Seismic stability of natural slopes subject to progressive failure

Stabilité sismique des talus naturels sujette à une rupture progressive

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SYNOPSIS A new model is presented for the assessment of the safety of natural slopes of clays exhibiting a degradation in dynamic shear strength during seismic loading. This involves a definition of local failure and a mechanism for failure progression from failed to non-failed slope regions. The randomness associated with the earthquake loading as well as the initiation and progression of failure are considered. The model can predict the extent of the region within the slope mass that is most likely to experience strength reduction during an earthquake and the corresponding safety measure of the slope in overall sliding. The applicability of the model is demonstrated in a numerical example and the obtained results are presented and discussed.

INTRODUCTION

Limit equilibrium procedures (deterministic or probabilistic) that are currently employed to evaluate the safety of natural slopes in seismically active areas are based on the assumption that slope failure is an instantaneous phenomenon occurring simultaneously with the application of the seismic loading. Thus, the progressive failure behavior of natural slopes (triggered by seismic activity) prior to their overall sliding has not yet been considered by current methods of analysis.

This study presents a model for the progressive failure of natural slopes composed of clays exhibiting strength reduction under seismic loading. The term "progressive" is used to denote failure propagation along a critical surface under the action of a uniform seismic load. As the latter is applied to the soil mass rapidly, slope stability is examined using total stress analysis. A probabilistic formulation is incorporated into the model in order to account for the uncertainty in the peak ground acceleration (random variable) and the variability along the critical surface of the undrained dynamic strength. The latter is defined as the sum of the sustained and uniform cyclic shear stresses that correspond to specified strain level and number of cycles. Its numerical values can be obtained using cyclic tests that simulate field conditions (Seed et al., 1966; Ishihara et al., 1983).

MECHANISM OF FAILURE PROGRESSION

An important consequence of the strength degradation of clays during seismic loading is the enhanced tendency of this behavior to propagate within the slope medium. Local failure may occur when the shear strength along the weakest and/or most stressed segment of the slope mass becomes equal to, or smaller than, the induced shear stress from combined static and seismic loads. In this case, the available strength is decreased and excessive loads (due to stress levels in excess to the reduced strength) can no longer be taken by the failed segment and are transferred to non-failed regions. The latter may in turn fail under the influence of the increased stresses and transfer loads once more to neighboring regions, and so on. In this manner, an initiated local failure progresses sequentially within the slope mass through a combination of increased stresses and material strength degradation, and may eventually lead to overall slope failure.

ATTRIBUTES OF THE PRESENT MODEL

The model presented in this study provides a probabilistic, two-dimensional, limit equilibrium, seismic stability analysis of clay slopes subject to progressive failure. Specific features of the model are as follows:

(1) It accounts for the spatial variability of the undrained shear strength within the slope mass. This is achieved through a stochastic modeling of strength in which its statistical values (i.e., mean, variance, autocorrelation function) are expressed as functions of the length of the critical failure surface. Such a formulation was presented for the first time by Vanmarcke (1977) in connection with a three-dimensional (non-progressive) stability analysis of embankment slopes. The procedure employed in this study is the one applied by Asaoka et al. (1981) in order to determine the (non-progressive) short-term reliability of slopes under static and seismic conditions.

(2) It considers the undrained strength degradation of clays caused by earthquakes. The peak and residual values used correspond to the first and thirtieth significant cycles, respectively, and are in accordance with the findings of previous investigations on the subject (Ellis et al., 1967; Ishihara et al., 1983).

(3) It accounts for the randomness in the magnitude of the peak horizontal acceleration (α_h) experienced by the slope during a seismic event. The statistical values of α_h are determined through a seismic hazard analysis of the area of the slope and depend on the seismic history of the area, the type of earthquake source involved, and the distance between the latter and the site of the slope (Grivas, 1978).
(4) It considers that failure progression occurs along the critical slip surface and employs the customary approach of dividing the soil mass above the slip surface into a number of slices. Thus, the base of each slice is a segment of the potential slip surface and failure may start at any such segment and then extend successively to other slices.
(5) It considers that strength reduction (from peak to residual) at a slice occurs when its undrained dynamic strength is exceeded by the combined static and seismic shear stresses.
(6) It determines the most probable failure progression path. This is achieved on the basis of the probabilities of failure for each slice and the transition probabilities of failure progression that account for the distribution within the slope mass of excessive load resulting from successive slice failures.
(7) Finally, it evaluates the probability of slope in overall sliding during each stage of failure progression.

