Determination of stability characteristics

Détermination des valeurs de portances

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SYNOPSIS Equations for the bearing capacity factors N_q und N_c are derived on basis of the slip line method, taking into account an inclined load, an inclined foundation base, and an inclined ground surface. It is analysed, how far it is possible, to divide these factors into base values $N_{q,o}$ und $N_{c,o}$ on the one hand and into inclination factors for load, the foundation base, and the ground surface on the other hand. The bearing capacity factors N_γ are calculated numerically. The limitations of the bearing capacity equation are considered.

PRELIMINARY REMARK

This contribution cannot give a complete description of the calculation method outlined below. Interested readers are referred to another publication of the author (PREGL, 1983) which also contains similar derivations for active and passive earth pressure.

INTRODUCTION

The methods to determine the bearing capacity of shallow foundations given in the literature may be arranged roughly into the following four groups:

- The limit analysis with static discontinuities determines statically admissible stress fields which may be separated by static discontinuities.
- The limit analysis with kinematic discontinuities considers failure mechanisms consisting of rigid sliding blocks (line ruptures).
- The slip line method is based on the concept of continuous plastic areas (zone ruptures).
- The limit equilibrium methods provide approximations for rigid block failure mechanisms.

So far there has been no clear-cut decision for any one method, possibly because precise results for testing the theoretical methods have become available only recently (cf. MUHS/WEISS, 1975; BÄTCKE/SIMONS, 1983). Yet such results can supply only some indications, due to the many factors involved. From the theoretical point of view, the method using the fewest approximations must be given preference.

SLIP LINE METHOD

The elementary principles of this method were established by, i.a., KÖTTER (1903) and SOKOLOVSKI (1960). For a static solution, we begin with the equilibrium equations

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{zx}}{\partial z} - \gamma_x = 0$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \sigma_z}{\partial z} - \gamma_z = 0$$

and Coulomb's failure condition (for c = 0)

$$\frac{\sigma_x + \sigma_z}{2} \sin \phi - \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2} = 0$$

According to the Lower-Bound Theorem, we thus get the lower bounds for the limit load. The limit loads calculated in this way will thus not be greater than the actual values. When applying this method to the design of shallow foundations, we get safe designs since the lower bound limit load is acting as the ultimate possible bearing pressure.

The bearing capacity of shallow foundations can be observed by means of the plastic wedge model (Fig. 1).

We assume that one boundary of this wedge (the "first boundary") has a known and evenly distributed load acting on it:

$${}^1\sigma = {}^1a$$

$${}^1\tau = {}^1\sigma.\tan({}^1\delta)$$

${}^1a$ normal stress at first boundary

${}^1\delta$ inclination angle of load at first boundary

${}^2\delta$ inclination angle of friction stress at second boundary

a adhesion at second boundary

Thus, static preconditions are given for the first boundary. The respective loads are identified by a prefixed index ^{1}. The load at the other wedge boundary (the "second boundary") depends on the movements of a body contacting the wedge at this side. We assume that this load satisfies

$$^2\tau = a + {^2\sigma}.\tan(^2\delta)$$

The stresses for the second boundary are identified by a prefixed index ^{2}. Accordingly, kinematic and static preconditions are given for this boundary. The evenly distributed weight γ_z (= γ) is acting within the wedge.

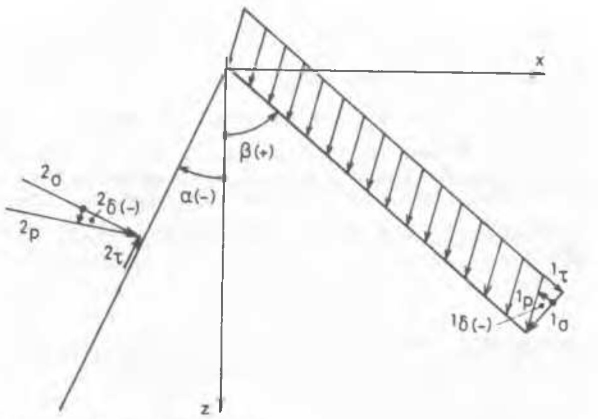
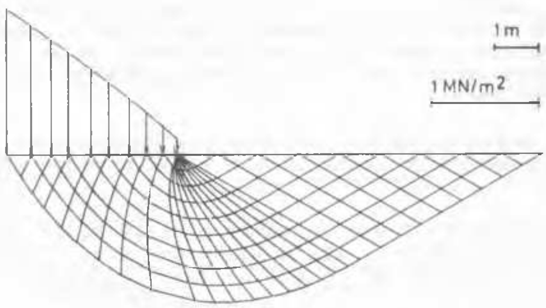


