Dynamic analysis and modelling of machine foundations

Analyses dynamique et modelage de machinerie fondations

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SYNOPSIS The severe consequences originated by machine vibrations upon mechanical equipment and personnel working conditions in industries and factories, together with the advent of new production processes that require machinery of a bigger size, have created an increasing interest for improving the dynamic analysis techniques of foundations and vibration isolation. On this line of research, the present work considers the modelling of soil-foundation systems and the evaluation of the flexibility functions through the method of finite elements. The results are compared with in situ measurements performed upon big machinery foundations and where special emphasis is given to the determination of the dynamic properties and geotechnical characteristics of the foundation soil. The results obtained clearly show the advantages of developing a methodology as the one mentioned that leads to more efficient designs for all types of foundations.

INTRODUCTION

At present, vibration problems in machine foundations are each day more frequent in mining operations and industrial plants. When vibrations exceed certain acceptable limits, it is necessary to incur in repairment and adjustment work usually expensive, and if not done on time, it results in significant damage in the machinery and sometimes it may lead to a partial shutdown of the industrial process.

Normally, machine foundation designs are based on empirical rules that do not consider the dynamic properties and characteristics of foundation soils. These conservative design criteria, besides the fact that they do not represent the real dynamic soil-foundation interaction, they do not permit to estimate the vibration levels of the machines.

Lately, more rational procedures are being used including aspects such as stratification, embedding and non-linear characteristics of the soil, thus permitting a better and more realistic modelling of the problem.

In spite of these analytical techniques such as those given by Wong and Luco (1976), Dasgupta (1979) and Medina (1980), the determination of the dynamic design loads and the selection of adequate safety factors are still scarcely studied subjects. It is also possible to observe a gap between the efforts performed to understand the dynamic soil-foundation interaction phenomenon and the efforts performed to really quantify the magnitudes of the dynamic loads involved. This is agravated when the information supplied by the machine manufacturer is insufficient.

The present study highlights the need for the largest possible amount of practical evidence, periodically instrumenting the foundations, thus permitting a continuous verification of hypothesis and design criteria.

COMPLIANCE FUNCTIONS FOR RIGID FOUNDATIONS

The displacements imposed by a dynamic exitation upon the foundation and the subsoil are normally described in terms of the compliance matrix $[C(\omega)]$. For a rigid foundation as in most machine foundations, the 6 x 6 compliance matrix relates the soil deformations to the forces exerted by the foundation upon the soil. The dynamic loads can be represented by a vector $\{F\} = \{F_x, M_y, F_y, M_x, F_z, M_z\}^T$ of 6 x 1 components, where F_x, F_y, and F_z represent the components resulting forces and M_x, M_y, M_z denote the components of the resulting moments about a point of reference in the foundation. Likewise, the general displacements of the foundation are expressed by $\{U\} = \{U_x, \theta_y, U_y, \theta_x, U_z, \theta_z\}^T$. Then,

$$\{U\} = [C(\omega)]\{F\} \qquad (1)$$

Usually, it is also common to refer to the impedance matrix $[K(\omega)]$, defined as the inverse of the compliance matrix. The compliance (impedance) matrix depends on the excitation frequency, on the geometry of the foundation, on the dynamic properties of the soil, and on the type of soil foundation contact. For a foundation with two vertical planes of symmetry (xz and yz planes) the impedance matrix can be written as

$$[K(\omega)] = Ga \begin{bmatrix} K^{xz}_{HH} & aK^{xz}_{HM} & 0 & 0 & 0 & 0 \\ aK^{xz}_{MH} & a^2K^{xz}_{MM} & 0 & 0 & 0 & 0 \\ 0 & 0 & K^{yz}_{HH} & aK^{yz}_{HM} & 0 & 0 \\ 0 & 0 & aK^{yz}_{MH} & a^2K^{yz}_{MM} & 0 & 0 \\ 0 & 0 & 0 & 0 & K^{zz}_{VV} & 0 \\ 0 & 0 & 0 & 0 & 0 & a^2K^{zz}_{TT} \end{bmatrix} \qquad (2)$$

where G is the dynamic shear modulus and a is a reference length. Each one of the elements appearing in the impedance matrix can be expressed in the form $k + i A_0 c$, where k and c correspond to the normalized stiffness and damping coefficients. The term $A_0 = \omega a/V_s$ represents a dimesionless frequency, with V_s being the velocity of the shear wave. In Eq. (2), $K^{xz}_{MH} = K^{yz}_{HM}$ and $K^{yz}_{MH} = K^{yz}_{HM}$. If the foundation has an arbitrary shape, the impedance matrix is transformed into a full matrix.

