983). Rather it appears to be a consequence of equilibrium being maintained on all potential mechanisms.The consistent position of the locus of maximum force found for compact frictional soil walls reinforced by uniformly spaced metallic strips should not be expected in fills of lower shear strength or for reinforcements which can develop their design forces over short bond lengths.

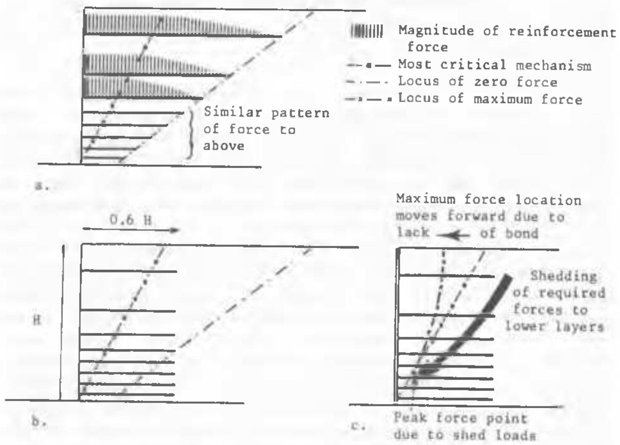


Fig.4. a.Ideal reinforcement layout and forces
b. and c. Truncated layout and resultant force redistribution (Case $\phi = 35°$)

COMPARISON WITH MEASURED BEHAVIOUR

Centrifuge models

Conventional metallic strip reinforced walls have been tested by Bolton and Pang (1982). Four identical models were constructed and detailed reinforcement force measurements taken. Results are given at 63 g acceleration, close to failure for the 202 mm high model walls, or the equivalent 12.73 m high prototype . The test parameters are summarised in Table 1. The sand of unit weight 16.9 kN/m^3 had a measured average peak strength ϕ = 45°, and bond coefficient with the reinforcement μ = 0.6, measured at a stress level equivalent to 63 g in the model.

These values of parameters were used in a procedure II analysis to calculate the force distributions in the reinforcement layers. Results at three levels below the wall crest are compared with the measured forces, Fig. 5. Good agreement was found for layers away from the influence of the wall base. In the lower reinforcement layers horizontal loading of the soil by facing panels constrained at their contact with the base would provide part of the required force,thus reducing the amount needed from the reinforcement. Bolton and Pang have commented correctly that extreme care should be taken before relying on this phenomenon to reduce the required reinforcement forces at the wall base.

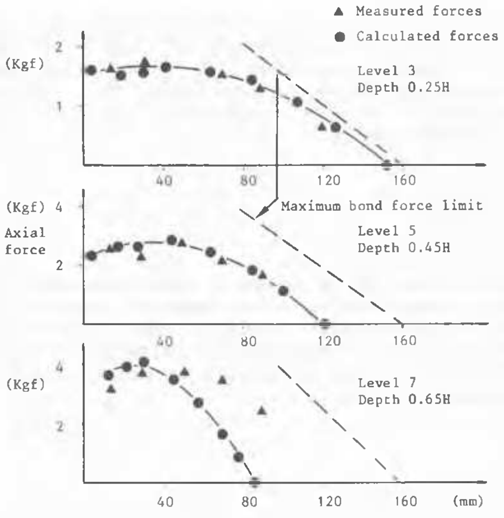


Fig.5. Comparison of measured and calculated reinforcement forces for centrifuge tested models

Dewsbury trial wall, Yorkshire

A 6 m high trial wall with rigid full height facing beams and reinforced with orientated grid reinforcement (Tensar SR2) was built in the summer 1983 using a compacted sandstone fill, Fig. 6a. Measurements were made of outward face movements.

The distribution of force in each grid reinforcement layer was calculated using the procedure II analysis and assumed mobilised strengths for the compacted granular fill ϕ' = 35° and 40°. The overall extension of each layer was calculated from the force distribution and a simplified linear load extension relationship for Tensar SR2 with 30 kN/m load causing 10% extension , which represents behaviour under sustained loading in the trial, (see Netlon Limited 1984). A straight line was fitted through the set of calculated extensions to represent the constraint of the rigid full height facing panels. The results are shown in Fig. 6b, and bound the measured movements. It is interesting to note the relatively small magnitude of outward movement , even though relatively extensible reinforcement was used .