Fig. 1: Plastic wedge

The static solution is reached by looking at three separate areas (areas 1-3), and represented by a field of so-called static characteristics. For each of these areas we start with known stresses at their boundaries. The elements located along the characteristics meet the failure condition $\tau = c + \sigma'.\tan\phi$. Fig. 2 shows an example of presenting the results of such a calculation.



$\gamma = 20$ kN/m^3, $c = 0$, $\phi = 30^0$,
$\alpha = -90^0$, $\beta = 90^0$, $^1p = 10$ kN/m^2, $^1\delta = {^2\delta} = 0^0$

Fig. 2: Contact pressure distribution on reaching the bearing capacity and associated static field of characteristics.

EQUATION OF BEARING CAPACITY

If, for calculating the bearing capacity, we start with the three-term equation

$$q_g/\cos\alpha = \gamma x N_\gamma + qN_q + cN_c \qquad (1)$$

(Fig. 3), we make the following three, usually approximated, assumptions:

- Along the foundation base, the stresses from weight show a linear increase, while stresses due to $q = \gamma.D.\cos\beta$ (D = foundation depth) and c remain constant.
- There is no interdependence between the actions of γ, q, and c (superposition principle).
- The individual factors are independent of the respective impact quantities γ, q, and c; this means that the factors are only functions of the remaining variables, i.e. the angles ϕ, α, β and δ_s.

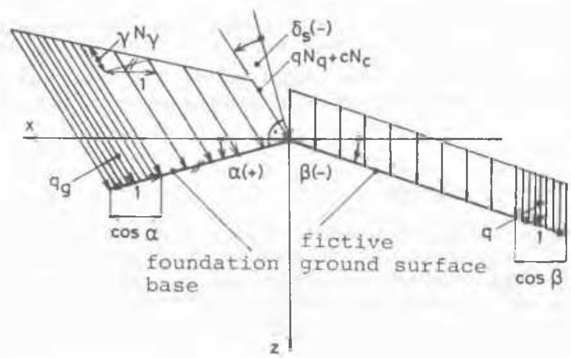


Fig. 3: Diagram for the bearing capacity equation

The factors may be determined in several ways:

- Tables (e.g. DIN 4017 T.1; VSS, 1966; PREGL/KRISTÖFL, 1983),
- analytical functions (cf. equations 9 and 16),
- factors for a basic case and inclination factors i_γ, i_q, i_c', and i_c'', ground factors g_γ, g_q, and g_c' and base inclination factors t_γ, t_q, and t_c'.

In the last case, equation 1 becomes

$$q_g/\cos\alpha = xN_{\gamma,o}i_\gamma g_\gamma t_\gamma + qN_{q,o}i_q g_q t_q +$$
$$+ c(N'_{c,o}i'_c g'_c t'_c + N''_{c,o}i''_c) \qquad (2)$$

Here the inclination factors take into account the impact of inclination δ_s of the contact pressure (this angle corresponds to angle $^2\delta$ of Fig. 1), the ground factors incorporate the impact of ground inclination β, and the base inclination factors include the impact of base inclination α. These factors depend only on the respective impact factor (δ_s, β or α) and the friction angle ϕ. With this method, the number of tables required is reduced to seven, as compared to the numerous tables for the bearing capacity factors N_γ, N_q and N_c. However, the factors i_γ, g_γ, and t_γ are only approximations.

The following derivations are valid only for such cases, where the first and third plastic areas do not intersect, i.e. if we get a central fan, which applies to capacity calculations in any case.

The factors N_q and N_c may be deduced from the stresses at the singular point (point of origin of the system of coordinates in Fig. 1), while the factors N_γ are determined from the distribution of stresses q_q along the foundation base. The computation is made for constant values of α, β, ϕ, and δ_s.

