The rheology of subsidence and swelling soils

La rhéologie des sols affaissants et gonflants

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SYNOPSIS. The paper offers the results of the experimental studies made to establish the regularities of the rheology of loess subsidence and swelling clayey soils. It shows that despite the substantial variation in the nature of deformability of these soils, when moistened, the temporal variation of their stressed-strained state obeys to Boltzmann-Volterr's theory of linear hereditary creep. The paper suggests the equation of the deflection of the beam-type foundation rested on subsidence and swelling soils, written with due regard for the rheological peculiarities of their deformability. A simple procedure proposed for the experimental studies of long-term compressions and softenings of swelling soils makes it possible to establish the regularities of their creep.

1. Rheology of Subsidence and Swelling Soils

The deformations caused by subsidence in the loess soils and those of swelling in the clayey soils represent the non-equilibrium rheological processes which result in the temporal expansion of deformations at constant moisture content and pressure. In order to interpret the regularities of these processes the long-term studies have been carried out at the Soil Mechanics Department of the Azerbaijan Civil Engineering Institute, and as the result of these studies there have been obtained the families of subsidence and swelling variation curves at constant values of compacting pressures, the stress variation curves at constant values of deformations caused by subsidence and those of swelling, as well as the isochronic curves of subsidence and swelling /1,2,3/. The said regularities have been constructed for various extents of moisture content, constant throughout the experiment, i.e. from the initial moisture content to the water-saturated state of subsidence and swelling soils.

The validity of the results of the experimental studies has made it possible to interpret the stressed-strained state of loess and clayey soils, when moistened, with the appropriate theories of creep. The possibility of applying the general interpretation of the theory of ageing proposed by J.N.Rabotnov has been considered as well.

The further study of the problem in question has shown that Boltzmann-Volterr's theory of linear hereditary creep /4/ seems to be most suitable for the general approach to the solution of the problem of rheology of subsidence and swelling soils:

$$\delta(t) = \pm \frac{1}{E_0}\left[\sigma(t) + \int_0^t K(t-\tau)\sigma(\tau)d\tau\right] \qquad (1)$$

with the nucleus of the Abel type $K(t-\tau) = \beta(t-\tau)^{-\alpha}$ /4/. With the constant values of compacting pressure and moisture content of loess and clayey soils the variation of temporal relative subsidence and swelling can, according to (1), be expressed by a common formula:

$$\delta(t) = \pm \frac{\sigma_0}{E_0}\left(1 + \frac{\beta}{1-\alpha} t^{1-\alpha}\right) \qquad (2)$$

Figure 1 illustrates the experimental points and the theoretical curves of subsidence and swelling constructed in accordance with the formula (2) for various values of compacting pressures. Similar comparisons have been made in the study of all the values of moisture content of loess and clayey soils used in the experiments. The most reasonable error of the formula (1) does not exceed 10-20%.

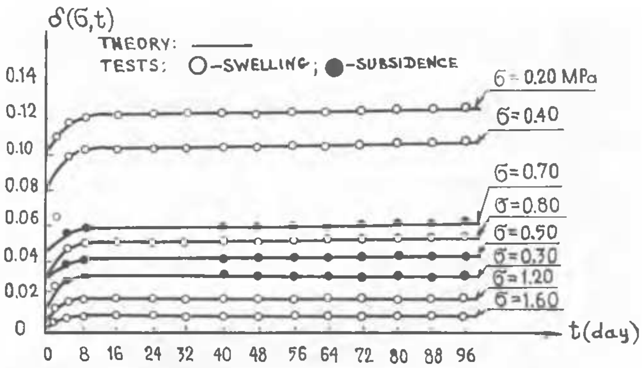

Fig.1. Theoretical and Experimental Relativeship of Temporal Variation of Relative deformations caused by subsidence and Swelling.