PROBABILITY OF LOCAL AND OVERALL FAILURE

In Fig. 1 is shown schematically a cross-section of a slope of height H and angle β and the corresponding critical slip surface. The slope mass above the slip surface is divided into n slices (Fig. 1a) and the force system acting on each slice is determined (Fig. 1b).

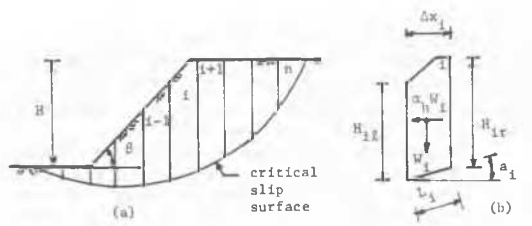

Fig. 1 Slope Mass Subdivision Into Slices

Failure of the i-th slice is defined as the event whereby its capacity C_i (available peak strength) becomes less than or equal to the demand D_i (applied stress). If SM_i denotes the safety margin of the i-th slice, defined as $SM_i=C_i-D_i$, then the i-th slice fails if $SM_i=C_i-D_i<0$, in which $C_i = c_iL_i$, $D_i = W_i\sin a_i + a_hW_i\cos a_i$, W_i is the weight of the i-th slice, L_i and a_i are the length and inclination with the horizontal of its base, respectively, α_h is the peak horizontal ground acceleration (in g's), and c_i is the local average of the undrained dynamic shear strength calculated along the length L_i, i.e.,

$$c_i = \frac{1}{L_i}\int_0^{L_i} c_u(\ell)\, d\ell \qquad (1)$$

The quantity $c_u(\ell)$ in Eq. 1 denotes the undrained strength expressed as function of the length ℓ of the slip surface.

The probability of failure p_i of the i-th slice is obtained as $p_i = P[SM_i < 0]$ and is evaluated under the assumption that SM_i is normally distributed with mean value $\overline{SM_i}$ and variance $Var(SM_i)$. These are determined from the expression for SM_i as

$$\overline{SM_i} = \bar{c}\, L_i - W_i \sin a_i - \bar{\alpha}_h W_i \cos a_i \qquad (2)$$

$$Var(SM_i)=\sigma_u^2 k_i\int_0^{h_i}\int_0^{h_i} \rho(\Delta z)dzdz'+\sigma_{\alpha h}^2 W_i^2\cos^2 a_i \qquad (3)$$

in which $\bar{c}$, σ_u^2 and $\bar{\alpha}_h$, $\sigma_{\alpha h}^2$ are the mean values and variances of the strength along the slip surface and the peak ground acceleration, respectively, $h_i=\Delta x_i\tan a_i$, Δx_i is the width of the i-th slice, and $k_i=1+(1/\tan^2 a_i)$.

Furthermore, if SM denotes the safety margin of the entire slope, then the probability p_f of an overall sliding of the slope is equal to $p_f = P[SM < 0]$ in which SM is also a normal variate. The mean value $\overline{SM}$ and variance $Var(SM)$ of SM are equal to

$$\overline{SM} = \sum_{i=1}^{n} \overline{SM_i} \qquad (4)$$

$$Var(SM)= \sum_{i=1}^{n} Var(SM_i)+2 \sum_{i=1}^{n-1} cov(SM_i,SM_{i+1}) \qquad (5)$$

in which $cov(SM_i,SM_{i+1})$ denotes the covariance between the safety margins of successive slices and is equal to

$$cov(SM_i,SM_{i+1})=\sigma_u^2(k_ik_{i+1})^{1/2}\int_0^{h_i}\int_0^{h_{i+1}} \rho(\Delta z)dzdz'$$

$$+ \sigma_{\alpha h}^2 \; W_iW_{i+1}\cos a_i\cos a_{i+1} \qquad (6)$$

where k_i is defined in Eq. 3, k_{i+1} is the same quantity for slice i+1, and $\rho(\Delta z)$ is the autocorrelation function of strength along the vertical direction. This is expressed as function of the vertical distance $|z-z'|$ between two points at depths z and z' and the correlation length c (Asaoka et al., 1981) in the form $\rho(\Delta z) = \exp(-|z-z'|/c)$.

