Session 1: Soil mechanics – Property characterization and analysis procedures
Séance 1: Mécanique des sols – Caractérisation des propriétés et procédures de calcul

Introduction to Session 1 A: Constitutive relationships for soil behaviour

T.ADACHI, G.GUDEHUS & H.POOROOSHASB

The work of the ISSMFE Subcommittee on Constitutive Laws is documented in the volume

Constitutive Laws of Soils

published by the Japanese Society of Soil Mechanics and Foundation Engineering 1985. Professor Murayama, Chairman of ISSMFE Subcommittee, could not participate in Session 1 A, unfortunately. He had begged Prof. Gudehus to act as Chairman of Session 1 A, and Prof. Poorooshasb to act as Discussion Leader. Prof. Adachi was Secretary of the subcommittee.

It was stipulated to concentrate on two aspects:
- determination of constitutive parameters
- experience with application.

The following experts were invited to deliver short contributions:
Prof. Matsuoka, Prof. Lade, Dr. Kolymbas,
Prof. Oka, Prof. Frydman, Prof. Juarez-Badillo.
The subsequent discussion is open.

Chairman's report, Session 1A

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The work of the Japanese task group led by Professor Murayama is greatfully acknowledged. Research experts are referred to their Report.

The discussion has again shown that the field of Constitutive Laws needs further work and clarification. We are still far away from an agreement on adequate formulations and respective tests. It appears that the various elasto-plastic and rate-type relations can cover the same observed behaviour. Sophistications are needed for reloading and loading to the side (among other effects).

The cry for simple relations must not be ignored. Constitutive laws should contain a low number of parameters which can be determined in routine tests. Physical and mathematical restrictions should be observed so that the boundary value problems are well-posed.

Session 1 A had about 100 participants.

TRIADH and its application: Calculating to predict the behaviour of Verney Dam during first impounding
TRIADH et son application: Calcul prédictif du comportement à la mise en eau du Barrage du Verney

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The TRIADH constitutive law for soils is a derivative of the ADH law developed by the Ecole Centrale des Arts et Manufactures in Paris, and is a descendant of the Cambridge models (Cam-Clay and Granta-Gravel). It is an isotropic, incremental elastoplastic law incorporating strain hardening.

It supposes a yield surface marking the end of reversible phenomena (Figure 1) which is mobile with respect to strain hardening. It includes the notion of critical state, and allows for the stress surfaces of these states to depend on a single parameter associated with the angle of residual friction.

The main characteristics of TRIADH are:

- its ability to represent two hardening variables, volumetric hardening and deviatoric hardening; and,
- consideration of the first three stress invariants: a notable function of the third invariant is to allow good appreciation of the influence of intermediary principal stress.

The laws which preceded TRIADH suppose a critical state surface with the space diagonal as a cylindrical-symmetry axis, which leads to overestimation of the angle of residual friction in directions other than those of triaxial compression. TRIADH, on the other hand, because it takes account of the third stress invariant, gives a surface reflecting real results and which corresponds to the surface proposed by Lade and Duncan (1975).

TRIADH takes account of the main mechanical fill material properties that experience has brought to light. It is perfectly suited to calculation of the different phases of embankment dam construction and impounding, as is illustrated in the following example.

Verney Dam was analyzed using the TRIADH law in order to calculate the behaviour of the structure during the first impounding of the reservoir. The reservoir has since been fully impounded and we can evaluate the prediction calculation.

Verney Dam is a forty-two metre high compacted alluvial fill dam with the particularity of being built on an eighty metre thick bed of very permeable alluvium and morainic material. It is rendered watertight by a bituminous concrete upstream facing which is extended into the foundation by a forty-five metre deep diaphragm wall.

The information gathered from instrumentation during diaphragm wall and dam construction was used to calculate predicted dam behaviour, and especially that of the facing and diaphragm wall, during impounding.

Two construction-phase calculations were carried out, the first using linear elastic laws, the second the TRIADH non-linear laws for the different materials. Both calculations very well match observed deformations.

However, despite the fact that the parameters for both calculations were based on the same data, their predictions for behaviour at the impounding stage were remarkably different.

Figure 2 shows deformation as measured, and as calculated by both analyses for the impounding phase.

Monitoring during impounding has closely confirmed the quality of predictions given by the TRIADH law, which contrasts strongly with the poor representation of actual behaviour obtained with elastic laws.

Non-linear calculation provided a useful benchmark for interpretation of instrument results during impounding, this analysis has shown that non-linear behavioural laws like TRIADH reproduce actual soil behaviour sufficiently well, at least for non-cyclic loading conditions, for them to give accurate predictions.

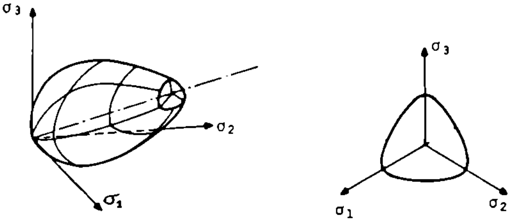

Fig.1 TRIADH failure surface

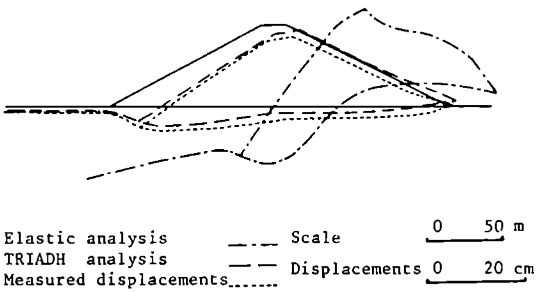

Fig. 2 Displacements after impounding

Hyperbolic undrained stress paths
Les chemins de contraintes hyperboliques

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INTRODUCTION

Evidence has been presented in Paper 1/A/23 by Hartford and Kirwan that, for well graded soils, the initial portion of the effective stress path, where the mean effective stress, p', is decreasing, forms part of a hyperbola. This concept is extended in this submission.

Hyperbolic Stress Paths

If there is a linear relationship between pore pressure, u and stress ratio, q/p', then a little mathematical manipulation leads to the conclusion that the undrained effective stress path is part of a hyperbola. Since the specimens are normally consolidated, it is a natural step to examine the data within the framework of Critical State Soil Mechanics. Fig. 1 demonstrates that both of the most widely used critical state models overestimate the pore pressures in undrained tests on the residual soil.

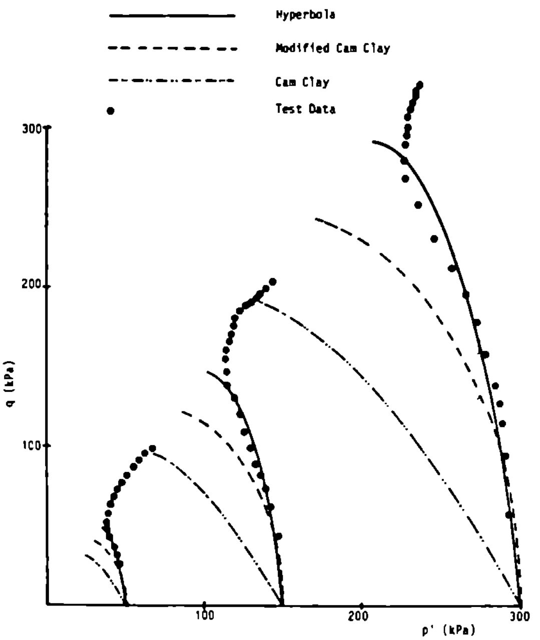

Fig. 1: Hyperbolic Stress Paths.

Prediction of Strains

It is possible to use critical state theory to predict the strains in triaxial tests. The hyperbolic undrained stress path defines the undrained cross-section of the stable state boundary surface in p', q, v space. The intersection of this surface with a vertical plane above a swelling line defines the yield surface for the soil which can be used to calculate plastic and elastic strains during any kind of test (Fig. 2).

Comments

1. The strains based on the hyperbolic undrained effective stress path clearly underestimate the measured strains. The prediction could be improved by including elastic shear strains which have been taken as 0.
2. Research work at Imperial College, London has shown that conventional methods of measuring strains in the triaxial test are very inaccurate. It is conceivable that the measured stiffness underestimates the true stiffness by at least an order of magnitude.

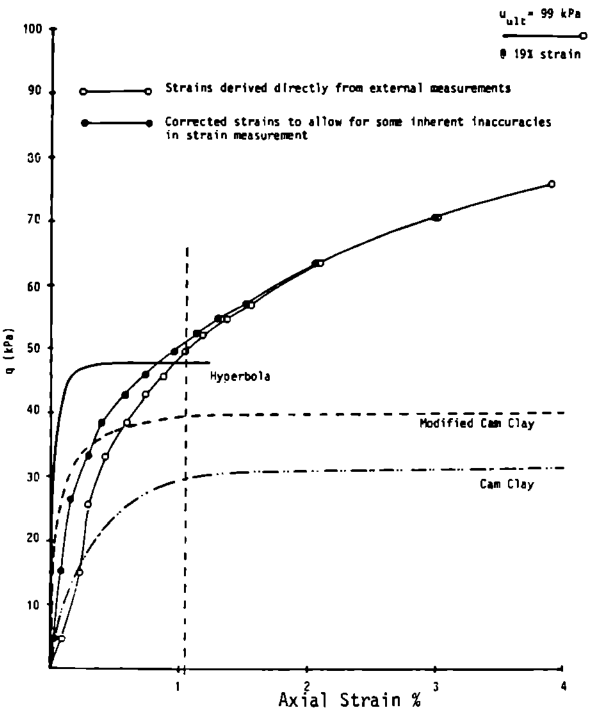

Fig. 2: Predicted Stress-Strain Behaviour.

A selection of the reasonable model for elastoplastic deformation

Une sélection du modèle raisonnable pour la déformation élastoplastique

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SYNOPSIS A consideration as to which one is most reasonable among various models of elasto-plastic deformation proposed in the past is given and the extended subloading surface model is concluded to be most recommendable.

INTRODUCTION

Various models for the elastoplastic deformation have been proposed in the past, though we have only to find one reasonable model. A due consideration has not been done, however, as to which model is most reasonable.
Here, we take up the five well-known models, i.e., the multi- (Mroz 1967), the infinite- (Mroz *et al.* 1982), the two- (Dafalias & Popov 1975, Krieg 1975, Hashiguchi 1981, 1985), the single- (Dafalias & Popov 1977, Mroz 1980) and the subloading- (Hashiguchi 1977, 1980, 1985, Dafalias & Herrmann 1980) surface models. They are explained in the author's state-of-art report (Constitutive Equations of Soils, Proc. Discussion Session A1, 11th ICSMFE, 1985, pp.25-65 & 127-130). Then we examine the adequateness of them from the viewpoint of expressibility of the two fundamental elastoplastic characteristics, i.e., the smooth (gradual) elastic-plastic transition and the Masing rule. In order to do it concisely, we consider the uniaxial loading behavior of the perfectly-plastic material.

PREDICTION OF UNIAXIAL STRESS/STRAIN CURVE

The uniaxial stress/strain curves predicted by the above-mentioned models are depicted in Fig.1, where the marks O, Δ and X mean sufficient, insufficient possibilities and impossibility, respectively. As known from this figure, the subloading surface model only can express both the smooth elastic-plastic transition and the Masing effect. Here, the extended subloading surface model is used, in which the center of similarity of the assumed surfaces moves with a plastic deformation. The explicit equations are written in pp.127-130 of the above-mentioned report, while the plastic strain rate equation should be replaced as follows:

$$\dot{\underset{\sim}{\varepsilon}}^{P} = \langle \mathrm{tr}(\overline{\underset{\sim}{n}}\dot{\underset{\sim}{\sigma}})/\mathrm{tr}[\overline{\underset{\sim}{n}}\{\frac{1}{n}\frac{F'}{F}\overline{\kappa}\{\overline{\underset{\sim}{\sigma}} + (1-R)\hat{\underset{\sim}{s}}\} + \overline{\underset{\sim}{a}} + (C + \frac{\zeta}{|\tilde{\underset{\sim}{\sigma}}|})(\frac{1}{R} - 1)\tilde{\underset{\sim}{\sigma}}\}]\rangle\overline{\underset{\sim}{n}}$$

$$\overline{\underset{\sim}{a}} \equiv A\,\mathrm{tr}\overline{\underset{\sim}{n}}\cdot\underset{\sim}{1} + B\,\mathrm{tr}(\overline{\underset{\sim}{n}}\frac{\overline{\underset{\sim}{\sigma}}}{|\overline{\underset{\sim}{\sigma}}|})\frac{\overline{\underset{\sim}{\sigma}}}{|\overline{\underset{\sim}{\sigma}}|}$$

where C and ζ are material constants.
In conclusion the subloading surface model is recommendable among the above five models.

