Residual displacements of retaining structures under earthquake loading

Déplacements résiduels des murs de soutènement soumis à chargement sismique

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SYNOPSIS A finite element model is proposed to determine the residual movements of retaining structures caused by seismic excitation. A displacement based adaptation of the Newmark-β Method is used to solve the equations of motion in the time domain. Residual displacements are computed from true hysteretic behavior, which in addition, eliminates the need to resort to viscous material damping. The model is a valuable tool for assessing the damage potential of retaining structures subjected to earthquake loading. It can be incorporated into a probabilistic framework to perform reliability studies.

INTRODUCTION

Since the development of the Mononobe-Okabe Equation (1926,1929), research efforts for the analysis of retaining structures subjected to earthquake loading have followed along two lines. The first has concentrated on developing deterministic techniques for predicting the lateral pressure distribution or load acting on the structure. Although these techniques are commonly used for design, they are limited in that they do not provide information regarding the displacements of the structure. The second line of research, undertaken more recently, has been directed at establishing methods for determining the displacement of the structure due to an acceleration time history. The displacement models proposed to date have been limited by restrictive assumptions regarding the initiation and mode of movement of the retaining structure and the constitutive behavior of the backfill material.

Recently, the authors' work in reliability analysis and damage assessment of retaining structures has prompted the need for a method to compute residual wall movements due to both strong motion events and events of modest magnitude. Existing models to date do not adequately consider permanent strain caused by moderate excitation. A finite element model is proposed herein to account for the distortions caused by moderate earthquakes.

EXISTING MODELS

Recently, a number of models have been proposed to compute the permanent displacements of retaining structures subjected to earthquake loading. Elms and Richards (1979) adapted the sliding wedge analysis developed by Newmark (1965) for dams and embankments to the case of a gravity retaining wall. They assumed that permanent translation of the retaining structure relative to the ground initiates when the ground acceleration exceeds a critical value, and ceases when the absolute velocities of the wall and ground become equal. The critical acceleration is determined by considering limit equilibrium of the inertia force of the wall, the base resistance force, and the lateral earth pressure force computed using the Mononobe-Okabe Equation. After the acceleration time history of the wall relative to the ground is computed, the permanent translation is obtained by double integration.

Nadim and Whitman (1982) have incorporated similar concepts into a finite element model. The model uses linear isoparametric quadrilateral soil elements and slip elements in the backfill and at the base and back of the wall. The equivalent linear method is used to obtain strain compatible moduli and damping ratios in the soil elements. Permanent translation of the wall is initiated when the shear stress in the slip elements exceeds the Mohr-Coulomb failure criterion. The analysis is performed by integration of the equations of motion of the system.

These two displacement models have four assumptions in common: (1) failure occurs on a predetermined plane; (2) permanent wall displacements initiate when the stresses exceed the Mohr-Coulomb failure criterion; (3) the effect of wall rotation can be neglected; and (4) the flexibility of the base soil is not considered. The major difference between the models is that the second permits elastic deformation of the backfill but the first does not.

PROPOSED FINITE ELEMENT MODEL

A typical soil-structure mesh for the proposed finite element model is illustrated in Figure 1. The soil deposit and backfill are discretized into linear isoparametric quadrilateral elements. The retaining structure is modeled either as a flexible system or as a rigid body. Quadrilateral elements or beam-column elements are provided to represent a flexible wall.

A hyperbolic constitutive law is used to model the non-linear constitutive soil behavior over a wide range of seismically induced strains.

Unloading and reloading portions of the stress-strain curves are determined using the Masing criterion (1926). The acceleration time history applied at the base of the soil profile is discretized into a large number of time intervals. A numerical integration technique is used to compute the displacements, strains, and stresses as a function of time. Within each time step of the integration, the soil moduli are adjusted iteratively to be compatible with the initial and final stress-strain states of that time step. The details of the computational scheme are described in the following section.