Different methodologies have been used to obtain accurately or approximately, the elements of $[K(\omega)]$. Techniques of finite discretization of the continuum have facilitated the study in the cases where the exact analytical solution is difficult to obtain (embedded foundations, stratified soils, etc.). In the following paragraphs we present a method of direct finite elements for the linear soil-foundation interaction.

FORMULATION OF THE PROBLEM OF LINEAR SOIL-FOUNDATION INTERACTION THROUGH THE METHOD OF FINITE AND INFINITE ELEMENTS.

The method consists on modelling the region near to the structure or foundation with finite elements and the distant region with infinite elements. The dynamic stiffness of the region distant from the foundation is represented by infinite elements in the frequency domain. These elements permit to model the geometrical energy radiation, absorbing the compression, shear and Rayleigh incident waves.

The formulation of this technique is common to the classical method of finite elements: discretization of the continuous, assumption of shape functions, error minimization through weighted residuals or variational principles; obtaining a set of algebraic equations characterizing the problem, etc. Accordingly, the proposed technique permits a one-step direct solution of the soil-foundation interaction, still maintaining the flexibility of the classical finite element method (Medina, 1980). In practice, the dynamic soil-foundation interaction problem is generally non-linear (non-elastic), specially for the foundation soil, the present formulation, is restricted to elastic or viscoelastic linear tridimensional systems.

Figure 1 shows the discretization of an embedded cilindrical foundation into finite and infinite elements.

The displacements U^e in both types of elements are approximated by

$$U^e(x) = N^e(x) r^e \qquad (3)$$

where $N^e(x)$ contains the assumed shape functions in point x within element e, with

$$N^e(x) = \begin{bmatrix} N_u(x) & 0 & 0 \\ 0 & N_v(x) & 0 \\ 0 & 0 & N_w(x) \end{bmatrix}^e \qquad (4)$$

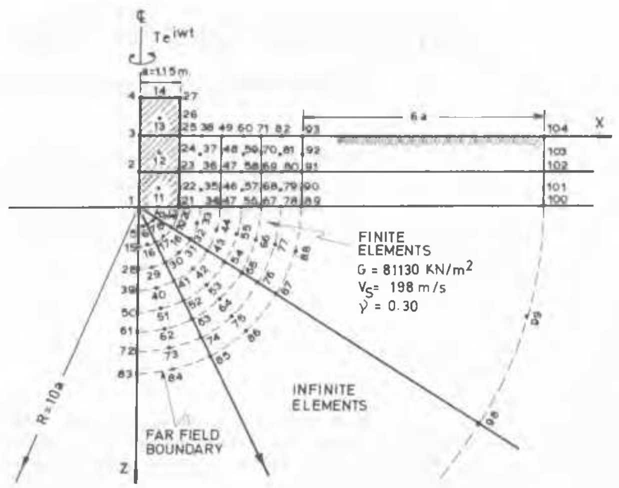

Fig. 1 Modelling of embedded foundation through finite and infinite elements.

$N^e_y(x)$ contains the assumed shape functions at point x within element e corresponding to the displacements component y(y=u,v,w).

For finite elements

$$N^e_u(x) = N^e_v(x) = N^e_w(x) \qquad (5)$$

Based on the linearity of the model, the system's response can be obtained in two steps. In the first step the response of the frequency domain for harmonic solicitations of the $e^{i\omega t}$ type is obtained. In the second step the time response through Fourier synthesis over the frequency range present in the solicitation is obtained. Thus, for a system subjected to dynamic excitations that can be represented by a solicitation vector $P(\omega)$, the solution of displacements of the system as a function of frequency can be obtained from

$$K^*(\omega)\, r(\omega) = P(\omega) \qquad (6)$$

where

$$K^*(\omega) = K(\omega) + i\omega C(\omega) - \omega^2 M(\omega) \qquad (7)$$

is defined as the dynamic stiffness matrix of the system. K, C and M are the stiffness viscous damping and mass matrices, respectively, defined in the usual manner. For example, for element e, the rigidity and consistent mass matrices are

$$K^e = \int_{V^e} [B^e]^T D^e B^e \, dV \qquad (8)$$

and

$$M^e = \int_{V^e} [N^e]^T \rho^e N^e \, dV \qquad (9)$$

respectively, where ρ^e is the mass density of the element, D^e is the constitutive relationship matrix and $[B^e]^T$ makes a relation of deformations with nodal displacements within the elements, defined as in Eq. 10.