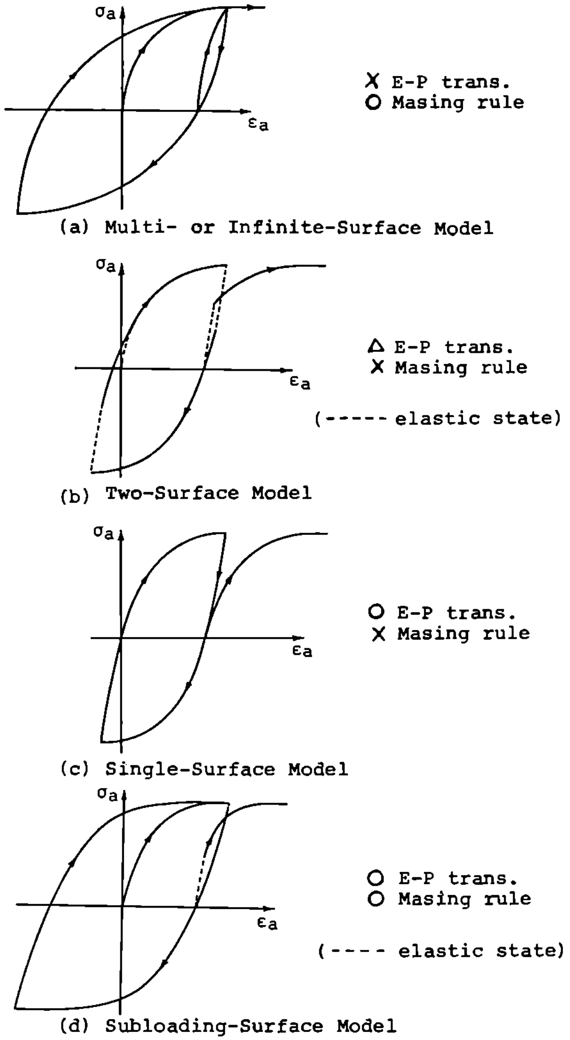

Fig.1 Prediction of Uniaxial Stress/Strain Curve. (E-P trans.: smooth Elastic-Plastic Transition)

The choice of framework for theoretical models for soils

Choisir une famille de modèles théoriques des sols

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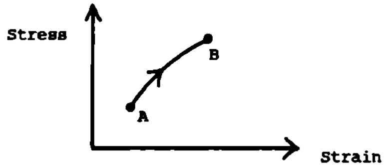
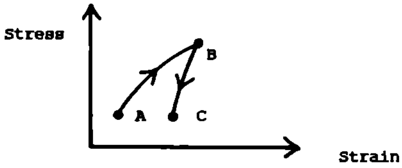
Constitutive models for soils vary not only in their detail, but also fall into a variety of fundamentally differing classes. Three different classes of model are outlined below. All three classes can successfully describe the non-linear behaviour observed for real soils subjected to monotonic loading. Two of the classes are able to describe unloading realistically, and only one is able to describe a subsequent reloading in a realistic manner (with a suitable choice of the detail of the model).

A non-linear loading curve may be successfully described by a non-linear elastic model as illustrated schematically:



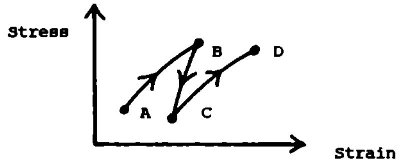
A small section of the loading curve AB is shown. On unloading from B the path BA is retraced. This behaviour is unrealistic for describing real soils (with the exception of a strictly limited range of stresses for over-consolidated soils). Except for a few limited applications the framework of non-linear elasticity is not therefore suitable for describing soil behaviour.

A more realistic unloading behaviour can be achieved using either a hypo-elastic mode (in the terminology of Fung (1965)) or an elastic-plastic model. This is shown as the path ABC:



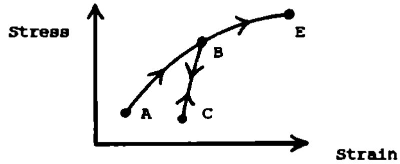
where the section BC is always steeper than AB.

For modelling loading and a single unloading either of the above frameworks may therefore be realistic. If, however, a reloading is made after a small unloading then the two theoretical frameworks result in a quite different response. The hypo-elastic model results in the path ABCD, with CD approximately parallel to AB:



This behaviour is quite unrealistic for a real soil subject to a small unloading. The problem is often obscured by examining only large unload-reload cycles. The hypo-elastic framework is not therefore suitable for the construction of general stress-strain models for soils. Endochronic models also fall into this category, although some endochronic models avoid this problem (at some considerable cost in terms of complication) by making use of some features drawn from plasticity theory.

Elastic-plastic models result in a different behaviour on a small unload-reload loop:



The material loads and unloads along ABC. On reloading the path BC is retraced, and on further loading the path BE (a continuation of the curve AB) is followed. This pattern models most closely the character of the response of a real soil during a small unloading-reloading cycle. It is therefore concluded that of the three possible frameworks described above for theoretical soil models, only elastic-plastic behaviour is suitable for constructing realsitic general stress-strain models.

Elastic-plastic theory is not without its problems. For large unload-reload cycles it is, for instance, less well suited to describing hysteresis than the hypo-elastic models. The sudden transition from elastic to plastic behaviour is often not encountered in real materials. Rather complex models within the elastic-plastic framework have been devised which circumvent both these problems. The criticisms of plasticity theory centre mainly, however, on points of detail. The non-linear elastic and hypo-elastic (or endochronic) theories describe behaviour which is fundamentally different from that of real soil.

Comment on this topic was made in Theme Lecture 1, and further discussion is given by Houlsby (1981).

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Discussion on: 'The occurrence of creep during consolidation'
Discussion sur: 'La présence de fluage durant consolidation'

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While the magnitude of consolidation deformations in the field may be reliably estimated from the results of small scale laboratory tests for most soils, the suggestions that creep does not occur during consolidation (Jamiolkowski et. al, 1985) and that the void ratio at the end of primary compression is a unique function of the effective confining pressure (Mesri and Choi, 1985) are misleading generalizations, objectionable on both practical and theoretical bases. The results of oedometer tests on 2.5 cm. thick specimens have been used for years to satisfactorily predict consolidation deformations in the field. However, much less success has been reported in predicting the development and dissipation of excess pore pressures. Some of the reported discrepancies between observed and predicted pore pressures can only be attributed to creep effects during consolidation. Furthermore, the assumption of a unique end of primary void ratio implies that soil behavior depends upon total stress as well as effective stress and suffers from several other inconsistencies.

Mitchell (1984), Crooks et. al (1984) and Tse (1985) document and discuss in detail field cases of "anomolous" pore pressure behavior, wherein the induced pore pressure is greater and the rate of pore pressure dissipation less than predicted by conventional consolidation theory. These anomolies are attributed to a variety of factors, including the Mandel-Cryer effect, diffusion from zones of higher pore pressure, non-linearity and non-homogeneity of soil compressibility, and permeability anisotropy and non-homogeneity. However some of the cited anomolies cannot possibly be explained by any of these effects and can be attributed to creep effects. Crooks et. al cite 11 cases in which pore pressures continued to increase after the end of load application, in some cases to a magnitude greater than the increase in total stress. A pore pressure increase of such magnitude can only be attributed to creep effects. Similar pore pressure anomolies have been observed in centrifuge model tests of embankments founded upon compressible soils (Bassett et. al, 1981). "Delayed" failures which occur days to weeks after the end of loading or excavation have been attributed to this type of pore pressure rise. Tse cites the case of an embankment at Rangsit, Thailand wherein pore pressures increased again after all initial excess pore pressures had dissipated. Creep generated pore pressures are commonly observed in the laboratory in consolidated undrained deviatoric creep tests and in isotropic, anisotropic, and one-dimensional consolidation tests due to the arresting of secondary compression.

The assumption that the void ratio at the end of primary consolidation is a unique function of the confining pressure implies that total stresses as well as effective stresses control soil behavior and that the soil somehow "knows" when the excess pore pressures have dissipated. A major inconsistency in any numerical analysis using a primary/secondary diffusion type of consolidation model is the need for an arbitrary definition of a small value of excess pore pressure as corresponding to the end of primary consolidation. Even with such a primary/secondary model, logic dictates that unless the distribution of excess pore pressure is absolutely uniform within the consolidating soil when the specified pore pressure is achieved, elements of soil within the consolidating layer adjacent to a drainage boundary will commence secondary compression before elements further from the boundary, and thus some size effect is inevitable. The magnitude of this effect, however, is apparently quite small and of no practical significance for most soils.

Theoretical analyses using an immediate/delayed consolidation model described by Borja and Kavazanjian (1985) indicate that the pore pressure distribution is nearly uniform towards the end of consolidation and that size effects in laboratory specimens are smaller than the scatter expected in the void ratio of natural soils and, perhaps, smaller than the resolution of laboratory tests for measuring void ratio. In an analysis of the consolidation behavior of the I-95 highway embankment using this model, described in these proceedings by Kavazanjian et. al (1985), little difference was observed in deformations between the results of analyses in which creep was ignored and analyses in which creep strains were scaled according to the coefficient of secondary compression. However, pore pressure response predicted by these two analyses started to diverge towards the end of the consolidation period. This analysis led to speculation that, in many cases, volumetric creep during consolidation may be a self compensating process.

The assumption that the void ratio at the end of primary consolidation depends only on the applied total stresses and the steady state pore pressure may, like total stress stability analyses, be expedient for practical purposes. However, the unqualified statement that creep is of no significance during consolidation suffers from theoretical shortcomings and may, in some cases, be dangerously misleading.

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A generalized hypoelastic constitutive law
Une loi rhéologique hypoélastique généralisée

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The constitutive law presented here has been developed along the framework of its versions published since 1978. Although the present version is much simpler than the previous ones, their basic idea is maintained: To describe the rate independent part of the material behaviour by representing the co-rotational stress rate $\overset{\circ}{\mathbf{T}}$ as an isotropic tensor valued function of the stress $\mathbf{T}$ and of the stretching $\mathbf{D}$:

$$\overset{\circ}{\mathbf{T}} = \mathbf{h}(\mathbf{T}, \mathbf{D})$$

This function must fulfil the following restrictions:

1. Rate independence requires $\mathbf{h}(\ )$ to be positively homogeneous of the 1.degree (but not necessarily linear) in $\mathbf{D}$.
2. Irreversibility (i.e.internal energy dissipation) requires $\mathbf{h}(\ )$ to be non-linear in $\mathbf{D}$.
3. Some experimental evidence referring to asymptotic properties of sand points to $\mathbf{h}(\ )$ being homogeneous in $\mathbf{T}$.

Note that the constitutive law presented here departs completely from the framework of elastoplasticity. Thus, usual notions such as yield surface, flow rule, normality, decomposition of strain into elastic and plastic parts etc., do not appear.

The constitutive law reads

$$\overset{\circ}{\mathbf{T}} = C_1 \cdot \frac{1}{2}(\mathbf{TD}+\mathbf{DT}) + C_2 \cdot \mathrm{tr}(\mathbf{TD}) \cdot \mathbf{1} + \left(C_3 \mathbf{T} + C_4 \frac{\mathbf{T}^2}{\mathrm{tr}\mathbf{T}}\right) \cdot \sqrt{\mathrm{tr}\mathbf{D}^2}$$

The degree of homogeneity in $\mathbf{T}$ should be 1 as long as the incremental stiffnesses are observed to be proportional to the actual stress niveau. This seems to be the case for many sands in a wide stress range. The law can be easily modified in order to describe a granular material which does not exhibit the above property. In this case, i.e. if the incremental stiffness $E_{ijkl} := \partial\sigma_{ij}/\partial\varepsilon_{kl}$ is proportional to $|\mathrm{tr}\mathbf{T}|^a$, $a \neq 1$, then the law should be modified as follows:

$$\overset{\circ}{\mathbf{T}} = |\mathrm{tr}\mathbf{T}|^{a-1} \cdot \left\{ C_1 \cdot \frac{1}{2}(\mathbf{TD}+\mathbf{DT}) + C_2 \mathrm{tr}(\mathbf{TD}) \cdot \mathbf{1} + \left(C_3 \mathbf{T} + C_4 \frac{\mathbf{T}^2}{\mathrm{tr}\mathbf{T}}\right) \sqrt{\mathrm{tr}\mathbf{D}^2} \right\}$$

In the above formulation $\mathbf{h}(\mathbf{T},\mathbf{D})$ is homogeneous of the degree a in $\mathbf{T}$.

Calibration:

The 4 material parameters C_1, C_2, C_3, C_4 can be determined by fitting a calculated stress strain curve to an experimentally obtained one. It is remarkable that this calibration procedure needs only the results of one single conventional (i.e. with constant lateral stress) triaxial compression test. Thus, according to this law, the whole mechanical behaviour is revealed by a triaxial loading, which also suffices to determine the unloading characteristics. From the plots of $\sigma_1-\sigma_2$ vs. the axial strain ε_1 and the volumetric strain ε_v vs. ε_1 the following quantities can be picked up:

1. the initial slope of the stress strain curve $E := d(\sigma_1-\sigma_2)/d\varepsilon_1$
2. the initial dilatancy $d\varepsilon_v/d\varepsilon_1$
3. the limit condition $\sin\phi := \left(\frac{\sigma_1-\sigma_2}{\sigma_1+\sigma_2}\right)_{max}$
4. the dilatancy at the limit condition $(d\varepsilon_v/d\varepsilon_1)_{limit} =: \tan\beta$

The requirement that the constitutive law fits the 4 test data stated above yields a system of 4 linear equations, the solution of which are the values of C_1, C_2, C_3, C_4. E.g., for Karlsruhe medium sand the following triaxial test data by Hettler and Vardoulakis (1984,Geotechn.34,No.2, 183-198) have been used

$E=200\sigma_2$; $d\varepsilon_v/d\varepsilon_1(\varepsilon_1=0)=-1$; $\phi=43°$; $\beta=36°$

to obtain

$C_1=-200$; $C_2=-47.9$; $C_3=36.4$; $C_4=-252.7$.

Recent accurate experiments show that, contrary to a widespread opinion, no lateral strain occurs at the beginning of a triaxial strain compression test, i.e. $d\varepsilon_v/d\varepsilon_1(\varepsilon_1=0)=-1$. This result seems to hold for all types of granular media. Taking this into account, the following calibration formulae can be derived:

$$C_1 = E/\sigma_2$$

$$C_2 = C_1 \cdot \frac{a}{d}\left[1+b-\frac{3}{a+2}(1+ab)\right]$$

$$C_3 = -C_1 \cdot \frac{1}{cd}\left[(2b-a)(a+b)+\frac{3ac}{a+2}(ab+1)\right]$$

$$C_4 = 3(C_2-C_3)$$

where

$$a := \frac{1+\sin\phi}{1-\sin\phi}\ ; \quad b := \frac{1}{2}(1+\tan\beta)\ ; \quad c := \sqrt{1+2b^2}$$

$$d := \frac{a-1}{a+2}\left[(2b-a)(2a+1)+3ac\right]$$

These formulae enable the calibration directly from the values E, ϕ, β.
Strictly speaking, the material parameters C_1, C_2, C_3, C_4 are not constants but functions of e (i.e. density dependent). The analytical expressions for these functions can be written down as soon as one knows the e-dependence of E, ϕ, β.