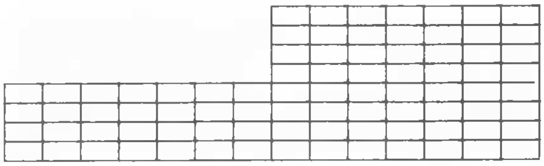

Fig. 1: Typical Mesh

As the constitutive soil model is inelastic, the stress-strain time history at any point in the soil is hysteretic, thus resulting in material damping of the soil-structure system. Therefore, artificially defined viscous damping terms, common to existing analytical models, are unnecessary.

COMPUTATIONAL SCHEME

The selection of the numerical integration scheme was guided by the need for economy of computational effort and adaptability to various constitutive soil models. A technique was developed to couple a displacement based adaptation of the Newmark-β Method with the Gauss-Seidel iteration procedure. The displacement based method of computation retains the advantages of the conventional acceleration based Newmark-β Method, i.e., unconditional stability of the computed motion for appropriately chosen integration parameters, and it has the additional advantage of imposing no restriction on the size of the time step. The acceleration based method, however, requires iterative improvement of the accelerations occurring within each time step, and convergence is not guaranteed unless the time step is less than a specified value proportional to the inverse of the highest natural frequency of the system.

The expressions for the acceleration based method in incremental form are

$$\dot{y}_{t+h} = \dot{y}_t + (1-\delta)\,\ddot{y}_t\,h + \delta\ddot{y}_{t+h}\,h \qquad (1)$$

$$\Delta y_{t+h} = \dot{y}_t\,h + (1/2 - \beta)h^2\,\ddot{y}_t + \beta h^2\,\ddot{y}_{t+h} \qquad (2)$$

$$M\ddot{y}_{t+h} + C\dot{y}_{t+h} + K\,\Delta y + S_t = f_{t+h} \qquad (3)$$

where δ and β are integration parameters and

y_t = displacement vector at time t

h = size of the time step

$\Delta y_{t+h} = y_{t+h} - y_t$

M = lumped mass matrix of the system

C = diagonal matrix of damping coefficients (used with the radiation boundaries)

K = incremental stiffness matrix

f_{t+h} = vector of nodal loads (equal to the product of the base acceleration at time t+h and the lumped mass terms)

S_t = vector containing the total nodal stiffness forces existing at time t

For any given time step, the value of the acceleration vector, y_{t+h}, is initially assumed. Corresponding velocities and displacements are computed using equations (1) and (2). These computed values of velocity and displacement are substituted into equation (3), which is then solved for the acceleration vector. Generally, several iterations are necessary to obtain agreement of the computed acceleration vector with the assumed acceleration vector.

The displacement based formulation is derived by substituting the expressions for the velocity y_{t+h} (equation (1)) and the acceleration $\ddot{y}_{t+h}$ (obtained from equation (2)) into equation (3). Subsequent rearrangement of terms, gives

$$K^*\Delta y_{t+h} = f_{t+h} + f^* - S_t \qquad (4)$$

with

$$K^* = K + \frac{1}{\beta h^2}M + \frac{\delta}{\beta h}C$$

$$f^* = M\left[\frac{1}{\beta h}\dot{y}_t + \left(\frac{1}{2\beta} - 1\right)\ddot{y}_t\right] + C\left[\left(1 - \frac{\delta}{\beta}\right)\dot{y}_t + \left(1 - \frac{\delta}{2\beta}\right)h\,\ddot{y}_t\right]$$