BEARING CAPACITY FACTOR N_Q

The bearing capacity factor N_q is derived for c = 0. The origin of coordinates as shown in Fig. 1 is a singular point, i.e., individual points can be distinguished which, while showing the same coordinates (0,0), have different states of stress, due to the direct stress $\sigma_m = (\sigma_1+\sigma_2)/2$ and the direction χ_σ of the larger main stress to the z-axis. Of the infinite number of states, we are interested only in those associated with stresses ^1p and ^2p at the first and second boundaries.

According to the relations for conjugate stresses p and p' in the plastic state (Fig. 4), we have for the normal stress σ_m at the singular point for the first boundary (cf. Fig. 1 and 3)

$$^1\sigma_m = \frac{^1p}{\cos(^1\delta) - \sin\phi.\cos(^1\gamma)} \qquad (3)$$

and for the second boundary

$$^2\sigma_m = \frac{^2p}{\cos(^2\delta) + \sin\phi.\cos(^2\gamma)} \qquad (4)$$

where

$$\sin(^1\gamma) = \frac{\sin(^1\delta)}{\sin\phi}$$

$$\sin(^2\gamma) = \frac{\sin(^2\delta)}{\sin\phi}$$

and $\quad ^1\delta = \beta - \pi/2$

The inclination of the major principal stress at the first boundary is calculated from

$$^1\chi_\sigma = \beta + \frac{^1\gamma - {}^1\delta}{2} \qquad (5)$$

and at the second boundary from

$$^2\chi_\sigma = \alpha - \frac{^2\gamma + {}^2\delta}{2} + \frac{\pi}{2} \qquad (6)$$

The connection between normal stresses $^1\sigma_m$ and $^2\sigma_m$ here is given by (cf. G. de JOSSELIN de JONG, 1979)

$$^2\sigma_m = {}^1\sigma_m e^{-2\Delta\chi_\sigma \tan\phi} \qquad (7)$$

where

$$\Delta\chi_\sigma = \alpha - \beta - \frac{^1\gamma - {}^1\delta}{2} - \frac{^2\gamma - {}^2\delta}{2} + \frac{\pi}{2} \qquad (8)$$

If equations 3 and 4 are incorporated into equation 7, which is then resolved for ^2p, we get

$$^2p = {}^1pN_q(\phi,\alpha,\beta,{}^2\delta) \qquad (9)$$

with

$$N_q(\phi,\alpha,\beta,{}^2\delta) = \frac{\cos(^2\delta)+\sin\phi\cos(^2\gamma)}{\cos(^1\delta)-\sin\phi\cos(^1\gamma)}\, e^{*}$$

$$* = -2(\alpha-\beta-\frac{^1\gamma-{}^1\delta}{2} - \frac{^2\gamma-{}^2\delta}{2} + \frac{\pi}{2})\tan\phi$$

If a function y of the variables $x_1, x_2, \ldots, x_n$ can be out into the form

$$y = f_1(x_1)f_2(x_2,x_1) \ldots f_n(x_n,x_1) \qquad (10)$$

then we also have

$$y = f_{1,o}(x_1,x_{2,o},\ldots x_{n,o})g_2(x_2,x_1,x_{2,o},\ldots x_{n,o}) \ldots g_n(x_n,x_1,x_{2,o},\ldots,x_{n,o}) \qquad (11)$$

$f_1, f_2, \ldots f_n$ are functions of the variables given in parenthesis. $f_{1,o}$ is the function calculated from equation 10, if we substitute the variables $x_2, x_3, \ldots, x_n$ by the constant values $x_{2,o}, x_{3,o}, \ldots, x_{n,o}$ characteristic for a certain basic case; then this function contains only the variable x_1. The functions g_i, like functions f_i, show two variables each, namely x_1 and x_i; and in addition they contain the constant values $x_{2,o}, x_{3,o}, \ldots, x_{n,o}$. These functions characterize the connection between any extended case with two variables and the basic case with only one variable.