The constitutive law has been checked by comparing numerically simulated element tests with real ones. The results are quite satisfactory as regards loading-unloading-reloading. By its very nature, the present formulation cannot be expected to describe shakedown and viscous effects. The latter can be comprised by adding further terms.

Pore pressure generation in soils containing a visco-elastic pore fluid

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When establishing constitutive equations, intended to describe the mechanical behavior of soils, it is usually tacitly assumed that the Terzaghi's law of effective stress remains strictly valid. However, as shown in the following, while this law in its original form is sufficient for ordinary, water-saturated soils, it needs a slight modification, when the soil is cemented, or when its pores are filled with a highly viscous or visco-elastic matrix. The differences between the two cases are illustrated in Figs. 1 and 2.

Figure 1 shows schematically, in a (p,q) plane, how the separation of applied stresses into effective stresses and pore water pressures is usually performed in the case of a water-saturated soil, such as a normally consolidated clay or a loose sand, consolidated to p_o', when it is subjected to a strain-rate-controlled, undrained triaxial compression test with $\sigma_1 > \sigma_2 = \sigma_3$.

When, in a σ_3 = const. test, a total stress increment $\Delta\sigma_1$ (O'A) is applied to the soil, its skeleton will be stressed along the effective stress path (O'B'), which implies an increase in the stress difference Δq and a decrease in the effective mean normal stress $\Delta p'$. As the water can support only hydrostatic stresses, the change in the pore water pressure will then be given by the Terzaghi's law

$$\Delta u_w = p - p' \qquad (1)$$

If a similar analysis of stress partition is made for a soil containing a highly viscous or visco-plastic matrix, such as a frozen soil or a sand-asphalt mixture, the result will be slightly different. The difference is due to the fact that a visco-plastic matrix can support not only hydrostatic stresses, but also, at least temporarily, a certain portion of the applied shear stress. (Ladanyi, 1974, 1981, 1985).

Figure 2 shows how the stress partition is likely to occur in tha latter case. Because the matrix carries now a portion of Δq, the soil skeleton will be stressed less than in a similar, water-saturated soil, the effective stress point attaining only B" on the effective stress path (O'B') valid for the applied strain rate. The hydrostatic pore pressure change in the matrix, Δu_m, generated by Δq is still given by Eq.(1), but it is smaller than Δu_w in the same soil when water-saturated. Consequently, one can also write a partition equation for shear stresses

$$\Delta q_m = \Delta q - \Delta q_{soil} \qquad (2)$$

which is analogous to the effective stress equation (1), valid for hydrostatic stresses. If (Fig.2), after applying Δq at a rate $\dot{\varepsilon}_a$, and attaining the effective stress point "B", the soil is allowed to creep under Δp = const., the stresses will be gradually transfered to the soil skeleton, with the stress point following a path such as B"B. The creep will be attenuating, leading eventually to a complete transfer of shear stresses to the soil skeleton (i.e., $\Delta q_m \to 0$), and of hydrostatic stresses to the pore matrix: $\Delta u_m \to \Delta u_f$, given by Eq.(1).

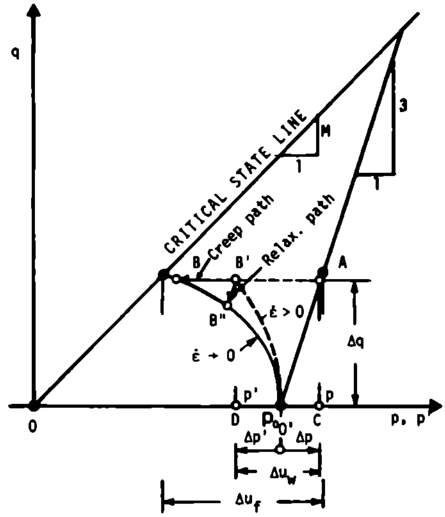

Fig.1. Stress partition in a CU compression test, with a water-saturated, normally consolidated soil.

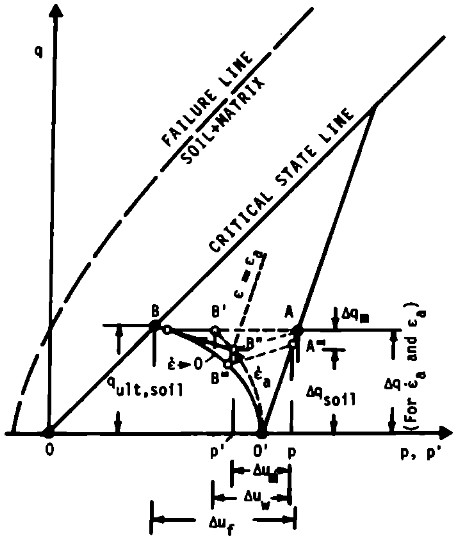

Fig.2. Stress partition in a CU compression test, with a normally consolidated soil, containing a viscous fluid in its pores.

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Constitutive model for normally consolidated or lightly overconsolidated soils

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Constitutive models of soils behaviour have two fundamental goals. The first, and more important from a scientific viewpoint, is to describe the observed behaviour of soils under the most general stress conditions in order to achieve a better understanding of such a behaviour. Since soil behaves in a very complex manner, it comes at no surprise that soil models which are most successful in getting a good quantitative agreement between prediction and experimental evidence, enjoy often a non-trivial structure and are characterized by a large numbers of constitutive parameters.

The second goal is that of providing a rational framework that can be employed for solving boundary value problems of practical importance. To limit the cost of computing time to an acceptable level the use of sophisticated, all-purpose models is virtually impossible, and simpler more economical models ought to be used. In addition, in engineering practice the knowledge of the mechanical characteristics of the foundation soil is obtained by means of sampling, in situ testing, and sometimes, but not frequently, by performing very simple laboratory tests. It bears than non sense to employ constitutive models characterized by a high number of parameters, since only very few of them could be identified with acceptable accuracy from such a scanty experimental evidence.

A compromise should then be found between the need of accuracy of the mathematical description and the necessity of simplicity of the structure of the model, joined to a limited number of constitutive parameters.

In the authors's opinion, the model of Nova and Wood (1979) in its generalized version, Nova (1984), achieves this compromise for normally consolidated or lightly overconsolidated soils. In fact the model is able to describe reasonably well the behaviour of virgin sand and normally consolidated clay in different types of tests, although its structure is simple - a classical elastic plastic strain-hardening model - and the number of constitutive parameters is limited to 8. Moreover they are directly or indirectly linked to traditional soil parameters such as, for istance, the limiting friction angle or the 'elastic' shear modulus. This allows an easy determination of constitutive parameters even on the basis of very simple tests.

As an example, consider the deposit of silty clay at Porto Tolle, which was extensively studied by Jamiolkowski et al. (1980). From the results of oedometric tests it was possible to determine 4 of the 8 parameters. 3 of them were taken from the results of triaxial compression in undrained conditions, but they could have been determined from the available in situ tests performed as well. The last one was simply guessed on the basis of previous experience gained on similar soils.

The influence of the limited length of the probe on the pressuremeter curve was then studied (Borsetto et al. (1983)). The calculated curve compared favourably with

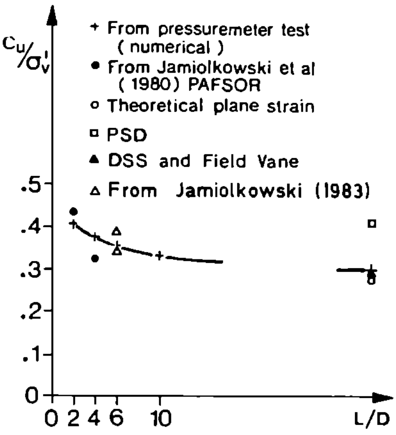

Fig.1 Influence of length-to-diameter ratio on undrained strength.

that measured by means of a PAFSOR (Ghionna et al. (1982)) which was not known a priori. Moreover, by applying the method of Wihdle and Wroth (1977), it was possible to backfigure the value of the undrained strength and Young modulus as a function of the ratio between the length L and the diameter D of the probe. It turned out that, whilst the Young modulus is virtually not affected by the aspect ratio, the undrained strength c_u decreases at a very slow rate with increasing L/D. In practice, even a Camkometer, that has an aspect ratio equal to 6, overestimates the strength of 24%. As shown in fig.1, the values of c_u backfigured from experimental data are in close agreement to calculated values. It is worthnoting that the values obtained by means of a Camkometer were not known in advance (Jamiolkowski (1983)).

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Contribution to discussion Session 1A

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D.RICHARDSON, The City University, London, UK

This discussion considers soils which are assumed to display a continuous yield surface and the relationship of the plastic strains to this yield surface. The terminology and notation used follow Atkinson and Bransby (1978).

Although the normality criterion is highly convenient mathematically, there is no prior reason why it should hold generally and a number of experimental examinations have been undertaken to determine the nature of the flow rule for soils. In order to test the normality criterion directly from experimental data it is necessary, firstly, to separate elastic and plastic components of strain and, secondly, to identify the current yield surface. Neither of these requirements is easy to achieve with the result that previous experimental examinations of the normality criterion for soils are largely inconclusive.

An alternative indirect procedure for examining the normality criterion in soils has recently been proposed by Atkinson and Richardson (1985). This method requires that soil samples are subjected to a special stress path test and the data plotted in a simple manner; it is not necessary to separate elastic and plastic components of strain or to identify the current yield surface.

Invariants of stress and strain for a cylindrical sample are defined as $q' = (\sigma_a' - \sigma_r')$, $p' = \frac{1}{3}(\sigma_a' + 2\sigma_r')$, $\varepsilon_s = \frac{2}{3}(\varepsilon_a - \varepsilon_r)$ and $\varepsilon_v = (\varepsilon_a + 2\varepsilon_r)$ where the subscripts a and r refer to the axial and radial directions. It is assumed that a constitutive equation for soil can be written

$$\begin{Bmatrix} \delta\varepsilon_s \\ \delta\varepsilon_v \end{Bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{Bmatrix} \delta q' \\ \delta p' \end{Bmatrix} \qquad (1)$$

where $A = \dfrac{\partial\varepsilon_s}{\partial q'}$, $B = \dfrac{\partial\varepsilon_s}{\partial p'}$, $C = \dfrac{\partial\varepsilon_v}{\partial q'}$, and $D = \dfrac{\partial\varepsilon_v}{\partial p'}$

If it is assumed additionally that total strains are the sum of their elastic and plastic components (i.e. $\underline{\delta\varepsilon} = \underline{\delta\varepsilon}^p + \underline{\delta\varepsilon}^e$) then equation 1 becomes

$$\begin{Bmatrix} \delta\varepsilon_s \\ \delta\varepsilon_v \end{Bmatrix} = \begin{bmatrix} A^e & B^e \\ C^e & D^e \end{bmatrix} \begin{Bmatrix} \delta q' \\ \delta p' \end{Bmatrix} + \begin{bmatrix} A^p & B^p \\ C^p & D^p \end{bmatrix} \begin{Bmatrix} \delta q' \\ \delta p' \end{Bmatrix} \qquad (2)$$

Now, for a soil which is elastic $B^e = C^e$ and for a soil for which the normality criterion applies $B^p = C^p$. Hence for a soil which is elasto-plastic with normality we have

$$\frac{\partial\varepsilon_v}{\partial q'} = [B^e + B^p] = [C^e + C^p] = \frac{\partial\varepsilon_s}{\partial p'} \qquad (3)$$

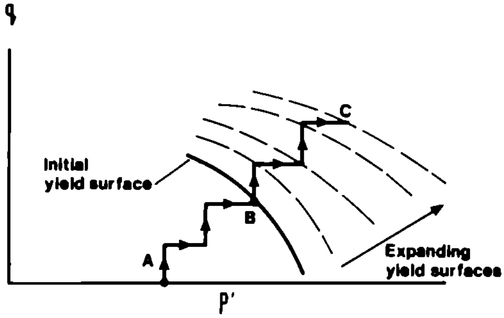

Fig. 1. The special stress path adopted.

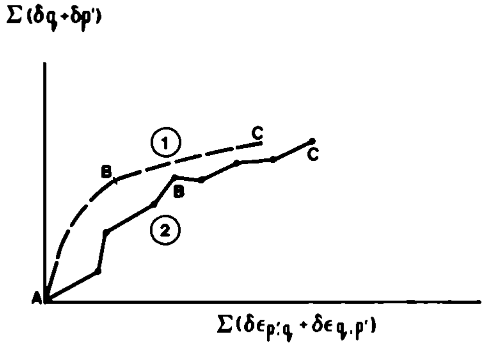

Fig. 2. Typical results.

Figure 1 illustrates a stress path A-B-C consisting of successive stages for which $\delta p' = 0$ and $\delta q' = 0$. The path A-B is inside the current yield surface and the path B-C expands the yield surface as plastic yielding and hardening occurs. For a stage with $\delta p' = 0$ we have $C = \delta\varepsilon_v/\delta q'$ and for a stage with $\delta q' = 0$, $B = \delta\varepsilon_s/\delta p'$. Thus B and C may be evaluated from observations of total strains and a soil sample is elasto-plastic with normality if B = C throughout the loading path A-B-C.

Figure 2 illustrates two possible sets of data in which increments of stress are plotted against increments of strain as $\delta q'$ against $\delta\varepsilon_v$ for $\delta p' = 0$ stages and as $\delta p'$ against $\delta\varepsilon_s$ for $\delta q' = 0$ stages. The gradients for each increment are given as 1/B and 1/C respectively and B = C if the resulting stress-strain curve is smooth. Thus the curve (1) indicates behaviour which is elasto-plastic with normality while the curve (2) indicates behaviour which is neither elastic nor normal.

Further details of the theory and experimental procedures are published by Atkinson and Richardson (1985) who also give the results of tests on isotropically consolidated reconstituted samples of three soils. Their results indicate that London Clay and a glacial soil were elasto-plastic with normality while for Speswhite kaolin it appeared that the normality condition was not well observed.