Hence, the problem of soil-foundation interaction, is reduced to find the displacements $r(\omega)$ of the system, as a function of the frequency for dynamic excitations that can be represented by a Fourier amplitude excitation vector P (ω).

$$[B^e]^T = \begin{bmatrix} \partial N_u/\partial x & 0 & 0 & \partial N_u/\partial y & 0 & \partial N_u/\partial z \\ 0 & \partial N_v/\partial y & 0 & \partial N_v/\partial x & \partial N_v/\partial z & 0 \\ 0 & 0 & \partial N_w/\partial z & 0 & \partial N_w/\partial y & \partial N_w/\partial x \end{bmatrix}^e \qquad (10)$$

Nevertheless, the infinite elements require selecting the shape functions, so as to adequately represent the different types of waves being propagated through the medium (Medina, 1980).

OBSERVED AND PREDICTED RESPONSE OF A FOUNDATION: APLICATION EXAMPLE

The installation of a new grinding unit of the National Cement Industry (Industria Nacional de Cemento) required accurate subsoil studies that would permit an adequate design of the respective foundations.

Based on a complete geotechnical survey of the sector, a model of the subsoil with the global stratigraphy of the area was obtained and the mechanical and dynamic properties were determined in field tests. Parallel to that, vibration amplitude and frequency measurements were performed on foundations of neighbor grinding units that were similar to the new projected machinery. This information not only permitted to model and perform the dynamic analysis of the foundations but also permitted to verify the adequacy of the model presented.

Geotechnically, the area of study is a sedimentary deposit of 15 m. of fine and intermediate sands, on top of a base of marine and shell sands that, in turn, rests on sandstone at a depth of approximately 17-18 m. The ground water table has been detected at a depth of 2 m. during a time that can be considered as with the minimum amount of water.

Figure 2 summarizes the soil profile where the upper 2 meters of slightly silty, moderately compacted fine sand are underlain by a uniform layer or dense medium and fine sands.

According to the model shown in Fig. 2, the limit pressure P_L (Menard, 1975) would be in the order of 1800 kN/m^2 or higher; and the modulus E_m would have a representative value of 8000 kN/m^2. Between 13 and 14.5 m. depth the sand turns very fine and silty, sometimes turning into a nonplastic very silty sand. Under the latter there is fine sand with a big amount of marine shell sands and below, cemented sandstone, allowing high recovery samples, an RQD = 87% as an average.

Crosshole and Downhole tests were performed at the site of installation of the new machinery, in order to determine the dynamic properties of the foundation soils.

In order to visualize the set of geophysical data obtained, Fig. 3 shows graphs of the P and S waves velocities given together with

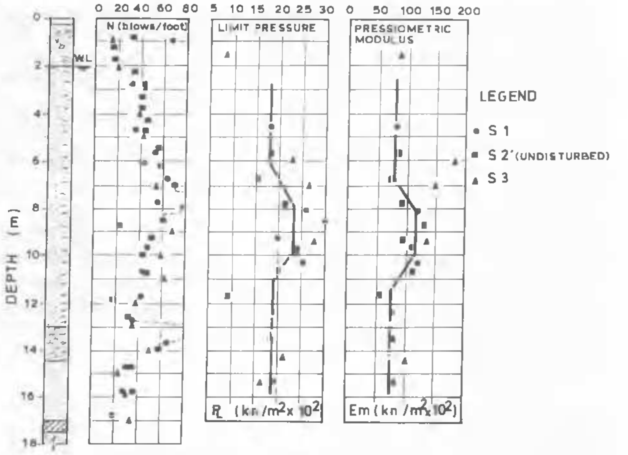

Fig. 2 Summary of Blow Counts, Limit Pressures and Pressiometric Modulus.

the Poisson's ratio and shear modulus inferred from them.

The modelling through finite and infinite elements used is ilustrated in Fig. 1. Such mesh has 104 nodes and contains 21 conventional axisymmetric elements with 6 or 9 nodes each one, and 5 infinite elements.

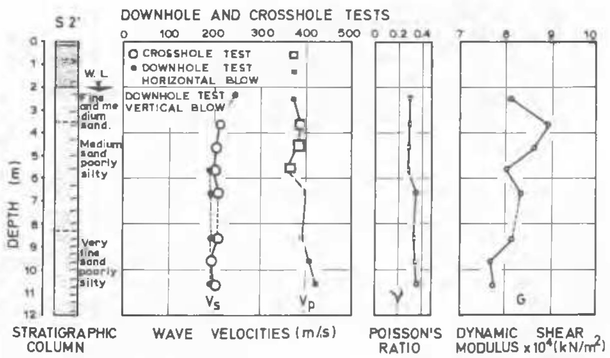

Fig. 3 Depth Distribution of Wave Velocities, Poisson's Ratio and Dynamic Shear Modulus.