Figure 2 shows the curves of compacting pressure variation in the course of subsidence and swelling. The pressure variation curves have made it possible, in accordance with $R(t) = \pm \frac{1}{\delta E_o} \frac{d\sigma(t)}{dt}$ which proceeds from the integral relation of the theory under consideration:

$$\sigma(t) = E_o \left[\delta(t) \pm \int_0^t R(t-\tau)\delta(\tau)d\tau \right] \qquad (3)$$

to construct the resolvent curves of Kernel of creep R(t) at various extents of soil moisture content (Fig.3). Similar curves have been constructed on the basis of the deformability law accepted. The comparison of these curves proves that the application of the said law

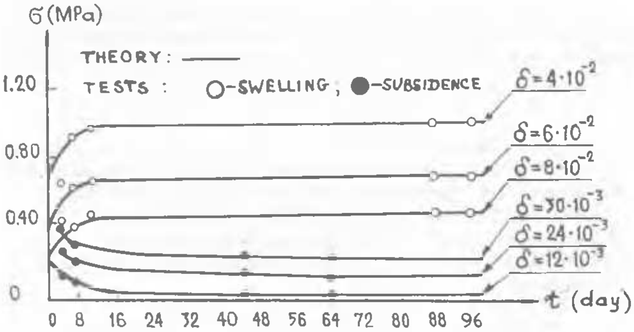

Fig.2. Compacting Pressure Variation Curves for Subsidence and Swelling

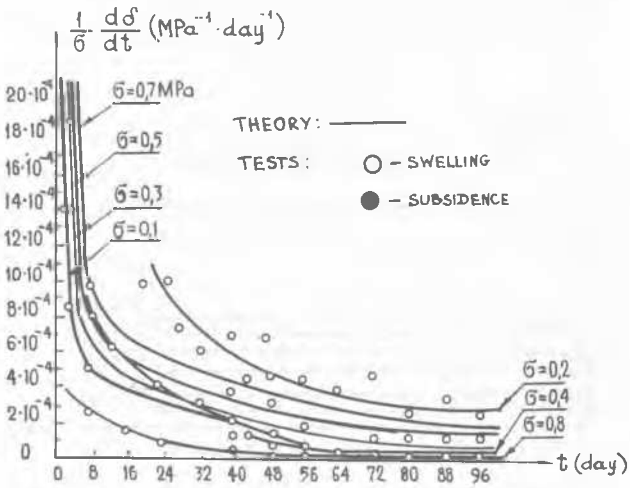

Fig.3. Resolvent Curves of Kernel of Creep (for Subsidence and Swelling) at Various Extents of Moisture Content.

of compacting pressure variation (3) also produces quite satisfactory results. Finally, it has been established that the regularities of the resolvent variation of the accepted Kernel of creep are in full conformity with the experimental data at all the considered values of moisture content of subsidence and swelling soils, and that proves that the accepted type of the Kernel of creep correlates well with its resolvent.

2. Deflection of the Beam-Type Foundations Rested on Subsidence and Swelling Soils

Considered below is the deflection of an elastic beam slab of finite stiffness and length, subjected to lateral load distribution:

$$\frac{EJ}{1-\nu_o^2} \frac{\partial^4 y(x,t)}{\partial x^4} = q(x) - \sigma(x,t) \qquad (4)$$

The behaviour of the foundation $\sigma(x,t)$ composed of loess subsidence soil of the second type or of swelling clayey soil is to be assumed as that obeying to the accepted rheological model:

$$y(x,t) - W(x,t) = \frac{1}{K(x,t)} \left[\sigma(x,t) \pm \beta \int_0^t \sigma(x,\tau)(t-\tau)^{-\alpha} d\tau \right] \qquad (5)$$

where k(x,t) is the variable-along-the-foundation instantaneous bed factor; w(x,t) is the equation of the subsiding or swelling contact surface of the foundation. Equations (4) and (5) make it possible to reduce the problem to the solution of the nonuniform integro-differential equation of the fourth order with a variable coefficient:

$$y(x,t) = W(x,t) + \frac{q(x)}{k(x,t)} \left[1 \pm \frac{\beta}{1-\alpha} t^{1-\alpha} \right] - \frac{EJ}{k(x,t)(1-\nu_o^2)} \left[\frac{\partial^4 y(x,t)}{\partial x^4} \pm \beta \int_0^t \frac{\partial^4 y(x,\tau)}{\partial x^4} (t-\tau)^{-\alpha} d\tau \right] \qquad (6)$$