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Report on discussion Session 1B: Numerical methods

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INTRODUCTION

The intention in organising this session was to concentrate not on numerical techniques as such, but on the practical applications of these techniques. Peck (1985), in an address to the closing session of the conference, commented that a subculture devoted to numerical computation has grown up in our midst. The session was planned with a similar point in mind. The aim was to emphasise the point that useful insight can be gained from simple numerical analyses.

A small panel was invited to make brief presentations on applications of numerical methods which illustrate what can be achieved with a relatively simple approach. Summaries of four of the six presentations made are included herein.

In the present context the meaning of simplicity is important, and it relates to the amount of effort and expense that is required for the successful execution of a numerical analysis. The major difficulty lies not in the computing resources needed but in the determination of soil properties. Thus a numerical method could be classed as simple if the soil parameters required are well known to the geotechnical community and can be determined by established methods.

The examples presented by Simpson and Rowe both illustrate how complex situations can be modelled with well established techniques and modest data. In each case it is necessary to be aware of an important aspect of soil behaviour. For the excavation in London clay a bilinear stress-strain relation was necessary. For very small stress changes, that is for the volume of soil some distance from the excavation, the modulus is very high. Nearer the excavation the stress changes are larger and the apparent modulus much smaller. This simple bilinear concept gives very satisfactory modelling of the observed displacement field about the excavation. In the case of the deep tunnel discussed by Rowe it is found that linear elastic modelling cannot predict both the maximum settlement at the ground surface and the shape of the settlement trough. The introduction of the possibility of plastic deformation in the soil adjacent to the tunnel is required to give a better estimate of the variation of the surface settlement above the tunnel. In these two cases it is found that good results are achieved by taking a relatively modest step beyond the conventional linear elastic analysis. Both of these applications of the finite element method have, in the words of Peck, "vastly improved our understanding of complex problems". From the point of view of the practical application of such methods to the design process it is important that in the two cases discussed the understanding has been achieved without recourse to an extensive set of "new" soil parameters.

The presentation by Sagaseta is a particularly elegant reminder that there are still situations where much simpler semi-analytical approaches are successful. In this case the requirement of zero volume change provides additional equations so that the displacement field can be estimated without detailed consideration of the constitutive behaviour of the soil.

Finally the presentation of Byrne illustrates how concepts that have been developed for water and air in the pore space of soil can be extended to something as far removed from traditional soil mechanics as oil sand.

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Stress, deformation and flow analysis in oil sand masses

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The oil sands of Alberta contain the largest known reserves of oil in the world. The problem is to extract the oil. Where the deposits are close to the surface - within about 50 m - they are simply excavated and the oil removed. At greater depths it becomes uneconomical to do this and in-situ recovery methods are used. These involve constructing shafts and tunnels in the oil sand. Two problems of concern are:

(1) What are the earth pressures on a shaft constructed in oil sand?

(2) What is the flow or production from a well in oil sand?

The oil in the form of bitumen has a high viscosity and consequently a low permeability. To increase production it is common to first heat the stratum by injection of steam or hot air. This aspect will not be addressed herein. The purpose here is to present a logical method of analysis for predicting the stresses and displacements adjacent to tunnels and shafts in oil sand as well as the flow of oil into such openings.

Oil sand is a very dense sand whose pore space is filled mainly with bitumen. The low effective permeability causes the oil sand to respond in an undrained manner to construction loading and unloading. The sand grains are not

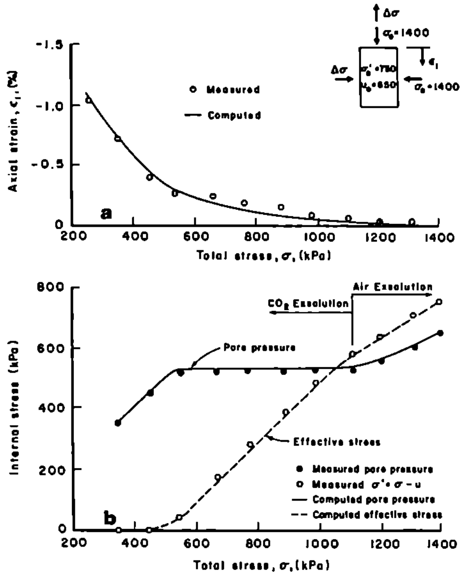

FIG. 1 : Undrained Unloading-Comparison of Observed and Computed Response-Gassy Soil.

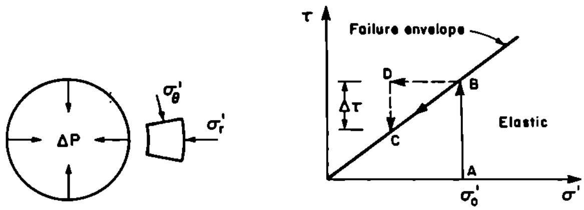

FIG. 2 : Stress Path for Element Adjacent to Shaft as Unloading proceeds.

cemented and consequently the sand matrix behaves much as any other very dense sand. The unusual behaviour of oil sand arises because of the very high dissolved gas content in the bitumen. Upon unloading such gas evolves causing the pore fluid to become flexible with the result that load is removed from the sand matrix while the pore fluid pressure remains high. When the stress in the sand matrix drops close to zero, large deformations and instability occur.

Method of Analysis

The method of analysis involves idealising both the sand matrix and pore fluid behaviour and coupling them in a finite element analysis.

The sand matrix is modelled as incremental elastic using stress dependent tangent Young's, E_t, and tangent bulk, B_t, moduli as described by Duncan et al., 1980. In addition, dilation was also included as described by Byrne and Eldridge, 1982.

The pore fluid effects the response because of:

(a) Its high viscosity leading to low permeability and an initial undrained condition.

(b) Its compressibility or stiffness which controls the share of the normal pressure taken by the fluid and sand matrix. This is important since it is the stress in the sand matrix that governs the deformation response. The compressibility of the fluid can be derived from the gas laws Cheung (1985), resulting in

$$\text{Compressibility} \qquad C_f = \frac{1 - S + SH}{u_a} + SC_L \qquad (1)$$

$$\text{Stiffness} \qquad B_f = \frac{1}{C_f} \qquad (2)$$

in which
S = degree of saturation
H = Henry's constant
u_a = the absolute fluid pressure
C_L = the compressibility of the liquid

The undrained response is obtained by adding the stiffness contribution of the sand matrix and pore fluid components under the condition of volume compatibility.

The sand matrix effective stress-strain relations are given by

$$\text{Sand Matrix} \qquad \{\Delta\sigma'\} = [D']\{\Delta\epsilon\} \qquad (3)$$

where $[D']$ contains E_t and B_t terms.

The pore fluid response is given by

$$\text{Pore Fluid} \qquad \Delta u = B_f(\Delta\epsilon_v)_f \qquad (4)$$

Compatibility requires that

$$(\Delta\epsilon_v)_f = \frac{1}{n}(\Delta\epsilon_v) \qquad (5)$$

or

$$(\Delta\epsilon_v)_f = \frac{1}{n}\begin{Bmatrix}1\\1\\0\end{Bmatrix}^T\{\Delta\epsilon\} \qquad (6)$$

in which n = porosity

and from (4) and (6)

$$\{\Delta u\} = \frac{B_f}{n}\begin{Bmatrix}1\\1\\0\end{Bmatrix}\begin{Bmatrix}1\\1\\0\end{Bmatrix}^T\{\Delta\epsilon\} = [D_f]\{\Delta\epsilon\} \qquad (7)$$

$$\text{now} \qquad \Delta\sigma = \Delta\sigma' + \Delta\sigma \qquad (8)$$

$$\text{from which } \{\Delta\sigma\} = \big[[D'] + [D_f]\big]\{\Delta\epsilon\} = [D]\{\Delta\epsilon\} \quad (9)$$

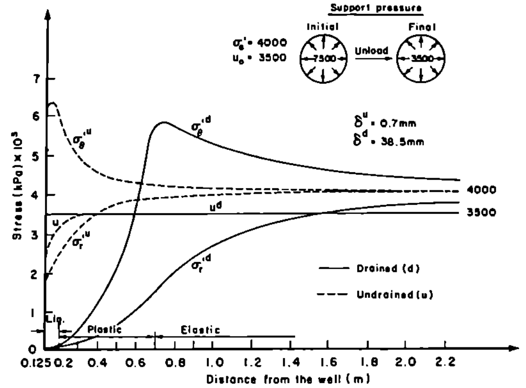

FIG. 3 : Stress and Pore Pressure Distribution under Undrained and Drained Conditions for a Fully Saturated Elasto-Plastic Material.

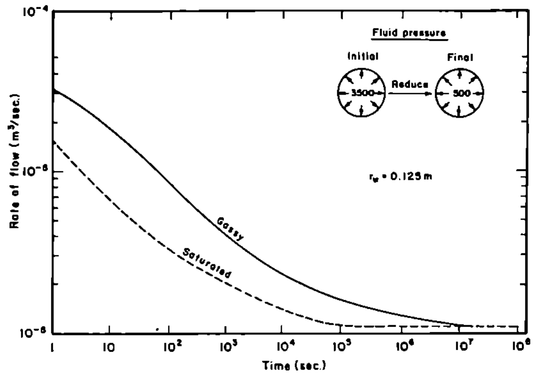

FIG. 4 : Variation of Flow Rate with Time for a Gassy and Saturated Soil.

Equation (9) is used in the finite element formulation so that the pore pressure is not an unknown in the solution of the system of equations but is obtained later from Eq. (7). This method, first proposed by Naylor (1974), appears to have been little used. We have found it to be far superior to other techniques tried. In particular, it is stable when the effective stresses go to zero and all of the load is carried by the pore fluid.

The undrained analysis procedure has been checked against the results of laboratory triaxial tests, performed by Sobkowicz (1982). Samples of sand whose pore spaces were filled with water, air and carbon dioxide were subjected to unloading, and displacements and fluid pressures observed. Carbon dioxide is very soluble in water (high Henry's constant) and acts in a similar way to the dissolved gases in bitumen. The sample was unloaded in steps from an initial condition of $\sigma_o' = 750$ and $u_o = 650$ kPa. The strains are small until the total stress drops to about 500 kPa at which point large displacements occur. The reason for this sudden increase in strain may be seen in Fig. 1b where the variation of pore and effective stresses with total stress are shown. It may be seen that gas exsolution causes the pore fluid to become flexible relative to the sand matrix, with the result that the effective stress drops rapidly while the pore fluid pressure stays high. When the total stress reaches 500 kPa, the effective stress drops to zero and so also does the skeleton stiffness, with the results that further unloading is taken by the pore fluid and leads to the large expansion strains shown in Fig. 1a. It may be seen that the predicted and measured response are in remarkably good agreement.

Shaft or Well Bore Problem

The unloading of soil associated with a shaft or well bore in sand follows the stress path ABC for an elastic-plastic stress-strain law. Using the incremental elastic approach the tangent stiffness, E_t is set to zero at point B and the predicted path in Fig. 2 will be BD which violates the failure condition. The over stress $\Delta\tau$(DC) is removed by a load shedding technique in which element nodal forces equivalent to $\Delta\tau$ are computed and applied as load vectors to the global system while keeping the element stiffness $E_t \approx 0$. In this way elements subjected to such overstress shed or transfer their load to adjacent elements. This approach was found to work remarkably well and gives excellent agreement for both stresses and displacements when compared with closed form elastic-plastic solution reported by Hughes et al. (1977).

The computed radial and circumferential stress field around a well bore at a depth of 300 m is shown in Fig. 3. The initial horizontal stress, σ_o, was 7500 with σ_o'=4000 and u_o=3500 kPa. The stresses shown are for unloading to a support pressure of 3500 kPa for both undrained and fully drained conditions. It may be seen that the undrained condition has higher effective stress and is consequently more stable than the drained condition. This is so because of the drop in pore pressure associated with the undrained condition. For the drained condition a zero effective stress condition exists when the supporting fluid pressure is dropped to the in-situ pore pressure and the predicted deformations increase sharply for a further drop in support pressure (not shown). This finding is in agreement with closed form solutions, given by Risnes et al. (1982), for elastic-plastic (c' = 0) soil which indicates collapse of the well if the supporting fluid pressure is dropped below the in-situ fluid pressure causing inflow to the well.

From the point of view of the design of casings through oil sand strata the minimum long term pressure would be the in-situ fluid pressure. Short term undrained construction pressures could be much less than this particularly if gas has no time to evolve.

From the point of view of a production well such analyses indicate that a supporting screen must be installed before the fluid pressure in the well can be dropped below the in-situ pressure and inflow achieved. The predicted inflow rates based on a radial flow consolidation analyses for both gassy and non-gassy or saturated conditions are shown in Fig. 4. These rates are for a drop in fluid pressure from the in-sute value of 3500 to 500 kPa and with a screen to carry the effective load. It may be seen that the flow rates are higher by a factor of about $2^{1/2}$ for the gassy soil. But more important, the duration of flow for the gassy soil is much greater, so that the volume of flow is very much greater for the gassy soil. The steady state flow rate is very low and since the skeleton essentially does not change volume, the majority of flow is coming from expansion of the fluid due to the pressure drop. This contrasts with the usual consolidation process in soil where the majority of flow is caused by expulsion of essentially incompressible fluid due to compression of the soil skeleton.

In practice it is found that a screen inhibits flow into the well. Better production is obtained without a screen. However, sand does come into well and must be bailed out. The reasons for the better production without a screen may be:

(a) The screen clogs and inhibits flow.

(b) By bailing out the unscreened hole a larger well results which analyses show give rise to greater production.

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The effects of soil model on calculated settlements due to tunnelling

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Modern numerical methods have reached a point where they can be now used to provide a reasonable indication of the deformations which will be induced by tunnelling in soft clay provided that due consideration is given to the modelling of the construction process and soil behaviour. This present discussion is concerned with the modelling of soil behaviour with particular reference to the long term deformations caused by the construction of a shallow sewer tunnel using a full face tunnel boring machine in a soft silty clay at Thunder Bay, Canada (see Belshaw & Palmer, 1978; Palmer & Belshaw, 1980; Rowe & Lo & Kack, 1983; Lo & Rowe, 1982).

