Stability of cuttings considering variable cohesion and earthquake effects

Stabilité de tranchées en considérant une cohésion variable et des effets sismiques

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SYNOPSIS An analytical solution is presented for the stability of slopes and cuttings in soil possessing constant internal friction but linearly varying cohesion with depth. Use is made of a log spiral sliding mechanism. Earthquake effects are also presented in a pseudo-static manner, with some results being given for the yield acceleration in function of the soil parameters and the slope geometry.

INTRODUCTION

Limit Equilibrium and Limit Analysis

For a slope whose material properties are uniform with respect to strength parameters it is convenient to use a rigid block sliding mechanism in the form of a log spiral failure surface as in the classical limit equilibrium solutions of Rendulic (1935) and Taylor (1937). When there is full mobilization of internal friction simultaneously along the entire slip surface the condition is everywhere fulfilled that the unknown reaction force is colinear with the pole of the spiral and thus does not enter into the moment equation, which then suffices to investigate stability conditions. Taylor (1937) considered both circular (ϕ-circle method) and log spiral surfaces. Although the latter was theoretically more attractive he recommended the former due to ease of hand computations, a reason no longer relevant today. Taylor showed that the two approaches gave essentially the same critical slip surface. Thus the choice between a log spiral and a circular slip surface in simple limit equilibrium analysis is not a crucial one. For the method of slices, which has to be adopted in non-homogeneous ground conditions, the circular surface is obviously advantageous. On the other hand, for consideration of seismic forces the log spiral provides a simple analytical solution (Prater, 1979).

With the development of upper bound limit anlysis techniques it was recognized that for a rotational slip mechanism with two rigid blocks separated by a thin layer of c, ϕ material (c: cohesion, ϕ: angle of friction) a log spiral slip surface is necessary for fulfillment of the normality condition (i.e. associated flow rule) of the theory of plasticity (Chen, 1975). Failure with such a thin slip band is often observed in nature, but whether the kinematic slip condition required by plasticity theory applies is still an unsettled question. There is much evidence to show that dilatancy effects characterized by an angle ν (Davis, 1968) are much less than in a perfectly plastic material for which $\nu = \phi$, the angle of internal soil friction. Experimental evidence suggests that ν is approximately in the range $1/3\ \phi < \nu < 1/2\ \phi$. For a purely frictional material (non-dilatant) $\nu = 0$. The consequences of having non-associated plastic behaviour in stability problems still have to be worked out, but one general preliminary finding is that if the soil mass in free of kinematic boundary restraints there is very little difference in collapse loads (Davis, 1968). For example, Chen and Chang (1982) showed that for a plane failure mechanism in a vertical slope the associated and non-associated analyses give identical results. There is, however, a marked difference in the extent of the failure zone, at least in cases of passive failure (cf. Frydman and Baker, 1982).

As soil approaches, in the post peak range, the critical state ($\phi = \phi_{cv}$) the dilatancy reduces to zero. Some authors prefer, therefore, to adopt purely frictional behaviour with $\nu = 0$. For example, Goldscheider (1979) investigates composite kinematically admissible slip mechanisms with this assumption and suggests that to exploit the peak friction angle an average ϕ-value, i.e. $\phi = 1/2\ (\phi_{peak} + \phi_{cv})$, could be used.

It is clear from the above discussion that since soil is not perfectly plastic the advantages of limit analysis over limit equilibrium methods are more apparent than real. In fact, no exact analysis exists for the problem in hand.

Special Cases

i) $\phi = 0$ method for undrained failure.

This method applies to short-term failure in relatively impermeable soils, as opposed to the effective stress approach appropriate for long-term failure. Gibson and Morgenstern (1962) extended the $\phi = 0$ method to cuttings in normally consolidated clays, in which the cohesion varies linearly with depth. It appears that there would also be some practical usefulness for an analytical solution in the case of constant ϕ and cohesion varying as

$$c = c_o + a_o z \quad (1)$$

in which c_o is the cohesion at the ground surface, a_o is a constant and z the depth below the surface. Such a solution is presented below. The case also sometimes arises, e.g. in a slightly overconsolidated clay, in which the effective stress parameters are such that ϕ' is more or less constant, and c' varies linearly with depth (cf. Kjaernsli and Simons, 1962).

ii) Earthquake Loading.