Standard algorithms developed for static analysis can be used to solve equation (4) for the incremental displacements Δy_{t+h}. But due to the large mesh size associated with soil structure interaction problems, assembly of the stiffness matrix for each time step is often prohibitive. Therefore, the Gauss-Seidel Method of iterative solution is preferred to Gauss elimination for the solution of equation (4). The incremental displacements Δy_{t+h} are initially assumed, and then successively improved with each iteration using

$$\Delta y_i^! = \Delta y_i + \frac{\beta}{K^*_{ii}}\left[R^* - \sum_{j=1}^{N} K^*_{ij}\;\Delta y_j\right] \qquad (5)$$

where $R^* = f_{t+h} + f^* - S_t$

and N = the number of degrees of freedom

For each time step, the initial estimate of the displacements is made using

$$\Delta y_i = \frac{R^*}{K^*_{ii}} \qquad (6)$$

The method requires many multiplications of the left side of equation (4), which is accomplished on an element by element basis, thereby avoiding the assembly of the matrix K^*. Since convergence of the Gauss-Seidel Method is guaranteed for symmetric positive definite K^* and the Newmark-β Method is unconditionally stable for $\beta = .25$ and $\delta = .5$, the size of the time step is limited only by the required degree of accuracy.

The rate of convergence, ρ, of the acceleration based method, however, is proportional to the square of the natural frequency of the system and to the square of the size of the time step. If ρ is larger than 1, convergence is impossible. Since the natural frequency associated with the stiffest mode of motion of a finite element mesh is generally much greater than the frequency range of interest, the size of the time step must be chosen much smaller than that needed to obtain sufficient accuracy.

A second level of iteration exists for the moduli of the soil elements. For each element, within any given time step, the modulus is a secant value equal to the ratio of the increment in stress, Δτ, to the increment in strain, Δγ. These increments are computed from the stresses, τ_t and τ_{t+h}, and strains, γ_t and γ_{t+h}, at time t and t+h, respectively. Since the modulus is dependent on the dynamic shear strain amplitude, the incremental stiffness matrix K^* is dependent on both y_t and y_{t+h}. Thus, iterations are necessary to obtain secant moduli which are consistent with the initial and final stress-strain states. Equation (4) must be solved once for each iteration on the secant moduli.

To maximize the efficiency of these operations, it is desirable to have a well conditioned stiffness matrix, since the number of iterations required for convergence of the Gauss-Seidel Method increases with increasing degree of ill-conditioning. The elements used to model stiff inclusions, such as a concrete cantilever retaining structure, have stiffness terms which are orders of magnitude larger than the terms in the soil element stiffness matrices, and thus, generally ill-condition the stiffness matrix, K. Alternatively, the retaining structure can be modeled as a rigid body capable of translation and rotation. As the distortions of the structure are much smaller than those of the soil mass, the effect of neglecting the distortions of the structure is virtually nil. Structural deformations should be considered for flexible structures such as sheet pile walls and for such cases the K matrix is not severely ill-conditioned.

PROBABILISTIC FRAMEWORK

The statistical moments of the wall displacement, D, are approximated through a "point estimate" procedure which "replaces" the distribution of the random variables with point estimates (weights) at properly selected values. The statistical moments of the random variable D are calculated by expressions of the form (Rosenblueth, 1975)

$$E(D^m) = \sum_{i=1}^{k} p_i \; D_i^m \qquad (7)$$

where

$E(D^m)$ = the m^{th} moment of D
D_i = the i^{th} point estimate of D
p_i = the weight assigned to D_i
k = the number of point estimates

This technique requires evaluation of 2^n values of D, where n is the number of independent random variables. For example, if the displacement is assumed to be a function of the angle of internal friction φ and the cohesion c, then four computer runs are required, each using appropriate values of c and φ. The moments of D are estimated using

$$E(D^m) = \frac{1}{4}\left(D^m_{++} + D^m_{+-} + D^m_{-+} + D^m_{--}\right) \qquad (8)$$

where the notation D_{ij} indicates the permutations

$$D_{\pm\pm} = D\left[\bar{c}(1 \pm V_c),\ \bar{\phi}(1 \pm V_\phi)\right] \qquad (9)$$

where $\bar{c}$ and $\bar{\phi}$, and, V_c and V_ϕ are the means and coefficients of variation of c and φ, respectively. These operations can be performed for any selected location on the structure and then probabilistic statements can be made by assuming a reasonable probability density function for D.