The modelling parameters are summarized in Table I. The subsoil dynamic properties have not been corrected for the "deformation level" because the strain magnitudes induced by the Crosshole and Downhole tests are similar to those affecting the subsoil during the machinery operation (10^{-3}%).

TABLE I
Modelling Parameters

STRATUM	V_s	γ	ρ	G	ν	E	f
	m/s	kN/m^3	kg/m^3	kN/m^2	-	kN/m^2	Hz
1	198	20.3	2030	81130	0.30	210940	38
2	209	20.3	2030	89940	0.29	232040	38

The measurements taken on foundations with rheology and vibration characteristics similar to those proyected are shown in Fig. 4., where the relative influence to each displacement component can be observed. Likewise, it was possible to verify that there was a pronounced main vibration frequency common to all the orientations with an average of 38 Hz. In the same manner, the vibration records (see the upper right angle in Fig. 4) showed a phase delay of 10 ms. between one and other measurement direction. This practical information was incorporated in the modelling at the moment of assigning the working frecuency. Thereby, the modelling considered one stratum, characterized by the properties of Stratum 1 on Table I.

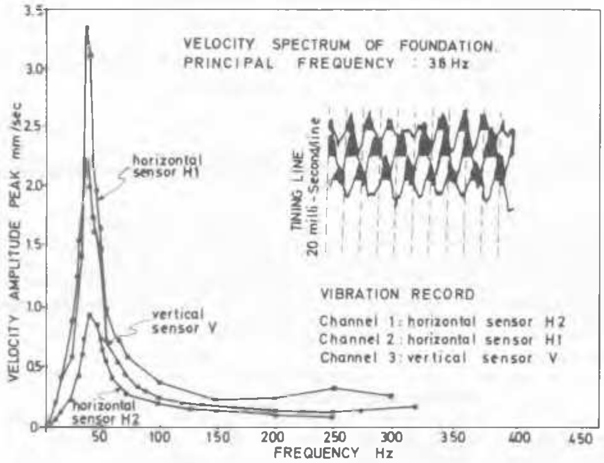

Fig. 4 Wave Velocity Spectrum in Vertical and Horizontal Directions.

Figure 5 shows an example of the results of the torsional vibration modes of an embedded foundation. The figure shows the real and imaginary parts of the torsional compliance function as well as the total angular displacement of the foundation for different magnitudes of the dynamic torque T. Total angular displacement is given by

$$\alpha = T \; |C_{TT} \; (A_0)| \tag{11}$$

Additionally, the experimental angular measurements calculated on the basis of the horizontal displacement amplitudes, measured at the end points of the foundation have been included.

DISCUSSION AND CONCLUSIONS

The methodology described above permits the calculation of dynamic excitations of foundations, by using a modelling which should include the latest developments of dynamic soil-foundation interaction's theory as well as measuring in situ with modern technologies. T values, necessary for the design of foundations were obtained by using Eq. 11, set for A_0 = 1.38 and α determined on a similar foundation.

According to the trend shown by the angular displacement curve α, there would be an amplification peak around the dimesionless frequency A_o = 0.9. The usual foundation design practice advises to go far from this zone, preferably acting upon the geometry of the foundation, so as to reduce the normal working vibrations.

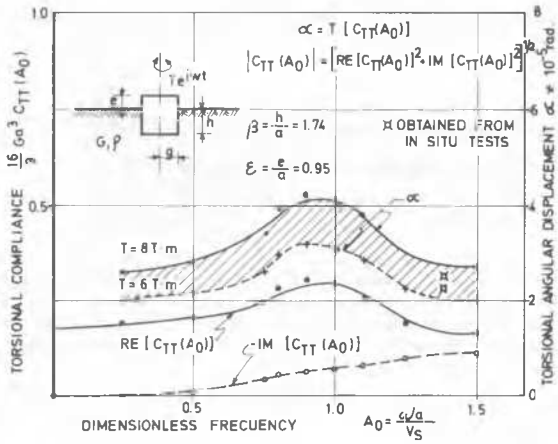

Fig. 5 Flexibility and Torsional Angular Displacement for Different Magnitudes of Dynamic Torque.

The type of excitation and embedding ratio chosen β = 1.74 corresponded to a real situation, nevertheless smaller embedding increases the torsional angular displacement.

The design approach presented here shows good agreement with the experimental data proving to be an efficient method for the design of isolated foundations and a tool for a better understanding of the dynamic interaction of machinery batteries with different vibration patterns common in industrial and mining operations.

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