Equation (6) can be solved by way of successive approximations, with the solution of the equation (7) being assumed as a zero approximation:

$$\frac{EJ}{1-\nu_o^2} \frac{\partial^4 y(x,0)}{\partial x^4} + K_o y(x,0) = q(x) + K_o W(x,0) \qquad (7)$$

which determines the shape of the deflection of the foundation rested on an instantaneously subsiding or swelling base. Another way of solution lies in the use of the quadrature formula reduced by Gauss, Chebyshev et al., and in the change of the Kernel applying Boubnov-Galerkin's degenerated method and of least squares. The said problem can be approximated to the solution of the equation:

$$\frac{EJ}{1-\nu_o^2} \frac{\partial^4 y(x,t)}{\partial x^4} + \frac{K_o}{1 + \frac{\beta}{1-\alpha} t^{1-\alpha}} \left[y(x,t) - W(x,t) \right] = q(x) \qquad (8)$$

by applying the mean-value theorem to the integral of the right-hand part of the equation (6) and by making use of the peculiarities of the temporal development of subsidence and swelling.

The construction of the general solution of the equation (8) implies no mathematical complications since the unknown function which determines the shape of foundation deflection has turned out to be dependent on time as the parameter of the generalized stiffness factor of the deforming soil of the foundation.

3. Protracted Compressure and Softening of Swelling Soils

The following types of protracted deformation are possible at the one-dimensional compression and softening of swelling clay soils (Mestchyan, 1980a):

a) the compression ($+\varepsilon_{ct}$) of soil with a natural humidity (W_o) under action of any compressional pressure (σ_1);

b) the compression ($+\varepsilon_{ct}$) of a partly or a completely saturated soil under ban of swelling, at $P_1 > \sigma_{s,o}$ (P_1 - external pressure, $\sigma_{s,o}$ - pressure of a free swelling);

c) the softening ($-\varepsilon_{ct}$) water-saturated soil (W_s) under ban of swelling at $P_1 < \sigma_{s,o}$ - in the cases of a partly or a completely unloading;

d) the compression of soil after a partly or a completely swelling;

e) the softening at a partly or a completely unloading of a compressed soil under ban of swelling after wetting;

f) the softening at additional wetting of a compressed soil with natural humidity.

The approximation of the family of creep curves for a swelling soil under action of one-dimensional compressure and softening one can make by the following correlation (Mestchyan, 1980a, 1983) in the two limiting states - state of natural humidity (W_o) and state of a completely saturation under ban of swelling:

$$\varepsilon_{ct} = C_c(t) \cdot F(P_1 - \sigma_{s,o}) \qquad (9)$$

In (9) $C_c(t)$ - is creep measure (creep under unit effective pressure $\sigma_1 = P_1 - \sigma_{s,o} = 1$); $F(P_1 - \sigma_{s,o})$ is function of effective pressure which satisfies the condition $F[(P_1 - \sigma_{s,o}) = 1] = 1$