Traditionally seismic effects are included in a pseudo-static manner. Due to the transient nature of earthquake motions this is less rational than an attempt to evaluate actual sliding displacements during the period of strong motion (Newmark, 1965; Franklin and Chang, 1977). Newmark's method presupposes block movement for the case in which liquefaction effects can be ruled out. It is this movement (mere centimetres or several metres) which enables the seismic stability of the slope to be evaluated. The method requires finding first of all the yield acceleration N, as it is only the downslope acceleration pulses exceeding this value which cause movement. For homogeneous slopes charts are available for N (Prater, 1981; Chang et al, 1984). Although in the first case limit equilibrium and in the second upper bound limit analysis methods were used the results are the same when using a log spiral slip surface. For the case of $\phi = 0$ and c proportional to depth a solution is given by Koppula (1984).

LOG SPIRAL STABILITY ANALYSIS

The slip surface is defined by the spiral equation:

$$r = r_a \exp(\theta \tan \phi) \quad (2)$$

r, r_a and θ being defined in Fig. 1.

Moment equilibrium gives

$$(1 \pm k_v)\, M_{weight} + k_h\, M_{inertia} - M_{cohesion} = 0 \quad (3)$$

in which k_h and k_v are the component seismic coefficients.

It is convenient to employ the following dependent variables:

$$m = \exp(\Theta \tan \phi)$$
$$d = r_a/H = 1/([1+m^2 - 2\, m \cos \Theta]^{1/2} \sin t)$$
$$j = t + \sin^{-1}(d \sin \Theta \sin t)$$
$$q = \pi - \Theta - j$$

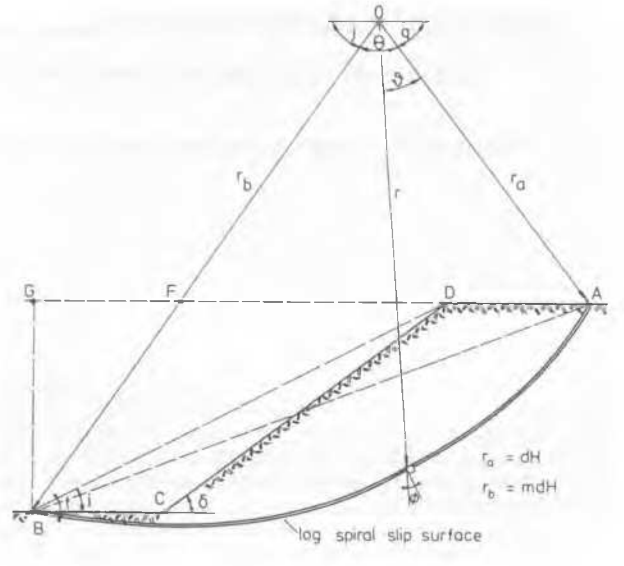

Fig. 1 Log spiral slip mechanism.

$$f = 1/(1+9 \tan^2 \phi)$$
$$b = \gamma H^3/3$$

in which H is the height of slope, γ the unit weight of soil and t is defined in Fig. 1. In the case of toe failure i = δ, the angle of the slope. The distance BC is defined by nH.

The analysis is, in principle, very simple. The main difficulty arises from the geometry of the sliding mass ABCD. The easiest method is to consider the total sectional area OAB and subtract the effect of the triangular sub-areas OAF, BDF and BED. There are three independent variables defining the geometry, i.e. the angles Θ and t and the factor n. Analytical expressions are derived for the moment terms (eqn. 3) and the most critical sliding surface is found by allowing Θ, t and n to vary.