PROGRESSION OF FAILURE

A Markov-like process is used to model failure progression in space with states given by the slope slices (Chowdhury et al., 1982). Having occurred in state i, failure will progress to the adjacent state j=i+1 or j=i-1 with probability equal to $p_{ij}= P[SM_j < 0 \,|SM_i < 0]$. With the aid of Bayes Theorem, this expression becomes

$$p_{ij} = \frac{P[SM_j \le 0 \text{ and } SM_i \le 0]}{P[SM_i \le 0]} \qquad (7)$$

The numerator and denominator of Eq. 7 are evaluated using the bivariate and univariate normal distribution, respectively. As failure initiates at the slice i with the highest probability of failure p_i, in a similar manner, failure is most likely to progress to the slice j = i-1 or j = i+1 that corresponds to the highest transition probability p_{ij}. Furthermore, as a failed slice has zero mean safety margin, any excessive loads must be taken by the slices that have not yet failed. In this study, it is assumed that the average excessive load from a failed slice is distributed among the non-failed slices in portions inversely proportional to their horizontal distance from the failed slice.

ILLUSTRATIVE EXAMPLE

The progressive failure of a slope located in the seismic environment of New York State is investigated under undrained conditions. The cross-section of the slope is shown in Fig. 2 together with the critical surface (determined from stability charts). It is assumed that failure progression may occur in seven stages that correspond to the seven slices of the slope mass (Fig. 2).

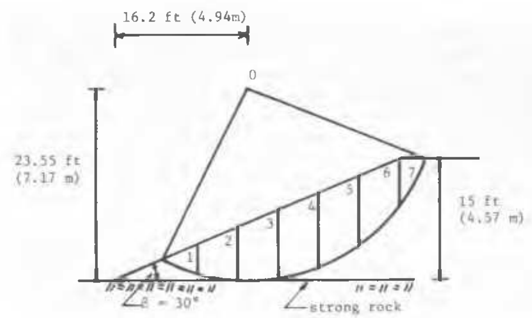

Fig. 2 Example Slope Cross-Section

The unit weight of the clay is γ = 120 pcf (18.24 kN/m^3) and the mean values and coefficients of variation of the peak (c_p) and residual (c_r) dynamic strength have been determined (e.g., from a series of cyclic, stress-controlled, triaxial tests) to be $\bar{c}_p$=600 psf (28.73 KN/m^2), V_p=20% and $\bar{c}_r$=450 psf (21.55 KN/m^2), V_r=26.67%, respectively.

The correlation length of strength is equal to c= 3.28 ft (1 m) determined as in Asaoka et al. (1981). Finally, the mean value and standard deviation of the peak horizontal ground acceleration are found from a seismic hazard analysis of the region of the slope (Grivas, 1978), to be equal to $\bar{\alpha}_h$ = 0.2 g and $\sigma_{\alpha h}$ = 0.47 g, respectively.

The results are summarized in three tables. Table I lists the values of the failure probabilities for each slice. Cases 1 and 2 correspond to a deterministic seismic load (α_h = 0.2 g) and autocorrelated (Case 1) and non-autocorrelated (Case 2) soil deposits. Cases 3 and 4 involve probabilistic seismic conditions ($\bar{\alpha}_h$=0.2 g, $\sigma_{\alpha h}$ =0.47 g) for autocorrelated (Case 3) and non-autocorrelated (Case 4) soil deposits. In all cases, the highest value for the local probability of failure corresponds to Slice 5, which is considered to fail first.

TABLE I

Initial Failure Probability of Each Slice

Slice No.	Failure Probability			
	Case 1	Case 2	Case 3	Case 4
1	0.376×10^{-3}	0.664×10^{-3}	0.594×10^{-5}	0.267×10^{-4}
2	0.100	0.102	0.419×10^{-3}	0.664×10^{-3}
3	0.195	0.195	0.789×10^{-3}	0.904×10^{-3}
4	0.394	0.394	0.125	0.140
5	0.512	0.512	0.544	0.536
6	0.375	0.382	0.221	0.268
7	0.234×10^{-4}	0.466×10^{-3}	0.356×10^{-5}	0.208×10^{-3}

Tables II and III list the values of the transition probabilities before (Table II) and after (Table III) local failure.