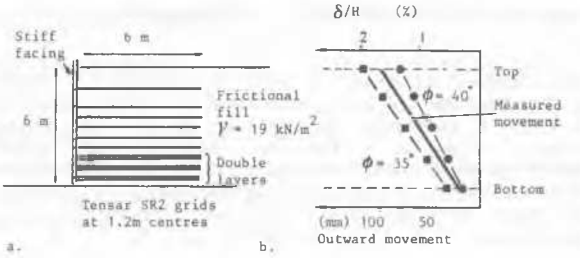


Fig.6. Dewsbury trial wall layout with measured and calculated face movements

Table 1. Details for the reinforcement in the model wall tests described (Bolton & Pang 1982)

Length	160 mm	vertical spacing	20 mm
width	4 mm	horizontal spacing	80 mm
Strength	4.8 kgf		

STRESS CONCENTRATION IN WALLS

For slope angles steeper than 80° a rigid block analysis for the reinforced zone indicates increased vertical stresses in the front half of the wall. Increased stresses have been directly measured , Schlosser et al (1983).The origin of the stress concentration may be identified with truncation of reinforcement length from the "ideal" case, Fig.4a. The truncated reinforced zone must support the wedge of unreinforced soil behind it, Fig. 4b. This results in overturning forces on the reinforced zone.

The results of limit equilibrium analysis clearly show increased force magnitudes in lower reinforcement layers towards the face , due to load shedding from higher layers, but these are less than would be anticipated from the local increase in vertical stress calculated with a rigid overturning block. The discrepancy could perhaps be resolved by a more sophisticated limit equilibrium formulation which examined moment equilibrium. Alternatively, the rigid block analysis may be unrealistically conservative.

For the present, however, it would be prudent to allow for extra reinforcement in the lower front portion of walls as indicated by the rigid block analysis to avert any possibility of local stress concentration triggering premature failure by reinforcements breaking. Such a provision is not required for slopes of 80° to 85° (depending on soil and pore water pressure parameters) for which the rigid block analysis does not indicate an increase in vertical stresses at the toe. There may be consequent savings on reinforcement quantities from building steeply sloping rather than vertical reinforced abutments .

COMPATIBILITY FOR EXTENSIBLE REINFORCEMENT

The response of granular soil to compressive loading may be represented by a relationship between mobilised frictional strength (or stress ratio) and tensile strain, Fig. 7a. Tensile testing of polymer reinforcements under sustained loading can provide a load extension relationship appropriate to long term sustained loading, which may be represented on an isochronous load extension curve, Fig. 7b.

The relationship between maximum gross required force and mobilised soil strength can be determined for a given soil geometry. Greater forces are required where there is lower mobilised shear strength. From the soil characteristics the calculated gross required force may be related to tensile soil strain as shown plotted in Fig. 7c.

In turn the gross available force for extensible reinforcement depends directly on the soil tensile strain, the reinforcement characteristics and the number of reinforcement layers. The gross available force may similarly be plotted against soil tensile strain as shown in Fig. 7c.

The critical state strength for the soil has been suggested for designs with extensible reinforcement, Jewell et al (1984), McGown et al (1984). The lesser of the long term strength under conditions in the ground, or the sustained load which would cause no more than 8% to 10% extension at the end of the design life, would be an appropriate limit to the maximum available force for polymer reinforcement . The compatibility curve shown in Fig. 7c indicates that with a lumped safety factor 1.3 to 1.5, designs based on this combination of parameters may typically reach equilibrium at relatively low reinforcement extensions of the order 2% to 4% . The resulting outward face movements would then be well within typical construction tolerances.

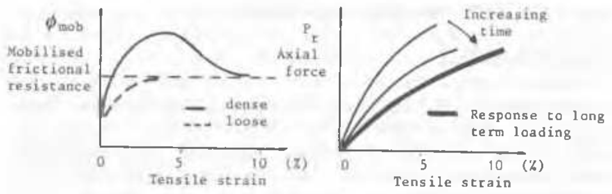
CONCLUSIONS

The findings presented in the paper confirm that routine limit equilibrium analysis can be effective in modelling the observed behaviour of reinforced soil, and for design. The separation of required and available reinforcement forces is helpful to the analysis. The following are significant .

1. Equal strength reinforcement layers should be spaced approximately as the inverse of depth to provide maximum and uniform benefit.

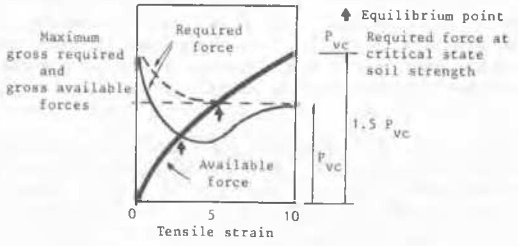
2. Ideally the reinforcement layers would extend back to the locus of zero force, Fig. 1. The practical (and more efficient) use of shorter reinforcement lengths causes load shedding to lower reinforcement layers.

3. The locus of points of maximum reinforcement force varies in position depending on the reinforcement length and bond characteristics, and the soil and pore water pressure parameters. The locus of maximum force appears not to represent a single worst slip mechanism, but rather to be the consequence of equilibrium on many potential slip mechanisms.



a. Soil charecteristics

b. Reinforcement charecteristics (Isochronous curves)



c. Compatibility curve

Fig.7. Soil and reinforcement charecteristics used to indicate strain compatibility

4. A compatibility curve has been shown to help match soil characteristics and polymer reinforcement properties so that sufficient reinforcement layers can be selected to provide equilibrium within allowable strain and deformation limits.

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