TYPICAL RESULTS

The geometry of the typical soil-structure mesh to be analyzed is illustrated in Figure 1. The soil elements are all 5 ft (1.52 m) high by 10 ft (3.05 m) long. The structure is a cantilever retaining wall modeled for rigid body motion. Its height is 25 ft (7.62 m) with an embedment of 5 ft. The base width is 20 ft (6.10 m), divided equally between the toe and heel. The bedrock is 40 ft (12.19 m) below the surface of the backfill. Horizontal and vertical motions can occur at every joint except at the far ends of the mesh where the vertical degrees of freedom are restrained. The soil is an overconsolidated sand deposit (K_o = 1) with an angle of internal friction of 40° and a unit weight of 110 pcf, which are used to define the parameters of the hyperbolic curve (Hardin and Drnevich, 1972). The base motion is the strong motion portion of a synthetic seismogram (Seed and Idriss, 1969) with a peak acceleration of .42g. The duration of the base motion is 4 seconds. The analysis was performed for a duration of 8 seconds.

Figure 2 gives the computed time histories of the horizontal displacement at the top and bottom of the wall. The shape of the displacement history after the excitation has ended is similar to free vibration of a viscoelastic system. The oscillation is about a reference level defined by the permanent displacement of the wall.

The analysis was performed using a time step of .02 seconds. If the response due to the high frequency content of the accelerogram is unimportant, a larger time step can be used, but generally, more Gauss-Seidel iterations will be needed. Note that when high frequency response is unimportant, coarser meshes can be used.

The Point Estimates procedure was used to determine the effect of a coefficient of variation of 10% for the angle of internal friction. For the synthetic seismogram scaled to a peak acceleration of .42g, a mean value of 5.3cm. and a coefficient of variation of 15.% were obtained for the permanent displacement at the top of the wall.

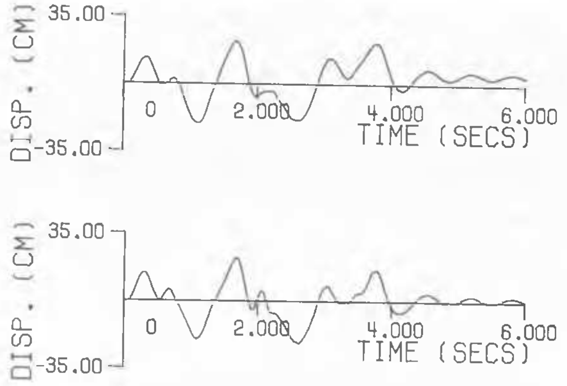

Fig. 2: Horizontal Displacement at top of wall (a) and bottom of wall (b)

RADIATION BOUNDARIES

The computer cost associated with the proposed time domain analysis is generally high for a large finite element mesh. Previous researchers have been able to use a reduced mesh size, without adversely affecting the computed response of the structure, by incorporating radiation boundaries into their analysis. Unfortunately, most boundary formulations available to date are frequency dependent, and thus, can only be used in the frequency domain. Recently however, good results have been obtained in the time domain for homogeneous linear visco-elastic halfspace models by evaluating the frequency dependent parameters of a Lysmer-Kulhemeyer boundary at the fundamental frequency of the soil-structure system (Bayo and Wilson, 1983). This approach can be used herein by assigning appropriate values to the damping coefficients at the boundaries of the mesh. However, since the hyperbolic constitutive law is non-linear, further study of the effect of the time variation of the fundamental frequency of the soil-structure system is necessary.

CONCLUDING REMARKS

A finite element model has been proposed to compute the residual displacements of both rigid and flexible earth retaining structures subjected to any level of earthquake loading. The equations of motion are solved in the time domain using a displacement based adaptation of the Newmark-β Method and the Gauss-Seidel Iterative Method. An iterative approach is used to obtain soil moduli that are compatible with the stress-strain states at any time.

At the time this paper is written (March 1984), a testing program is about to be undertaken, using a shaking table, in which measurements will be made of the displacements of model retaining walls under laboratory controlled conditions. These results will be used to check the developed analytical solution.

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