In particularly, the creep measure c(t) we can present in the form of:

$$C_c(t) = At^m \qquad (10)$$

The correlation between effective pressure and deformations have the form:

$$\varepsilon_{ct} = B(P_1 - \sigma_{s,o})^n \qquad (11)$$

and function $F(P_1 - \sigma_{s,o})$

$$F(P_1 - \sigma_{s,o}) = (P_1 - \sigma_{s,o})^n \qquad (12)$$

where A,B,m,n - are experimental parameters. Taking into account (10, 12) the condition (9) can be written as:

$$\varepsilon_{ct} = At^m(P_1 - \sigma_{s,o})^n \qquad (13)$$

From (13) it follows that when $W = W_o$ and $\sigma_{s,o} = 0$ the soil is compressed by external pressure which is equal to effective pressure ($P_1 = \sigma_1$). When $P_1 = \sigma_{s,o}$ the deformation of a creep vanished when $P_1 > \sigma_{s,o}$ the compression takes place at wetting. If $P_1 < \sigma_{s,o}$ the softening takes place reaching its maximal value at $P_1 = 0$.

In the right-side of Fig.4 the average experi-

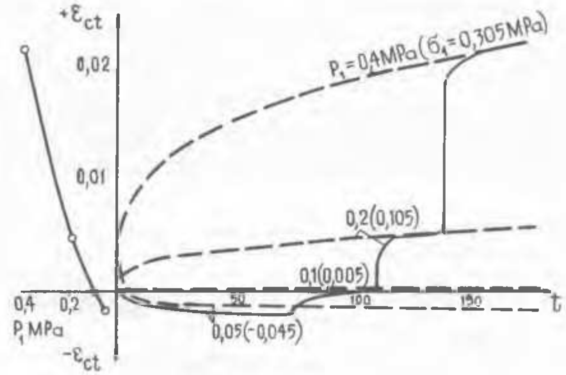

Fig.4. Experimental Creep Curves and Correlation $\varepsilon_{ct} - P_1$ (Continuous Line) and Creep Curves Plotted from (9) t-days.

mental curve of creep of soil is given in the case of an optimal humidity and maximal density (continuous line). The curve is obtained by means of the test of four twin-specimens (W_{opt}= 0.241, $S_{max.}$= 1940 kg/m^3, $S_{d,max.}$= 1560 kg/m^3, $\sigma_{s,o}$ = 0.095 MPa) under a stepped increasing external pressure. The specimens have been wetted after an action of the first step of pressure 0.05 MPa a twenty-four hours later.

As shown in Fig.4 at P_1 = 0.05 MPa first a softening takes place then (after wetting) the soil swelling under action of a negative pressure σ_1 = 0.05-0.095= - 0.045 MPa. At P_1 = 0.1 MPa, $\sigma_{s,o}$ = 0.095 MPa (the second step of a pressure), protracted compression takes place and so as a completely restoration of swelling deformation.

On the basis of soil creep curve in the left-side of Fig.4 the curve of correlation $\varepsilon_{ct} - P_1$ (continuous line) is given, which is approximated by function of type (11):

$$\varepsilon_{ct} = 0{,}103(P_1 - 0{,}095)^{1{,}3177} \qquad (14)$$

and for function of effective pressure:

$$F(P_1 - \sigma_{s,o}) = (P_1 - 0{,}095)^{1{,}3177} \qquad (15)$$

The creep measure, which is obtained by the method of one experimental curve (Mestchyan, 1980b) is presented by the following form:

$$C_c(t) = 0{,}022\, t^{0{,}2993} \qquad (16)$$

Creep curves for different pressures (dotted lines in Fig.4) is given by means of following expression of (9) type:

$$\varepsilon_{ct} = 0{,}022\, t^{0{,}2993} (P_1 - 0{,}095)^{1{,}3177} \quad (17)$$

From (13,17) it follows that under action of the same effective pressures ($\sigma_1 = P_1 - \sigma_{s,o}$) the creep deformations of the soil with natural humidity and of water-saturated soil under ban of swelling practically equal each other. For confirmation of the above-mentioned facts in Fig.5 the creep curves are given (dotted lines) for a soil of a natural structure (W_o = 0.3, ρ_s = 2760 kg/m^3, ρ = 1880 kg/m^3, $\sigma_{s,o}$ = 0.08 MPa). The retractions of tests is nine-times (for $W = W_o$) and three-times (for $W = W_s$). As one can see from Fig.5, the creep curves which are determined under equal values of effective pressure, coinside practically.