The expressions for a homogeneous slope and toe failure (Prater, 1979) are:

$$M_{weight} = M_1 + M_2 + M_3$$

where

$$M_1 = fbd^3\, [(m^3 \sin j - \sin q) - 3 \tan \phi\, (m^3 \cos j + \cos q)]$$

$$M_2 = -1/2\, bd^3 \sin^3 q\, (\cot^2 q - \cot^2 j)$$

$$M_3 = -1/2\, b\, [\cot^2 i - \cot^2 j - 3\, md \cos j\, (\cot i - \cot j)]$$

$$M_{inertia} = M_4 + M_5 + M_6$$

where

$$M_4 = fbd^3\ [(m^3 \cos j + \cos q) + 3 \tan \phi\ (m^3 \sin j - \sin q)]$$

$$M_5 = -bd^3 \sin^3 q\ (\cot q + \cot j)$$

$$M_6 = -1/2\ b\ (3\ d \sin q + 1)\ (\cot i - \cot j)$$

and the moment due to cohesive resistance

$$M_{cohesion} = \frac{c_o\ d^2\ H^2\ (m^2-1)}{2 \tan \phi}$$

For a failure surface passing below the toe of the slope a correction for the area BCD must be applied, namely for M_{weight}:

$$M_7 = -1/2\ bn\ (3\ m\ d \cos j - \cot i - n)$$

and for $M_{inertia}$:

$$M_8 = -bn\ (1 + \frac{3}{2}\ d \sin q)$$

The correction due to the cohesive component $a_o z$ in eqn.(1) is found to be

$$M_9 = +(a_o/\gamma)\ 3\ b\ d^3\ f\ [\exp(3\ \Theta \tan \phi)\ (3 \tan \phi \sin j + \cos j) - 3 \tan \phi \sin q + \cos q - \sin q\ (m^2-1)/(2\ f \tan \phi)]$$

All the moment terms can now be substituted into eqn.(3), in which the restoring moment due to cohesion is in equilibrium with the disturbing moments due to gravity and seismic inertia. In finding the yield acceleration there is no complication due to the factor of safety F, i.e. F = 1. If only horizontal earthquake effects are considered the acceleration causing slip is N = k_h.g where g is the acceleration due to gravity and from (3):

$$k_h = \frac{M_{cohesion} - M_{weight}}{M_{inertia}}$$

A minimum value of k_h is sought.

Sarma and Bhave (1974) show that the determination of yield acceleration is a possible alternative description of F, the static factor of safety, there existing a good correlation between the two values. The advantage of working with eqn.(4) is that no iteration is required unlike with F. For example, when using Taylor's (1937) stability charts if F is placed on both c and ϕ (i.e. F = F_{true}) three or four interations may be necessary. Taylor (1937) also assumed that ϕ is fully mobilized and F is determined with respect to c only. Eqn.(3) can likewise be rewritten in a form to find F for the case of pseudo-static loading in that the stability number N_s = $c_o/(F\gamma H)$ is determined. To determine the maximum value of N_s, however, the second cohesion component (a_o z) has to be kept constant.