For the Thunder Bay clay, the value of drained Young's modulus determined in triaxial extension is approximately twice the value in compression. If we now consider the stress paths actually followed by the soil and perform stress path tests for the stress paths above the crown and at the springline, we find that the appropriate modulus is much closer to the extension modulus than to the modulus in compression and consequently in analyses, the extension modulus should be the value adopted.

This assertion raises the question as to how important is the modulus of the soil. If we assume a perfectly elastic soil behaviour and adopt a realistic modulus profile, then the analysis would provide a very poor indication of the settlement for the Thunder Bay tunnel. One might be tempted to adopt a much lower Young's modulus above the tunnel and this does indeed increase the magnitude of the centreline settlement. However even if we reduced the modulus sufficiently to give a reasonable estimate of centreline settlement, the shape of the settlement trough would be quite at variance with observed behaviour. In fact, elastic analyses will not provide good estimates of settlement induced by the construction of lined tunnels in soft soils if these is significant plastic failure within the soil. The effect of local plastic failure can be easily considered by performing an elasto-plastic analyses. In addition to the elastic parameters we require the cohesion of the soil c, the angle of internal friction ϕ and a dilatancy angle ψ which will be zero for plastic deformation at constant volume. If we perform an elasto-plastic analysis (adopting a reasonable modulus), it is found that the local yield within the soil significantly increases the magnitude of the calculated settlement and gives rise to a steeper settlement trough which is more in accordance with observed behaviour.

These results indicate that if we adopt a realistic modulus profile for the soil, then we must also consider the effect of the plastic flow characteristics of the soil if we are to obtain a reasonable settlement trough. However ccepting the need for an elasto-plastic analysis, there still remains the question as to how critical is the determination of the modulus profile.

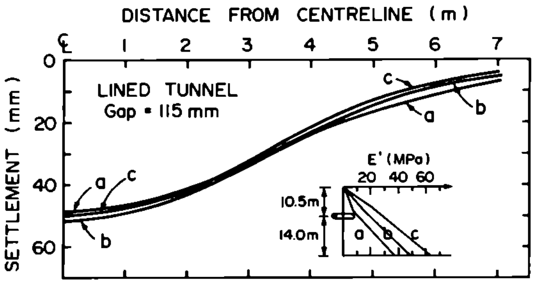

FIG. 5 : Settlement Profiles for the Thunder Bay Sewer Tunnel Elasto-Plastic Analyses.

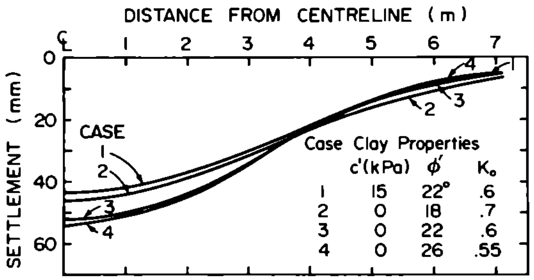

FIG. 6 : Effect of Strength Parameters and K_o on the Calculated Elasto-Plastic Drained Settlement Profiles.

Figure 5 shows the results from three elasto-plastic analyses which were performed for the modulus profiles (a), (b) and (c). Two observations can be drawn from these results. Firstly, modulus profile (a) gives the smallest centreline settlement and the widest settlement trough. The higher modulus profile (b) gives the greatest centreline settlement. Thus it is not necessarily conservative to adopt a low modulus. For soft soils and small to moderate loss of ground, the calculated settlement may be greater and the settlement trough steeper if the higher extension modulus is adopted.

At first sight this result may be surprising, however the explanation is really quite simple. For a decrease in mean stress as occurs in the critical areas above the tunnel, elastic deformations will involve elastic expansion for Poisson's ratio less than $1/2$. If a lower Young's modulus is adopted this implies a significant increase in the volume of soil above the tunnel which quickly fills the void around the tunnel and hence reduces settlement. Of course, in reality, this does not actually occur since in extension the actual elastic volume increase is quite small and so there can be significant plastic deformation of the soil as it moves in to fill the tail piece void. Thus use of the extension modulus gives more realistic settlement because it permits more plastic strain to occur. It should be noted that there is a limit to the magnitude of contained plastic deformations which is related to the modulus of the soil. Thus nothing is to be gained by assuming unrealistically high modulus values as denoted by profile (c). There is a rather interesting interaction between elastic and plastic deformation and the most realistic soil parameters. Grossly overestimating or under-estimating the modulus may both result in smaller calculated settlements.

The second observation is that in this case the effect of the assumed modulus profile on the calculated settlement is not great. Thus provided one makes a reasonable attempt to determine a realistic modulus variation with depth the calculated surface settlements are not particularly sensitive to the precise value of Young's modulus adopted. However this observation only holds true if there is a small or moderate loss of ground. The larger the loss of ground, the more critical is the distribution of modulus with depth because of the greater potential for elastic and plastic straining.

Figure 6 shows the effect of the strength parameters and K_o on the calculated drained settlement profile. In obtaining these results it was assumed that K_o was related to the angle of friction ϕ by typical correlations. Now if the strength parameters had been held constant while K_o was varied it would be found that the lower the value of K_o, the larger the settlement. Thus when K_o is related to ϕ, the maximum settlement and steepest trough is obtained for the highest friction angle with the correspondingly low value of K_o. In this case the effect of K_o dominates the effect of the friction angle ϕ and hence one needs to be a little careful in treating K_o as a dependent friction of ϕ. If in doubt, it is conservative to adopt a low value of ϕ and then use this to deduce a value of K_o by empirical correlation.

If we now adopt the actual soil parameters determined from a detailed laboratory and field investigation (Ng et al., 1985) and calculate the surface settlements, we can compare the observed and calculated settlement troughs for two values of the GAP parameter as shown in Fig. 7. Here the GAP is a measure of the lost ground (see Rowe et al., 1983; Rowe & Kack, 1983). A gap of 90 mm corresponds to the physical difference in diameter of the tunnelling machine and the lining. The gap of 120 mm allows for some additional lost ground due to movement into the face and overcutting.

Comparison of the calculated and observed settlement profiles indicates encouraging agreement and suggests that reasonable estimate of the deformations can be obtained using a relatively simple elasto-plastic soil model where the required modulus and strength parameters can be readily determined from conventional techniques.

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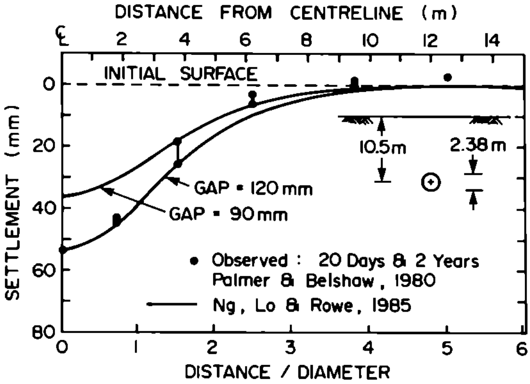


FIG. 7 : Observed and Calculated Settlement Profiles for Two Values of the GAP Parameter.

Direct analysis of undrained soil deformations

C.SAGASETA, Panelist, University of Santander, Spain

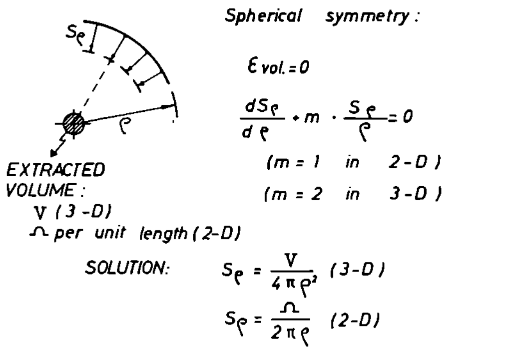


FIG. 8 : Undrained Displacement about a Single Cavity in an Infinite Medium.

In any boundary value problem, there is input, either in terms of forces or displacements imposed at some points. In order to obtain a solution a set of equations, formed by the equilibrium and constitutive equations, are required. The output results are the streses (or forces) and/or strains (or displacements) elsewhere in the soil mass.

The equilibrium equations are in terms of stresses only, whilst in the constitutive equations contain both stresses and strains. In cases in which the input and output are only stresses, it is possible to use only the equilibrium equations, eliminating the strains from the problem and so by-passing the complexities included in the constitutive equations. A classical example is the limit analysis method used in bearing capacity analysis. There are also some cases in which the input and output are in terms of strains (or displacements) only. Then it is possible to try a similar method, eliminating the stresses from the problem.

For incompressible materials, it happens that regardless the complexities of the constitutive law, the condition of no volume change must be fulfilled. So, for any soil model, the sum of the three stress-strain equations corresponding to the three normal strains will lead to the incompressibility equation:

$$\varepsilon_{ii} = 0$$

This equation is very simple and independent of the soil model, but is not sufficient for the determination of the three components of the displacement. However, in some cases, the symmetries of the problem provide the necessary equations, and in others, additional reasonable assumptions can complete the problem.

A brilliant example is the Strain Path Method, proposed by Baligh and his co-workers at the MIT for the analysis of deep penetration problems. In this case, there is an equilibrium equation, axial symmetry provides an additional equation, and the third comes from the assumption of irrotational deformation, which in some cases leads to the exact solution (Baligh, 1984) and in others to a reasonable first approximation.

In this contribution, a different application of this method is presented, for the evaluation of soil movements due to the collapse of small cavities near the surface. As it will be shown, the method can be extended to cases in which the displacements are known along a line or surface.

The basic case is the collapse of a small spherical cavity at infinite depth in a

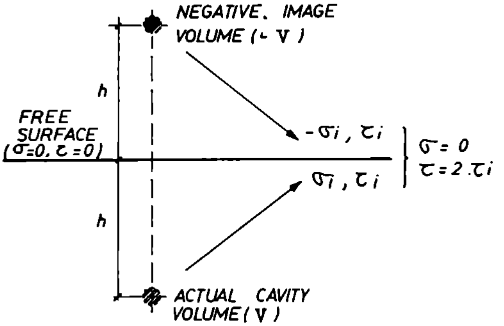
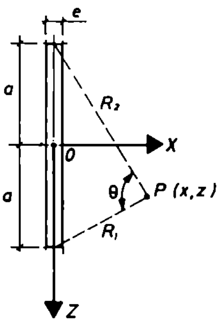


FIG. 9 : Method of Images used to Model the Presence of a Free Surface.



$$S_x = - e \cdot \frac{\theta}{2\pi}$$

$$S_z = e \cdot \frac{1}{2\pi} \cdot \ln \frac{R_1}{R_2}$$

FIG. 10 : Plane Strain Solution for a Rectangular Slot in an Infinite Medium.

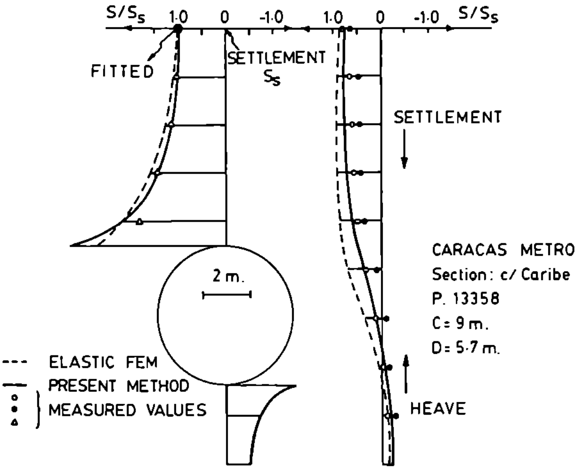


FIG. 11 : Observed and Calculated Vertical Displacement Adjacent to the Caracas Metro.

homogeneous incompressible soil, Figure 8. The spherical symmetry provides the necessary conditions for a complete solution. If the volume of the collapsed cavity is V, then the inward radial displacement at a distance ρ from the cavity is:

$$S_\rho = \frac{V}{4\pi\rho^2}$$

in three dimensions, and:

$$S_\rho = \frac{V}{2\pi\rho}$$

in plane strain.

If the cavity is near the surface, the problem is of a different kind, because the free surface requires a stress boundary condition ($\sigma = \tau = 0$). However, this condition can be partially fulfilled by ignoring the surface and considering a virtual cavity, a mirror image of the actual one with respect to the surface, and which expands a volume V, Figure 9. This will produce a condition of zero σ at the surface. The shear stress, τ, will not be cancelled, so the solution is only an approximation, but its effect will be local and small, unless the cavity is very close to the surface.

The single cavity problem can be integrated to any finite thin cavity within the soil. The integration can be of closed-form in many cases, and in others it can be performed numerically. An alternative in this latter case is the discretisation of the cavity into rectangular slots of uniform thickness, for which a simple closed-form solution exists, Figure 10.

As an example, this procedure has been applied to the evaluation of soil movements around tunnels in clay. The ground loss is represented by the collapse of a cavity along the tunnel perimeter and whose thickness is maximum at the crown and decreases linearly until zero at the invert. The absolute value of the thickness is obtained from the ground loss, if it is known, or by fitting the resulting soil movements to the measured value at some significant point.

Figure 11 gives a comparison for an experimental section in the Caracas Metro (Oteo and Sagaseta, 1982), the cavity thickness has been defined so as to fit the value of the surface settlement above the crown.

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Non-linear deformations in London clay adjacent to deep excavations

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During the early 1970's linear elastic finite element analyses were used to study the displacemetns of the ground around deep basements constructed in the London Clay. Using back analyses to derive the parameters, Ward and Burland (1973) computed the movements around the 18m deep excavation at the Houses of Parliament in London. Figures 12 and 13 show comparisons between their computed results and measurements publis;hed by Burland and Hancock (1977). In many respects the predictions were very good, but two problems were apparent:

(a) Although the magnitudes of displacement were computed fairly well, the pattern of movement around the excavation was much less well predicted, as is evident in Figure 13.