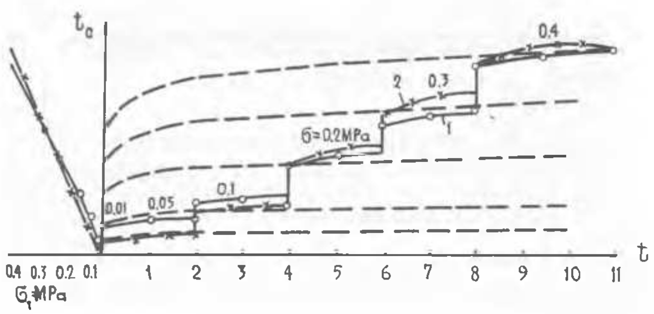

Fig.5. Creep Curves of Swelling Soil with Natural Humidity (9) and Water-Saturated Soil under Ban of Swelling (10), which are Determined under Action of Stepped Increasing (Equal Steps) Effective Pressure. t-Days. Curve of Correlation $\varepsilon_{ct} - \sigma_1$ and Its Approximation (Dotted Lines).

From it follows that the creep of swelling soil with natural humidity may be determined from results of tests of saturated soils under ban of swelling.

In Fig.5 by means of the dotted lines the curve of functions $\varepsilon_{ct} - \sigma_1$ and $\varepsilon_{ct} - t$ are shown for different effective pressures which are corresponded to correlations of (11) and (13) types. In order to describe the creep process of swelling soil for variable respect to time external pressure and humidity it is necessary to know a changeability of the swelling pressure in depend of.

Then the correlation (13) may be written as:

$$\varepsilon_{ct} = A t^m [P_{1,t} - \sigma_{s,o}(W_t)] \quad (18)$$

In conclusion note that the obtained results are just only in case of tests of thin specimens of soils under two-side using of void water, when compression of a soil on the whole depends from creep of its skeleton.

REFERENCES

1. Mustafayev A.A.,Chigniev G.D.,Nazirova G.P. On the Rheological Nature of Deformations of Swelling in Clayey Soils and Those Caused by Subsidence in Loess Soils. - "Bases, Foundations and Soil Mechanics", No.5, 1974.
2. Mustafayev A.A.,Chigniev G.D. The Relationship of Rheological Deformation of Swelling in Clayey Soils and the Methods of Forecasting It for the Foundation of Structures. - Reprinted from the Proc. of the First Nat.Sympos.on Expansive Soils. Held at H.B.T.I.,Kanpur, 19-21 Dec., 1977.
3. Mustafayev A.A.,Chigniev G.D. Rheology of Swelling Soils and Deformation Forecast . - Proc.of the IV Int.Conf.on Expansive Soils (USA), Denever, 16-18 June, 1980.
4. Mustafayev A.A.,El-Hansy R.M. The Forecast of the Clayey Soils Deformations of Swelling at the Foundation of Structures. - Proc.of the Vth Int.Conf.on Swelling Clayey Soils, Adelaide, Australia, May 1984.
5. Mestchyan S.R. Methodics of Determination of Compressional Creep of Swelling Clay Soils. - Proc. of the Third All-Union Sympos.on Soil Rheology, Yerevan,1980a, pp.279-284 (in Russian).
6. Mestchyan S.R. Methodics of Creep Deformation of One-Dimensional Compressure of Swelling Clay Soils at Primary Natural Humidity. - Dokl.Akad.Nauk Armyanskoy SSR, 1983, v.76, No.2, pp.65-70 (in Russian).
7. Mestchyan S.R. General Methodical Problems of Investigation of Clay Soils Creep. - Proc.of the Third All-Union Sympos.on Soil Rheology, Yerevan, 1980b, pp. 88-107 (in Russian).