Since equation 9 can be written in the form of equation 10, we may factorize N_q according to equation 2. The basic case to be introduced for the bearing capacity is defined by $\alpha = -\pi/2$ (horizontal foundation base), $^2\delta = 0$ (vertical contact pressure), $\beta = \pi/2$ and $^1\delta = 0$ (horizontal ground surface). From equation 9 we thus have

$$N_{q,o}(\phi) = \frac{1 + \sin\phi}{1 - \sin\phi}\, e^{\pi.\tan\phi} \qquad (12)$$

The inclination factor is determined by

$$i_q = \frac{N_q(\phi,{}^2\delta)}{N_{q,o}(\phi)} = \frac{\cos(^2\delta) + \sin\phi.\cos(^2\gamma)}{1 + \sin\phi}\, e^{(^2\gamma + {}^2\delta)\tan\phi}$$

the surface inclination factor by

$$g_q = \frac{N_q(\phi,\beta)}{N_{q,o}(\phi)} = \frac{1 - \sin\phi}{\sin\beta - \sin\phi.\cos(^1\gamma)}\, e^{(\beta + {}^1\gamma - \pi/2)\tan\phi}$$

and the base inclination factor by

$$t_q = \frac{N_q(\phi,\alpha)}{N_{q,o}(\phi)} = e^{-(2\alpha + \pi)\tan\phi}$$

BEARING CAPACITY FACTOR N_C

If calculating the normal stresses $^1\sigma_m$ and $^2\sigma_m$ at the singular point, on assuming a cohesion of c > 0, we get the equation in the form of

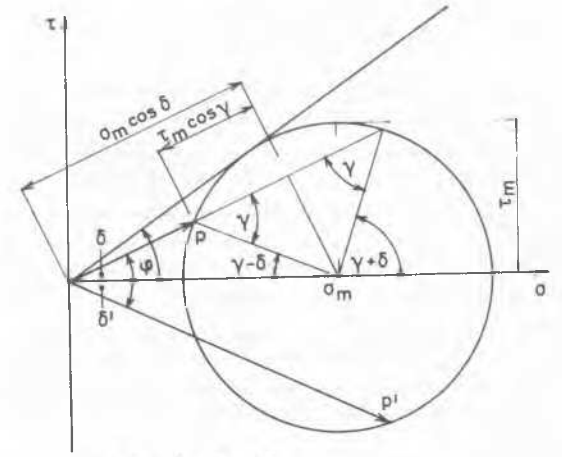
$$^2p = cN_c(\phi,\alpha,\beta,{}^2\delta)$$

only if we put ${}^1p = 0$ and $a/c = \tan({}^2\delta)/\tan\phi$. With these assumptions

$$N_c = \cot\phi\left[\frac{\cos({}^2\delta)+\sin\phi\cos({}^2\gamma)}{1-\sin\phi}\, e^{*} - \frac{1}{\cos({}^2\delta)}\right] \quad (13)$$

$$* = -2\left(\alpha-\beta-\frac{{}^2\gamma+{}^2\delta}{2}+\frac{\pi}{2}\right)\tan\phi$$

Since this equation is valid only for the above assumptions, the bearing capacity factor N_c is exact only for this case. The independance from the quantities c and ${}^1p$ of the bearing capacity factor N_c assumed on applying equation 1 consequently is an approximation.



Für p' > p gilt

$p' = \sigma_m(\cos\delta + \sin\phi.\cos\gamma)$

$p = \sigma_m(\cos\delta - \sin\phi.\cos\gamma)$

mit $\sin\gamma = \sin\delta/\sin\phi$

Fig. 4: Conjugate stresses

The bracket element of equation 13 comprises two terms, which means that this equation can be factorized into the form of equation 10 (similar to equation 9) only individually for each of the two terms:

$$N_c = N_c' + N_c'' = N_{c,o}' i_c' g_c' t_c' + N_{c,o}'' i_c''$$

For the basic case we have

$$N_{c,o}' = \cot\phi . N_{q,o}$$

and

$$N_{c,o}'' = -\cot\phi$$

Inclination factors:

$$i_c' = \frac{N_c'(\phi,{}^2\delta)}{N_{c,o}'(\phi)} = i_q$$

Surface inclination factor:

$$g_c' = \frac{N_c'(\phi,\beta)}{N_{c,o}'(\phi)} = e^{(2\beta-\pi)\tan\phi}$$

Base inclination factor:

$$t_c' = \frac{N_c'(\phi,\alpha)}{N_{c,o}'(\phi)} = t_q$$

BEARING CAPACITY FACTOR N_γ

It is not possible to derive any axact analytical expression for the bearing capacity factor N_γ, i.e. it must be computed numerically.

We have computed tables for the bearing capacity factors in the manner outlined above (PREGL/KRISTÖFL, 1983).

A comparison of the surface inclination factors g_γ and g_q with the values derived from model tests by BÄTCKE/SIMONS (1983) was published separately (PREGL, 1984).

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