(b) The stiffnesses derived by back analyses were between three and five times greater than had been measured in high quality laboratory tests on triaxial samples. This was disturbing since the laboratory tests themselves gave very consistent results and it therefore seemed likely that the laboratory measurements represented a real feature of the soil's behaviour.

There was a small amount of laboratory data which suggested that for very small strains following a rest period or an abrupt change of stress path the stiffness of the clay was much higher than it was at the larger strains normally measured in laboratory tests. It therefore seemed possible that the reason for the incorrect prediction of the pattern of movement around the excavations was that the behaviour of the clay was not linear. Rather, it had very high stiffness at small strain and smaller stiffness as the straining proceeded. These features have been discussed in detail by Simpson, Calabresi, Sommer and Wallays (1981).

In 1976 computations were required for the design of the basement of the British Library at Euston. This basement was bigger in area and deeper than the previous basements and its geological setting suggested that the coefficient of earth pressure at rest in the ~lay, K_o, would also be higher. It was there-ore considered questionable whether linear elasticity could be used to extrapolate from the field measurements around other excavations. At least the possibility of other types of behaviour had to be investigated.

As part of the design work for the British Library, Simpson, O'Riordan and Croft (1979) therefore developed a constitutive model for the behaviour of London Clay which had the following main features (illustrated in Figures 14 and 15):

(a) High stiffness at small strains using linear elastic parameters but with stiffness proportional to mean normal stress.

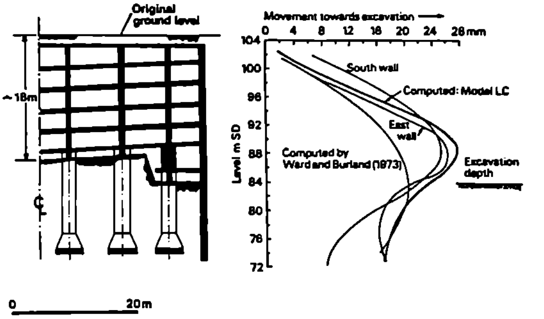

FIG. 12 : Computed and measured movements of the retaining wall at the Houses of Parliament.

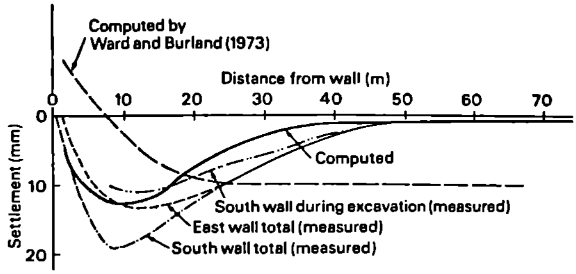

FIG. 13 : Computed and measured settlements outside the retaining wall.

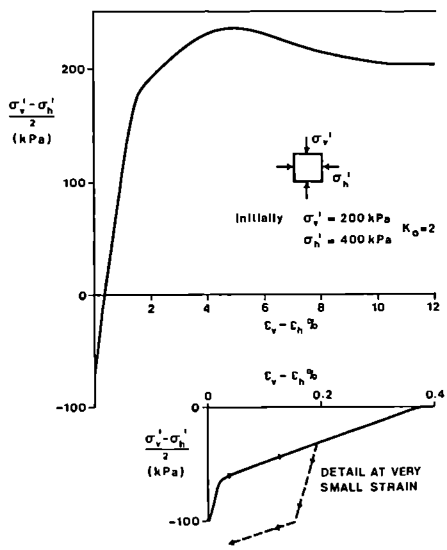

FIG. 14 : A typical stress strain curve for Model LC.

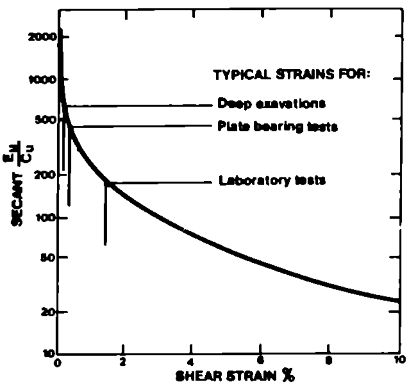

FIG. 15 : Variation of apparent stiffness with strain.

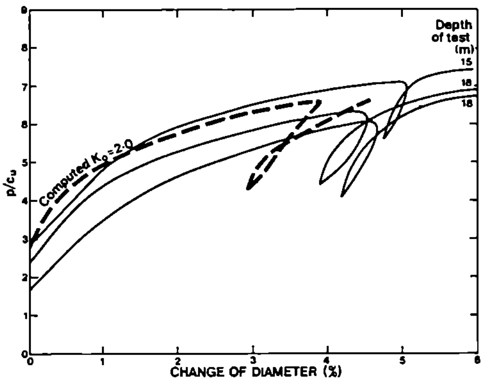

FIG. 16 : Computed and measured results from a pressuremeter test.

(b) An intermediate range of lower stiffness in which linear elastic parameters are again used.

(c) Plastic yield using a flow rule similar to that of Cam-clay but without the assumption of associated flow.

The model was named Model LC and was calibrated so that it would predict the stress strain curve derived from typical high quality laboratory tests. It was subsequently shown that the results of self-boring pressuremeter tests could be computed successfully (Simpson et al., 1981), as shown in Figure 16.

Independently researchers at Imperial College and other centres had been carrying out laboratory tests in which the behaviour at very small strains was measured accurately. It has been interesting to find that, at least qualitatively, their measurements have been consistent with the London Clay model.

The same model was used in finite element computations for the Houses of Parliament excavation. Figure 12 shows that the magnitudes of movement were again computed correctly, but this time the pattern of movement around the excavation was represented rather better, as seen in Figure 13.

The model was subsequently used for computations of the movements around the British Library at Euston. This project has a basement 25m deep which extends through the London Clay and 5m into the underlying Woolwich and Reading Beds. To date, field measurements are available for the first 5m of excavation and agreement between computed and measured movements is encouraging. Early indications suggest that movements as the excavation proceeds may be less than the computed values.

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Summary of Session 1C: Decision theory and probability
Factor of safety and risk analysis
Coefficient de sécurité et analyse des risques

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SYNOPSIS The selection of factors of safety in modern geotechnical engineering design depends on the methods of analysis and the calculation models, the ways in which factors of safety are defined, the assessment of appropriate geotechnical parameters, and the degree of risk. These key areas formed the basis for discussion in the Session. Examples were shown where factor of safety selection was meaningless when critical failure mechanisms had not been correctly identified. Other examples showed that prescribed factors of safety would not lead to a common degree of reliability, because of the different calculation models and input parameters assumed by different designers. Opinion was divided on the relative merits of single 'global' factors of safety compared with partial factors. Several examples were given of the complexity of assessing soil strength parameters from the many types of laboratory and in-situ tests available, and of the important effect this has on factor of safety selection. Risk analysis was shown to be potentially valuable in allowing objective assessment of key assumptions and judgements made in design. The use of statistical and probabilistic methods was the subject of vigorous debate.

1 INTRODUCTION

The primary aims of the Discussion Session were to focus on the problems of selection of factors of safety in geotechnical engineering and to discuss the potential contribution of risk analysis. Emphasis on practical applications was encouraged and the following three major topics were identified for discussion:

Topic 1. Influence of method of analysis and definition of factor of safety.

Topic 2. Influence of method of assessment of soil parameters.

Topic 3. Influence of consequences of failure and reliability of data (risk analysis).

Topic 1 covered (i) the identification of critical failure mechanisms in the design process, (ii) the difficulties when different types of analysis were applied to the same problem, and (iii) the question of how factors of safety should be defined, and, in particular, whether partial factors were preferable to single 'global' factors. Detailed analysis and selection of factor of safety were of limited value if the critical failure mechanism had not been properly identified. Different methods of analysis could often lead to different margins of safety even when identical parameters were used. Perhaps most complicated of all was the definition of factors of safety: how logical was it for a designer to opt for a single 'global' factor when different degrees of uncertainty were associated with different geotechnical parameters? Was it more logical to adopt partial factors which could reflect these differing degrees of uncertainty?

Topic 2 was concerned with the difficulties of assessing strength parameters. The appropriate factor of safety selected for a given problem depended on how the soil strength had been derived. The observed undrained shear strength, for example, depended on how it was measured, and may differ significantly when obtained from a triaxial, or direct shear apparatus or derived from a cone penetrometer, field vane or pressuremeter. Effects of anisotropy and progressive failure were further complications.

Topic 3 was associated with risk analysis and its relation to selection of suitable factors of safety for design. Risk analysis and its application needed to take into account, amongst other things, (i) the consequences of failure or of excessive deformations (ii) the expected loading and changes of geometry during the design life and (iii) the probability of error in the geotechnical parameters assumed in the design. Economic constraints were also important.

In order to obtain a worldwide view on the subject of factor of safety selection and risk analysis, the following speakers were invited to make short contributions to the Discussion Session and introduce particular points relating to the three identified topics:

Dr K Fujita	(Japan)
Ms Hynes-Griffin	(USA)
Dr R Lancellotta	(Italy)
Dr A W Malone	(Hong Kong)
Dr T L L Orr	(Ireland)
Dr A Uriel	(Spain)

2 INFLUENCE OF METHOD OF ANALYSIS AND DEFINITION OF FACTOR OF SAFETY

2.1 Critical Failure Mechanisms

Dr Fujita illustrated the importance of designers identifying correctly the critical failure mechanism by describing a catastrophic failure in Japan of a 23 m diameter temporary works sheet pile cofferdam. This was built in a river for construction of a bridge pier, was circular in plan and designed to retain 7 m of soft clay within the river bed and 6.5 m of water depth above the river bed. After installation, each ring-beam bracing the cofferdam was prestressed. When the excavation for the seventh ring-beam was almost complete (Figure 1), the sixth ring-beam failed in buckling. Complete failure of the cofferdam occurred, resulting in the loss of eight lives.

Considerable attention had been paid by the designers to possible failure of the H-section

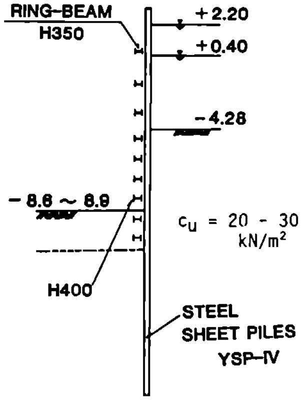

Fig.1 Section through wall of cofferdam immediately prior to failure.

ring beam by buckling. The mode of failure considered was buckling in-plane, that is to say, in a plane perpendicular to the axis of the cylindrical cofferdam. Taking into account the confining effects of the soft clay outside the cofferdam, the factor of safety against in-plane buckling of the ring-beam had been calculated to be 2.5. The failure was caused by buckling of the ring-beam but in an out-of-plane mode, i.e. in a direction parallel to the axis of the cylindrical cofferdam.

Dr Fujita's example showed that despite the effort devoted by the designers to ensuring a suitable factor of safety against failure by buckling, in the event the mode considered proved not to be the critical mechanism.

2.2 Methods of Analysis

An example was shown in the Introduction to the Session of the design of a diaphragm or sheet pile wall retaining stiff clay in a deep excavation, with the wall propped or anchored at one level. Different designers made different assumptions about the development of wall friction. Given that the clay beneath the excavation swelled in the long term as a result of stress-relief (and hence heaved relative to the wall), there was a case for assuming the angle of wall friction, δ, was equal to ϕ', the angle of friction in terms of effective stresses for the clay. Many designers, however, restricted the value of δ, somewhat arbitrarily, to ½ ϕ', or even to zero. Differences in assumptions such as this in the method of analysis could make a very large difference to any calculated factor of safety. For $\phi' = 25°$, for example, values of passive earth pressure coefficient K_p of 4.0 and 2.4 were obtained for assumed values of $\delta = \phi'$ and $\delta = 0$ respectively. Prescribed factors of safety, therefore, would not lead to similar margins of safety being obtained by different designers unless the calculation model and the assumptions in the calculations were well-defined.

Dr Malone described how, in Hong Kong, the Geotechnical Manual for Slopes (Geotechnical Control Office, 1984) stipulated minimum values of factor of safety for design of slopes in weathered materials, and also made recommendations on the stability calculation model to be employed (including recommendations on the method of measurement of the input parameters for the calculation).

A source of uncertainty which could lead to significantly different calculated factors of safety was the selection of trial slip surfaces. Dr Malone illustrated this by describing how, under humid sub-tropical climatic conditions, rock masses close to the surface of slopes could weather to give a complex pattern of fresh corestones and matrix material in various states of decomposition. The simplification of such geological complexity into a layered model distinguishing "soil" from "rockhead" was open to question and often resulted in different trial slip surfaces being assumed by different designers.

Another speaker argued that lower bound plasticity models gave inherently safe results and therefore the use of equilibrium stress fields combined with fully softened strengths could be used to guarantee the avoidance of a collapse condition. It was further argued that, with this approach, problems of selecting values of factors of safety could be overcome by assuming a 'worst credible' scenario: by adopting an inherently conservative method of analysis (the lower bound plasticity approach) and worst credible values for soil parameters, loads and water pressures, no factor of safety would be required when considering a collapse condition.

2.3 Definitions of Factor of Safety

In the Introduction to the Session, the design of a propped diaphragm wall in stiff clay was used as an example illustrating how a number of different methods of defining factor of safety led to different values of these factors being necessary to achieve the same penetration (and hence the same margin of safety). The depth of penetration, d, below excavation level was shown for different factors of safety (Fig 2). F_p and F_r were variations of the conventional single 'global' factor of safety expressed in terms of rotation about the prop, with the factor of safety defined as the ratio of the moment of the resisting forces to the moment of the activating forces. In contrast, a partial factor of safety F_s was applied to the soil strength, so that c' and tan ϕ' were each divided by F_s; d was then found for the case where the calculated resisting moment equalled the activating moment with the active and passive pressure calculated from the reduced strength parameters (Padfield and Mair, 1984).