TABLE II

Values of Transition Probabilities Before Local Failure

Transition		Joint Probability of Failure of Slices i and j	Probability of Failure of Slice i p_i	Transition Probability p_{ij}
From i	To j			
2	1	0.374×10^{-2}	0.1003	0.373×10^{-2}
3	2	0.1008	0.1949	0.5171
4	3	0.1950	0.3936	0.4955
5	4	0.3935	0.5120	0.7686
5	6	0.3746	0.5120	0.7317
6	7	0.235×10^{-4}	0.3745	0.627×10^{-4}

TABLE III

Values of Transition Probabilities After Local Failure

Transition		Joint Probability of Failure of Slices i and j	Probability of Failure of Slice i p_i	Transition Probability p_{ij}
From i	To j			
2	1	0.529×10^{-3}	0.1112	0.476×10^{-2}
3	2	0.1118	0.2199	0.5569
4	3	0.2087	0.4325	0.4825
5	4	0.4127	1.00	0.4127
5	6	0.4265	1.00	0.4265
6	7	0.566×10^{-4}	0.4364	1.297×10^{-4}

The probability of failure p_f and central factor of safety FS of the slope in overall sliding are found to be p_f=0.14, FS=1.74, and p_f=0.375, FS=1.56, before and after the reduction in the mean undrained dynamic strength of Slice 5, respectively.

EVALUATION OF THE PRESENT MODEL

A main characteristic of the presented model of progressive slope failure involves its ability to account for the reduction in the undrained shear strength of clays during earthquakes. It was considered that, when the peak dynamic strength was exceeded by the shear stress induced by the combined static and seismic loads, shear strength dropped to its residual value. The values for the peak and residual strength required by the model can be determined through cyclic testing in accordance with previous studies on the subject (Ellis et al., 1967; Ishihara et al., 1983).

The procedure used to calculate the values of the shear stresses along the critical surface involved the assumption of pseudo-static loading and rigid body material behavior. Although the shortcomings of this procedure are well known (Seed, 1979), the simplicity involved in its formulation continues to render it an attractive method of assessing the stability of slopes in many practical situations. This is particularly true for the case of natural slopes that are of concern in the present study. Moreover, the presented model is not dependent on a specific technique of determining shear stresses (the pseudo-static procedure was used for simplicity) and, in principle, any suitable method may be employed.
An important postulate of the presented progressive failure model is that local failure, while it reduces the safety of a slope, does not necessarily lead to an overall slope sliding. Thus, one can determine the extent of the failed region within a slope and the corresponding reduction in its safety. This is of practical significance as it can help decide whether there is a need for stabilization actions that will enhance the safety of a slope during future seismic activity. The presented illustrative example demonstrates the ability of the model to predict the extent of local failure within a spatially variable slope medium, and the reduction in the overall safety of the slope caused by an earthquake.

SUMMARY AND CONCLUSIONS

A model capable of describing the progressive failure of slopes in natural clays during earthquakes was presented. Important aspects of material behavior, loading parameters, and failure development and progression were discussed. The applicability of the model was demonstrated in an illustrative example.

The conclusions drawn from the analysis and results of this study are: (a) the present model provides a rational description of the seismic stability of clay slopes that accounts for strength degradation of the material, the spatial variability of strength, the uncertainty in the ground acceleration, and the details associated with the initiation and progression of failure; (b) the local failure probability is overestimated in non-autocorrelated deposits when $\overline{SM}_i > 0$ and underestimated when $\overline{SM}_i < 0$ (Table I); (c) local averages in autocorrelated soil deposits have smaller variance than in non-non-autocorrelated ones (Table I); (d) the probability of failure of natural slopes is underestimated if the variability in the ground acceleration is neglected (Table I); and (e) transference of excessive loads from failed to unfailed regions increases the probability of failure progression within the slope mass (Tables II and III).

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