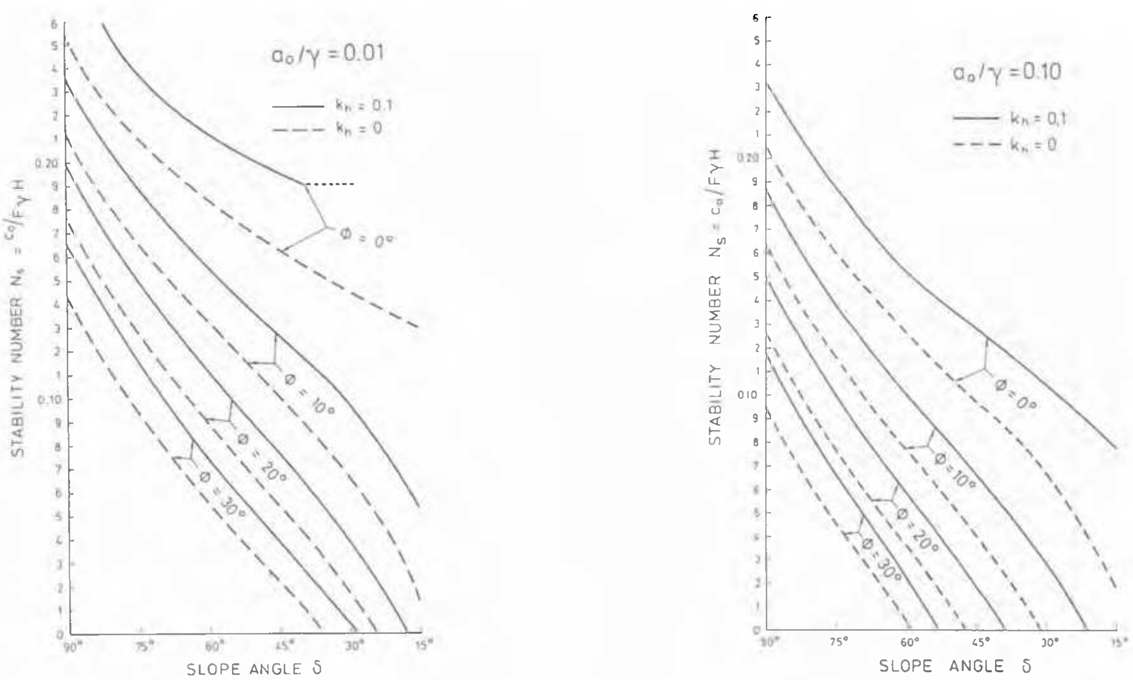

Fig. 2 Stability charts for (c, ϕ) soil with depth variation of cohesion showing influence of factor (a_o/γ) and horizontal seismic coefficient k_h.

The determination of critical values, either of k_h or N_s, would require setting the partial derivatives with respect to the two independent variables to zero. Alternatively, this may be done numerically in that a search is made as follows. For an initial value of t just less than i the angle Θ is varied to find a minimum value, the procedure being repeated for new (reduced) values of t. Increments of Δt and $\Delta\Theta$ of 1^o are adequate.

RESULTS

Analogous to Taylor's (1937) representation, charts can be produced each for a particular value of depth factor (a_o/γ). Due to the depth variation of c toe failure will generally apply. Due to space restrictions only two such charts are presented here (Fig. 2). The factor of safety F is introduced into the term $N_s = c_o/F\gamma H$, i.e. on the value of cohesion at the ground surface for a given increase of cohesion with depth $(a_o z)$. If a true factor of safety is required (i.e. with respect to both c and ϕ) then the charts must be used in an iterative manner. The influence of the factor (a_o/γ) is such that the required N_s-value for given F is reduced, as expected.

For comparison purposes the curves are also shown for a horizontal seismic coefficient $k_h = 0.1$, with $k_v = 0$.

Charts for yield acceleration (/ g) for horizontal seismic excitation only are presented in Fig.3 for a selected value of (a_o/γ). For further parameter variation the reader can readily program the given equations varying t and Θ as described previously.

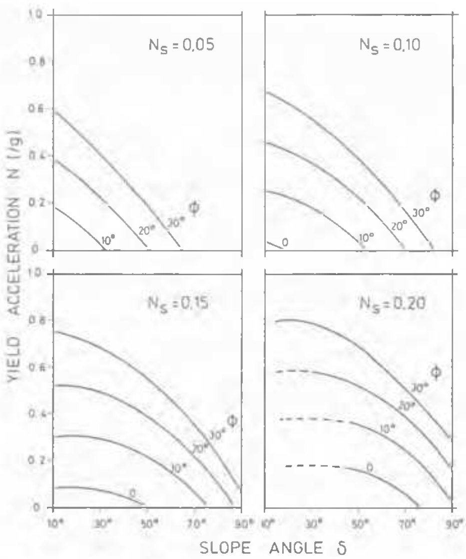

Fig. 3 Yield acceleration for selected slope angles δ and (a_o/γ)-value, i.e. $a_o/\gamma = 0.04$.

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