The values of the variously defined factors of safety must be significantly different to achieve the same depth of penetration. Any discussions on factor of safety selection should therefore be very clear about the definition. Of most significance was the difference between the single 'global' factor approach and the partial factor approach.

A speaker drew attention to the Danish Codes

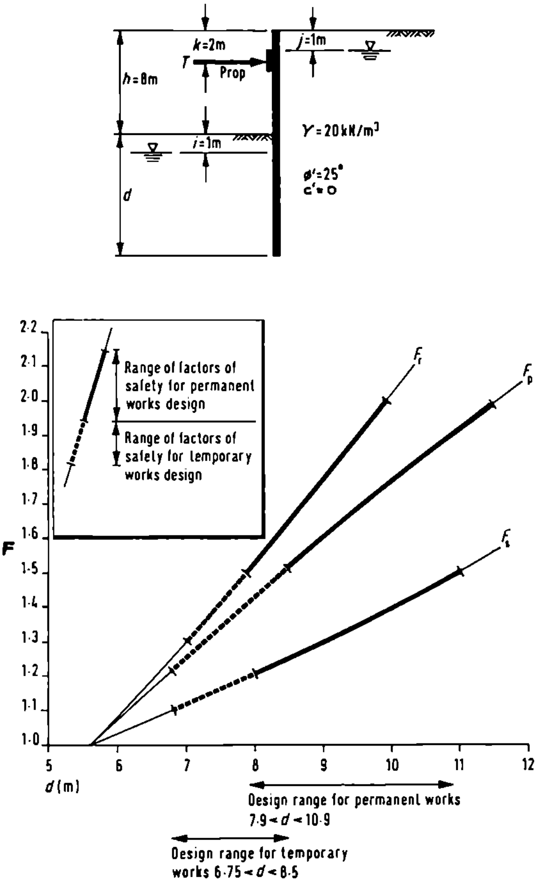

Fig.2 Factors of safety for different depths of penetration of a diaphragm wall in stiff clay

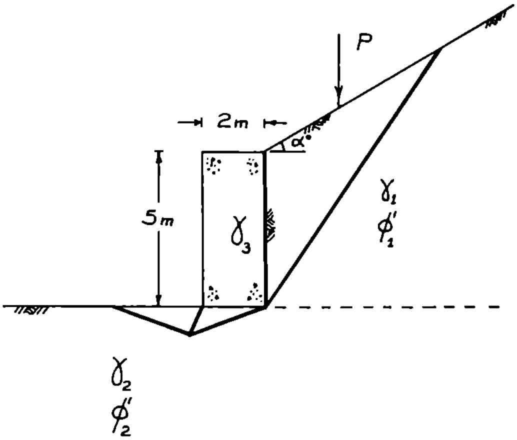

Fig.3 Stability of a gravity retaining wall.

of Practice formulated in terms of partial factors and asked for experiences of their use in practice. In response to this, experiences with the use of partial factors in Finland were described: it was felt by designers that the approach apparently gave more 'accurate' designs, although the designers did not generally calculate what equivalent single 'global' factors were obtained. Experience in Finland suggested that the use of the partial factor approach led to larger equivalent single 'global' factors than were obtained previously, and hence resulted in less economic designs.

Another speaker referred to the large body of experience acquired over many years by practising engineers using single 'global' factors of safety linked to particular methods of calculation. While accepting the validity of a partial factor approach for superstructure design, he considered that this approach could lead to very large variations in foundation engineering design, with significant cost implications.

A speaker pointed out the important difference between limiting deformations under working conditions and preventing collapse. For example, relatively large partial factors were often applied in stability calculations to limit deformations for a retaining wall or foundation, whereas smaller values were adopted to prevent collapse for other types of problem such as the stability of a cut slope or embankment.

Dr Orr illustrated the value of a partial factor of safety approach with the example shown on Figure 3. The stability of a gravity retaining wall resting on cohesionless soil and backfilled with granular fill had been investigated. It was assumed that the collapse condition would occur by the formation of a mechanism consisting of three rigid blocks sliding on straight slip planes as shown on the figure. The variable parameters in the analysis were the unit weight and angle of friction of the fill material (γ_1, Φ_1') and of the foundation soil (γ_2, Φ_2'), the unit weight of the wall (γ_3), and the surface load P. Choosing characteristic values for these parameters and assigning standard deviations, a reliability analysis was undertaken to determine the values for each of these variables for which failure was likely to occur.

The results of the analysis indicated that, for the particular set of assumptions, the parameter having the greatest influence on the reliability of the wall was Φ_2' and the next most influential parameter was Φ_1'. These parameters were also those with the greatest uncertainty. Dr Orr felt that it perhaps did not matter whether single 'global' safety factors or partial factors were used; but if partial factors were adopted, the greatest margin of safety should be applied to the soil strength parameters since these had the greatest variability. He also felt that a partial factor of unity was appropriate for the unit weight of soil in most stability calculations.

Dr Orr expressed the view that it was unlikely that prescribed factors of safety, either 'global' or partial, would lead to a common degree of reliability for all geotechnical structures. He therefore felt that it would be more sensible for design codes to give only general guidance concerning appropriate values of factors of safety. They should allow designers to use their engineering judgement and practical experience of the actual ground conditions, and to choose suitable factors consistent with the calculation model.

A speaker said that there was at present a lack of a pool of experience on which to base a change to a partial factor approach in geotechnical engineering. He believed that single 'global' factors had the advantage that they could often be related to local experience. Nevertheless, he felt that in circumstances where suitable partial factors could be selected, the method was an equally valid approach to design.

3 INFLUENCE OF METHOD OF ASSESSMENT OF SOIL PARAMETERS

A speaker argued that there was a need for clarification of the 'strength' used in calculations and called for strengths to be measured in a manner appropriate to the problem. He referred to the Bulk Strength, which was fully representative of the strength under conditions of uniform loading and was more applicable to foundation design. There was also the Average or Operational Strength, which allowed for anisotropy and progressive failure around a slip surface. This was the strength obtained, for example, in a back-analysis of a failure of a slope or embankment for which the pore water pressures were measured or could be estimated reasonably reliably. Finally there was Measured Strength, which was often used in design with appropriate factors of safety, because of previous experience. Bulk Strength, Average or Operational Strength and Measured Strength were usually three different quantities, and the use of factors in any form applied to soil strength therefore needed to be treated with caution. This view was endorsed by another speaker.

A contribution showed the results of a sensitivity study conducted on a propped diaphragm wall supporting an excavation in London Clay. The study examined the effect of different methods of stability assessment and input assumptions on the required depth of penetration. Significant changes in depth of penetration were obtained for ostensibly small alterations in design assumptions. In particular, attention was drawn to the marked sensitivity to the assumed value of the shear strength parameter c'. Reference was also made to the very different effective stress paths to failure followed in conventional laboratory testing compared with those experienced by elements of soil adjacent to a diaphragm wall at the base of an excavation in stiff overconsolidated clay; the soil was at or near a state of passive failure combined with a significant reduction in vertical stress.

The relevance of appropriate stress-path testing was also mentioned by another speaker when describing the selection of shear strength parameters for slope stability calculations. The consolidated undrained compression test with pore water pressure measurement was conventionally undertaken, although the stress path followed was very different from that experienced by an element of soil adjacent to a potential slip as the pore water pressure increased when the groundwater table rose.

The difficulties in assessing the shear strength parameter c' was illustrated by a speaker describing a case history of a road embankment which was constructed using a stiff London Clay fill. Specimens had been consolidated under different cell pressures in the triaxial apparatus and then subjected to conventional drained compression tests. The strength parameter ϕ' was found to vary between 16.2° and 25.0° with an average of 21.2°, and c' from 6.9 kN/m^2 to 56.2 kN/m^2 with an average of 30.8 kN/m^2. Faced with such a large range in these parameters, particularly in the case of c', the designers had a major problem in determining reliable factors of safety for the embankment. A rational method of estimating how c' varied with the water content of the clay was described in terms of Hvorslev's failure criterion; reference was made to a description of the method given by Wroth and Houlsby (1985).

In summary, selection of suitable factors of safety, methods of analysis and the methods of obtaining soil parameters for different categories of problem could not be separated. Factors of safety for one type of problem analysed a particular way using soil parameters determined from a certain laboratory or in-situ test method should not necessarily be applied to a different set of conditions. Correlations between different 'complete' design approaches were needed.

4 INFLUENCE OF CONSEQUENCES OF FAILURE AND RELIABILITY OF DATA (RISK ANALYSIS)

Dr Malone described the approach to slope stability design in Hong Kong. Most landslides there occurred during intense rainstorms, and so groundwater conditions in stability calculations should correspond to extreme rainfall conditions. Lower factors of safety were used when rarer rainfall conditions were assumed. Allowable factors of safety in Hong Kong depended on the consequences of failure. The minimum allowable values for a 10-year return period rainfall were as given in Table 1, based on the use of Janbu's simplified method. Dr Malone believed that this method may be somewhat conservative.

Lower factors of safety were permitted in cases where designers made use of back-analyses of existing cuttings which had withstood extreme rainstorms, and where values of soil parameters in the calculation model were known more reliably. In assessment of the stability of existing slopes, seasonal and storm piezometric responses were measured, and

TABLE 1

Minimum permissible factors of safety for design of cut slopes in Hong Kong (Geotechnical Control Office, 1984)

Factor of Safety	Consequence of Failure
1.4	High risk to life
1.2	Low risk to life
greater than 1.0	Negligible risk to life

storm seepage and surface water run-off observed. For calculations utilizing more reliable data such as this, a minimum factor of safety of 1.2 was permitted for the case of high risk to life.

Dr Uriel described two case histories concerning diaphragm walls supporting temporary and permanent excavations. In one case, complete collapse of the wall took place, whereas excessive deformations rather than collapse occurred in the other. The examples illustrated the importance of designers considering both ultimate and serviceability limit states, and the different factors of safety and degrees of risk associated with each of these. He mentioned that failures could often be attributed to inadequacy of the site investigation or to a misunderstanding of the likely failure mechanisms. Moreover, the factor of safety adopted in design could not cater for gross human errors. It was essential that proper account was taken in design of the mode of failure, the method of analysis, the reliability of soils information and variability of soil parameters, as well as any likely change in groundwater conditions. Similarly, risk analysis needed to consider the probability of an event, the vulnerability of the resisting elements and the consequences of failures.

Ms Hynes-Griffin described the use of risk analysis by the US Corps of Engineers in evaluating seismic stability of earth dams. Although a very simplified method was used, it provided an organized framework for assessing both risks and associated costs. The approach was particularly informative when considering dams that were judged to be marginal or unsafe when analysed by the conventional methods which were usually conservative. The approach to the risk analysis was to examine pool levels, the seismic hazard, the earthquake and post-earthquake stability of the dam (embracing all site characteristics and engineering analyses), the downstream consequences and the remedial actions. Pool level and ground motion studies were aimed at evaluating the probability of an earthquake occurring while the pool level was high. Engineering analyses were then used to determine the boundary between safe and unsafe combinations of pool level at the time of the earthquake and the peak acceleration at the site.

The risk analysis then proceeded to examine the downstream consequences of failure of the dam, with failure defined as catastrophic loss of the reservoir. Downstream consequences included economic and environmental losses as well as loss of life. Potential losses increased rapidly as the pool level rose. Ms Hynes-Griffin described how the risk analysis approach gave a framework for examining and selecting alternative remedial actions. If, for example, a dam was strengthened by adding a berm or by performing in-situ densification, losses would only be incurred at higher pool levels and the reduced level of risk could be quantified. Other alternatives which could be similarly evaluated were permanently lowering the pool level, purchasing downstream property and relocating the inhabitants, or installing effective early warning systems.

Dr Lancellotta showed an example in which the spatial variability of soil parameters was assessed. The scatter obtained in measurement of soil parameters was mainly due to (i) real spatial variation of these parameters and (ii) random errors introduced by measurement of the parameters. It was important to be able to separate these two components. Dr Lancellotta's example concerned the use of indirect statistical methods. He described SPT results obtained from 18 boreholes at a site on a thick deposit of sand. The SPTs were performed under carefully controlled conditions. In one area of the site, much more scatter was obtained than elsewhere. Dr Lancellotta's analysis showed that the variance due to random errors was similar for the two areas, but the largest variance was due to real spatial variability.

Two speakers gave examples of the application of probability theory to problems of stability and earth retaining structures.

A speaker described the application of probability methods for assessing the risk of failure of a 20 km length of highway embankment on soft ground. The analysis was performed in two steps: (i) analysis of soil strength data over the length of the project taking into account spatial variability, (ii) probabilistic analysis of embankment stability leading to an overall characterization of uncertainties (and, possibly, probability of failure). The principal variables considered were the heterogeneity of the soils, the embankment height, the spatial rate of change of the soil properties and the spacing and location of points where properties had been measured. The results obtained gave probability density distributions as a function of failure location and length of embankment, embankment height and slope angle.

Another speaker expressed the view that probability theory offered advantages which could not be achieved by conventional approaches to design. It should therefore be viewed as an additional tool by designers. In its simplest terms, risk of failure for a given factor of safety depended on the variability of the input parameters. The speaker referred to early work by Bishop relating to coefficient of variation of strengths, and he said that many parameters displayed a normal distribution, although other distributions, if known, could be accounted for.

Comments were made by a speaker on the usefulness of statistical analyses of data in design and on the relevance of probability theory. Statistical analysis of data was useful for comparative purposes, although signficant differences were usually obvious. There were dangers in the use of statistical methods as a means of deriving design parameters in limit state design. In the use of probability theory, it was essential to take full account of all information about parameters and methods of design. It was essential to have sufficient data. If a probability approach did not take account of all the relevant factors, it could represent a retrograde step from current design practice. The speaker believed that, while in principle data could be more vigorously synthesised by statistical and probabilistic approaches, in practice engineers lacked the necessary tools. At present it was 'engineering judgement' which did the synthesis of data and designers usually selected the 'worst credible' rather than the 'most likely' scenario.

The use of statistical and probabilistic methods in geotechnical engineering became the subject of vigorous debate. A speaker said he believed that probability theory applied to soils data was wrong in principle and had no valid mathematical basis. Taking as an example the angle of friction of a sand in dense and loose states, he pointed out that the value of ϕ' converged at critical state conditions; this was a minimum value and, as such, represented a cut-off to any 'normal distribution'. He provided an example of the application of a probability approach to derive design values for a sand in loose and dense states. If a normal distribution was assumed for ϕ', the approach could show that the design value for the sand in a dense state would be less than the critical state value, which conflicted with the known mechanical behaviour of sands.

Supporting the use of probability theory in design, another speaker said that he believed that the current approach to design was primarily based on experience of previous failures, and this led to the use of a very wide range of global factors of safety. Currently, probability theory was employed only when all else failed. In his view, for new types of structure for which no past experience of failures existed, probability failure theory represented the way forward, and he urged that engineers should familiarize themselves with the necessary theory and methodology. He drew an analogy between the reluctance of many engineers two decades ago to accept the advent of computers and the present reluctance of engineers to consider probability theory as a tool in geotechnical design.

In response to this, a speaker pointed out that the engineering profession preferred to adopt conservative models which were based on what was known. His involvment with the use of statistical methods in practical geotechnical engineering design indicated that the variables were often too complex to be adequately dealt with by a probability approach.

Another supporter of the use of probability theory agreed with an earlier speaker's point that a thorough understanding of soil mechanics was essential to design. The use of probabilistic methods did not provide an alternative to this. He viewed probabilistic methods as complementing existing knowledge and design techniques. He felt that experience had shown that such methods could be useful in routine practice, although he accepted that some caution was necessary in their application.

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Factor of safety and reliability of design of cuttings in Hong Kong (Oral discussion)

A.W.MALONE

This contribution is about the stability factors of safety which are used in Hong Kong in the design calculations for cuttings in the weathered mantle of slopes.

Most landslides in cuttings occur during intense rainstorms, hence groundwater conditions assumed in stability calculations correspond to an extreme rainfall condition. Lower factors of safety are used when rarer rainfall conditions are assumed. Factor of safety is also tied to consequences of failure, the following minimum values being stipulated in the Geotechnical Manual for Slopes for the 10-year return period rainfall case (Geotechnical Control Office, 1984) :

1.4 high risk to life

1.2 low risk to life

$>$ 1.0 negligible risk to life

It is understood that the value 1.4 was chosen as this value is often used for the slopes of large earthfill dams.

The Geotechnical Manual for Slopes also makes some recommendations on the stability calculation model to be employed and the method pf measurement of the input parameters for the calculation. However this falls short of the complete specification that would be needed to guarantee a consistent standard of safety between cases with equal calculated values of factor of safety. Alternatively, standard of safety can be expressed as probability of failure, and this can, in principle, be calculated by regarding the parameters as random variables. In considering the application of probabilistic approaches to cutting design in Hong Kong conditions, it is helpful to examine the three principal sources of undertainty in the design calculation : the selection of trial slip surfaces, groundwater conditions and shear strength.

Selection of Trial Slip Surfaces

Under Humid Subtropical climatic conditions igneous rock masses adjacent to the surface of slopes can weather to give a complex pattern of fresh corestones and matrix material in various states of decomposition. The simplification of such geological complexity into a 2-D layered model, one surface of which represents a sloping "rockhead", often results in trial slip surfaces being positioned unrealistically.

Selection of Groundwater Conditions

The phreatic surface corresponding to a design rainstorm is usually modelled at a fixed head above this imaginary sloping "rockhead" surface, or above the dry season phreatic surface if higher, without benefit of piezometric observations of storm response.

In reality a more complex hydrogeological system exists, better modelled as 2 or more interacting inclined aquifers (Geotechnical Control Office, 1982). Local geological effects like flow in natural pipes in steep slopes, or partial occlusion by dykes, can be important, as can local leakage from water services in steeply sloping urban areas. Hence design data ought to be obtained by extrapolation of measured seasonal and storm piezometric response within the aquifer of interest at the site of the proposed cutting.

Selection of Shear Strength Parameters

The mass shear strength will depend on the disposition of corestones, but it is conventional to rely in design on laboratory measurements of matrix shear strength. The shear strength of decomposed granite in particular will vary markedly with the state of decomposition, which ought therefore to be quantitatively defined, but rarely is. The consolidated undrained compression test with pore water pressure measurement is conventional, but this does not reproduce the stress path followed in an element of soil adjacent to a slip plane as failure is approached.

Existing Slopes and More Reliable Design

It is in the analysis of existing slopes that practice in Hong Kong has come under most scrutiny, because preventive works are often very costly and very inconvenient. It has been generally agreed that lower factors of safety may be adopted for existing slopes, whilst maintaining the same adequately low probability of failure. This can be achieved at the cost of greater reliability, as is well illustrated by the case history presented by Lambe in the First Terzaghi Oration (Lambe, 1985 fig. 21). The opportunity to adopt lower factors of safety in Hong Kong has recently been introduced for the analysis of existing cuttings which have withstood extreme rainstorms, in cases where values of the parameters in the calculation model are known more reliably. Thus, for assessment of the stability of existing slopes, seasonal and storm piezometric response can be measured in the prototype, and storm seepage and runoff observed. The actual geology can be determined in inspection trenches. Foundations of adjacent structures can be examined, input geometry can be based on topographic survey and soil densities can be measured in situ.

For design calculations which utilize this more reliable data a minimum factor of safety of 1.2 is stipulated in the Geotechnical Manual for Slopes for the high risk to life case.

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Safety and safety factors in geotechnical engineering
Sécurité et coefficients de sécurité en géotechnique

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The aim of good deotechnical design is to create structures that are sufficiently safe and economic. Safety factors are introduced to take account of uncertainties in the design elements which include the geotechnical and geometric data, the loads and the mathematical model.

Many geotechnical failures occur not due to the incorrect choice of safety factors but due to the occurrence of failure mechanisms not considered at the design stage. To ensure safety all possible mechanisms should be considered and there should be a thorough geotechnical investigation.

Generally the greatest uncertainity in geotechnical design occurs in the values of the soil parameters. This material uncertainity is normally much greater than the uncertainity in the loads. To investigate the effect of parameter variability, the stability of a gravity retaining wall resting on cohesionless soil and backfilled with granular soil has been analysed. It has been assumed that the collapse mechanism consists of three rigid blocks sliding on planar slip surfaces as shown in Fig. 1. The characteristic values of the parameters and their standard deviations are given in Table I.

The design values of these parameters at which failure is most likely to occur were obtained using a Level II type reliability analysis. The ratio of the characteristic to the design values gives the partial safety factors. The variations in these partial factors with the backfill slope angle are plotted in Fig. 2.

Fig.2 shows that the parameter with the highest partial factor is ϕ_2' (the foundation soil ϕ' value) and this is followed by ϕ_1' (the backfill ϕ' value). The partial factors for the surface load, P and the concrete unit weight, γ_3 are both approximately 1.0 while the partial factors for the unit weight of the foundation soil and the backfill are also close to unity.

This investigation shows that in this case the parameter having the greatest influence on the reliability of the wall is ϕ_2' and the next most influential parameter is ϕ_1'. These parameters are also the parameters with the greatest inherent uncertainity and therefore greater reliability will be achieved if the safety margin is applied mostly to these parameters. Factors of safety of unity are appropiate for all the other parameters in this example.

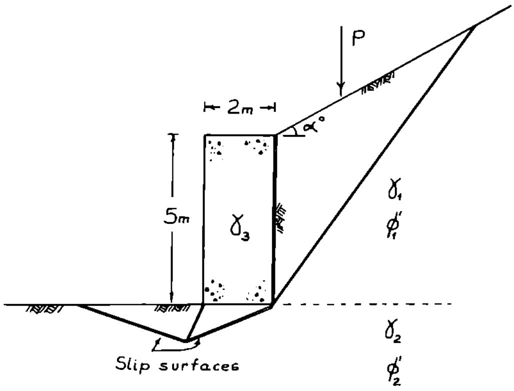

Fig. 1 Retaining Wall Collapse Mechanism

Table I Variables in Retaining Wall Analysis

	Variables	Characteristic Values	Standard Deviation	Variation %
Fill	ϕ_1'	35°	3.5°	10.0
	γ_1	18 kN/m³	1.5	8.3
Foundation Soil	ϕ_2'	35°	3.0°	8.6
	γ_2	18 kN/m³	1.5	8.3
Concrete in Wall	γ_3	24 kN/m³	1.0	4.2
Surface Load	P	50 kN	2.5	5.0

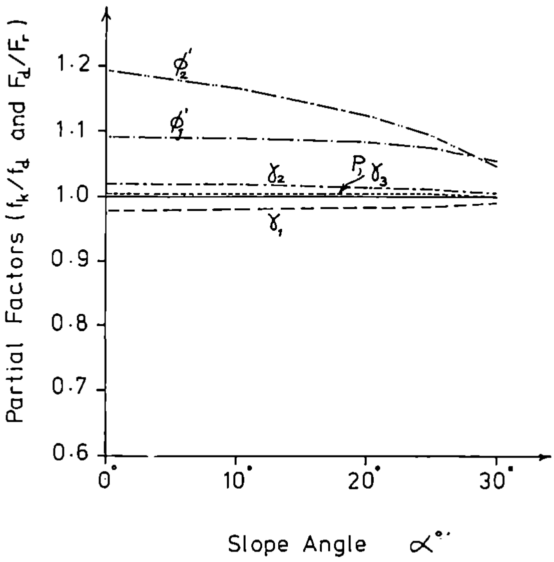

Fig. 2 Partial Factors v. Fill Slope Angle

Codes of practice should not prescribe precise values for safety factors but should give guidance as to the appropiate values. Prescribed safety factor values, whether total or partial, cannot lead to a common degree of reliability in all situations. The designer must use his engineering judgement and practical experience of the actual conditions and the mathematical model when choosing safety factor values to achieve the required margin of safety.

To summarise, greater safety will be achieved in geotechnical design by:

1. Carrying out a thorough geotechnical investigation,
2. Considering all possible mechanisms and design situations which could cause failure,
3. Using design values which take account of the relative uncertainities in each of the design elements and, in most situations, applying the greatest margin of safety to the soil strength parameters.

Contribution to discussion Session 1C

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Statistics, reliability and judgment

In the process of engineering design three items may prove to be useful:

1. Statistical analysis of data.
2. Reliability theory.
3. Engineering judgment.

It is essential that the separate identity of these three items is recognised, together with their differing merits and drawbacks.

Statistical analysis of data is useful for making comparisons between different sets of data, in order to determine how real any apparent differences might be. For example, statistical analysis may be used in comparing results from two different sites or from two different test methods used on the same site. Statistical analysis provides a means of quantifying these differences, though if the differences are of engineering significance this will usually be obvious without any detailed analysis.

In situations where it is required to derive design values for parameters, the use of simple statistical analysis or reliability theory may be very misleading, however.

If reliability theory is to be applied in geotechnical engineering with validity at least equal to that obtained by conventional approaches, then it must take account of all the available information normally used by engineers. This information is generally very complex and diverse and includes the following:

(a) Geological information about the site and its surroundings.
(b) Previous experience of the performance of structures in similar strata.
(c) The results of back analyses.
(d) The variability found in test results.
(e) An awareness of the paucity of data, since the amount of material sampled may be very small.
(f) Likely errors, both random and consistent, in the available test results.
(g) Conflicts and correlations between data from different types of tests.
(h) Knowledge of the physical limits on the possible ranges of parameters.
(i) Uncertainty or consistent errors associated with the method of analyses.
(j) Uncertainty in the boundaries between strata, the water levels, and other geometric parameters.
(k) Basic concepts and background information which affect the engineer's subconscious judgment, though he might have difficulty in remembering sources of information.

All these items are used, consciously or unconsciously by engineers. If these factors are not accommodated, then any new approach to analysis is bound to be inferior to our present procedures.

In principle it may be possible to use reliability theory to synthesise all this information in a rational way. In practice, however, it appears that the mathematical tools needed to produce this synthesis are not yet available, or, if they are, they are not yet accessible to engineers whose main discipline is the understanding of the physical nature of the ground. In the writer's opinion the best tool available at present to achieve the necessary synthesis of information is what is usually referred to as "engineering judgment". This is our most complex and most reliable probabilistic tool. The situation in a future generation may, of course, be quite different.

The importance of extreme values

In some design situations, such as the temporary stability of long embankments, failures in a small proportion of the total scheme are acceptable, or even desirable. However, in many other situations, including the design of foundations for most structures, it is necessary to ensure that failure is extremely unlikely. In considering the limit state of collapse, therefore, the designer's attention must be directed towards the worst that might credibly happen. Thus the parameter values required for limit state design will normally include some which are extremely adverse.

The required limit state values will often be so remote from the values that are most likely to occur in practice that extrapolation of data on the basis of means and standard deviations is not reasonable. Even if a probability density function incorporating all the relevant factors listed above could be derived, the area of interest would only be in the tail of the distribution, remote from the range of values identified by most of the data. In practical terms this means that it is the rare extremes, which may bear little relation to most of the test results, which must be considered most carefully in design.

In the writer's opinion it is therefore much more appropriate if the designer is directed to think directly about the worst credible values of the parameters. In doing this he is able to include all relevant information and to avoid unreasonable extrapolation from the mean of a frequency distribution.

Combinations of design values

Having considered directly the adverse range of values of parameters which could occur, together with a conservative analytical method, the designer requires a rational basis on which to select combinations of parameter values for calculations. At this point reliability theory may well have a useful role to play. A possible method was proposed by Simpson, Pappin and Croft (1981) and recent use of this method suggests that it provides a suitable framework for engineering decisions, whilst ensuring that the designer pays careful attention to the adverse limits of parameter values, calculation methods and design situations.

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