Influences on the Probability of Failures of Slopes

Influences sur la Probabilité de Rupture des Talus

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SYNOPSIS
On the basis of FRÖHLICH's method a stochastic variant is represented to express the static stability of slopes by probabilities of failure. The cohesion and the coefficient of fricton are random variables. The influence as of correlations between the shearing parameter as of assumption of normal, equal and empiric distribution respectivly of this parameters on the probability of failures is shown in an example.

INTRODUCTION

According to generally common definitions of safety factor, a slope with a coefficient of static stability of $S \geq 1.0$ proves in principle stable. However, practical experience again and again shows that even slopes which correspond to this coefficient fail.
Reasons for this are among others the neglecting of the stochastic qualitiy of the parameters, which are basic for the mechanic model, and incorrectnesses within the definition of the safety factor. Therefore the theory of sizing of different systems using the probability of failures begins to enter in engineering, and also in soil mechanics. Using the method of FRÖHLICH it is illustrated for a homogeneous slope, how the stability is to characterize when considering the stochastic character of the influencing parameters.
In extending the method of FRÖHLICH a safety distance S_M is formed as a difference between the parameters promoting and preventing a failure. This is the basis for the definition of probability of failure:

$$P_{Br} = P\ (S_M < 0). \qquad (1)$$

In this paper especially the influence of the used different density functions is shown.

STOCHASTIC MODIFICATION OF FRÖHLICH's METHOD

Only the influence of the shearing parameters on the probability of failure is investigated in the analysis represented here. It was possible to show in previous studies that the natural density of soil ρ_n does not have any important influence on the result because of its generally small variance. The same applies to the geometrical parameters slope height H and angle of slope β. No essentially higher probability of failures was yielded from calculations with variances of the height of slope $\sigma = \pm H/30$, $\sigma_\beta = \pm 0{,}33^\circ$ resp.

The following problems were investigated:

- the influence of correlations between the shearing parameters and
- the influence of assumption of normal distribution, equal distribution and empirical distribution of the shearing parameters on the probability of failures. Results of experimental investigations allow to utilize such distributions.

Investigations on the quantitative correlation of the shear parameters ϕ', c' showed, that there are strongly negative correlations between angle of internal friction and cohesion intercept, rangig from r = -0.8 to -0.9. For our material a correlation coefficient of r = -0.8 was found. The calculations were based on the equation for simple and linear correlation

$$r_{c',\tan\Phi'} = \frac{\sum_{i=1}^{n}(c'_i - \bar{c}')(\tan\Phi'_i - \overline{\tan\Phi'})}{\sqrt{\sum_{i=1}^{n}(c'_i - \bar{c}')\sum_{i=1}^{n}(\tan\Phi'_i - \overline{\tan\Phi'})}} \qquad (2)$$

c'_i, $\tan\phi'_i$ – sampling elements of c' and $\tan\phi'$;

$\bar{c}'$, $\overline{\tan\phi'}$ – mean values of c' and $\tan\phi'$.

The parameters necessary for the further calculations were taken from the statistic analysis (linear regression, χ^2 – adaption test for normal and equal distribution) of the shearing tests carried out in laboratory.

TABLE 1

Characteristic values of shear parameters for different assumptions of density function

normal distribution		
characteristic value	$\bar{m}$	σ^2
c' /kN/m²/	42.55	155.7
tanø'	0.5280	0.0065
equal distribution		
characteristic value	a	b
c' /kN/m²/	21.30	63.80
tanø'	0.3902	0.6658

The natural density of soil ρ_n = 2,04 g/cm³. On the basis of equation 1 the safety distance is to calculate according to

$$S_M = \frac{2\, r^2\, \alpha}{R\, .\, a} \cdot c' + \frac{r\, \cos\, \delta}{a}\, \tan\phi' - 1, \quad (3)$$

or

$$S_M = A_1 c' + A_2\, \tan\phi' - 1. \quad (4)$$

The used symbols are seen in figure 1.

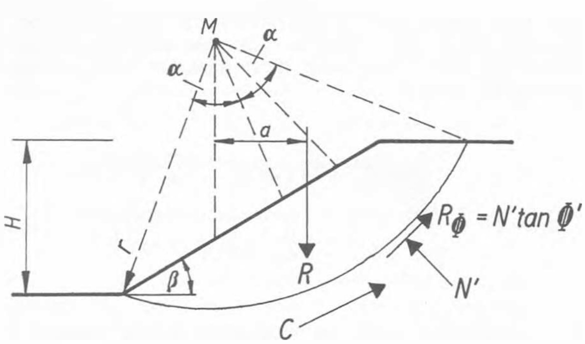

Fig. 1: Mechanical model for stability estimation

It has to be taken account that c' and tanø' are random variables with corresponding density functions, which parameters are available from table 1. The density function of the safety distance is a necessary factor for the calculation of the probability of failures. In our case, two random variables have to be summed up by a convolution of their density functions according to the rules of probability calculus.

For the condition of a normal distribution this was done for example by Stoyan et al. (1979) and Weber (1979). The resulting random variable is again normal distributed, having the mean value

$$\bar{S}_M = A_1\, \bar{c}' + A_2\, \overline{\tan\phi'} - 1 \quad (5)$$

and the variance

$$\sigma^2_{SM} = A_1^2\, \sigma_c^2 + A_2^2\, \sigma^2_{\tan\phi} - 2r\, A_1 A_2\, \sigma_c\, \sigma_{\tan\phi}. \quad (6)$$

In order to apply known tables the following normalization was used:

$$x = \frac{-\bar{S}_M}{\sigma_{SM}}, \quad P_{Br} = \Phi\, (x). \quad (7)$$

Where Φ denotes the standard normal density function. A similar practice is applied when assuming an equal distribution for the cohesion c' and the coefficient of friction tanø'. The values A_1 and A_2 of equation 4 are multiplied with the upper and lower limits b, a of the corresponding density function. The result are:

$$f(x_1) = \frac{1}{A_1\, b_{\tan\phi'} - A_2\, a_{\tan\phi'}} = \frac{1}{B - A}, \quad (8.1)$$

$$f(x_2) = \frac{1}{A_1\, b_{c'} - A_2\, a_{c'}} = \frac{1}{D - C}. \quad (8.2)$$

For this case solutions are only available for the convolution of independent values. The convolution of two equally distributed random variables leads to a trapezoidal density function.

1. $C + A > x > D + A: \quad f(x) = 0 \quad (9)$

2. $C + A \le x \le C + B: \quad f(x) = \frac{x - (C + A)}{(B - A)(D - C)}$

3. $B + C \le x \le A + D: \quad f(x) = \frac{1}{D - C}$

4. $A + D \le x \le B + D: \quad f(x) = \frac{B - D - x}{(B - A)\,(D - C)}.$

The corresponding probability of failures is resulting by integrating the density function from the corresponding lower limit up to x = 0, if necessary in intervalls.
The calculation of the probability of failures from an empiric distributions of the shearing parameter is based on the following equation

$$P_{Br} = n_i / n$$

n - total number of samples

n_i - number of samples with $S_M < 0$.

The shear parameters which belonged to the same sample where used in equation 4. By this existing dependences between c' and tanø' were taken into consideration.

EXAMPLE

In the discribed way probabilities of failures were calculated for four cases

(i) both random variables are normally distributed and independent (r=0);

(ii) the random variables are normally distributed and correlated (in our example r = -0.8);

(iii) the random variables are equally distributed

(iv) base of calculation is an empirical distribution

By using the characteristic data from table 1, the results for all four cases including the corresponding geometrical form of the slopes are listed in table 2. The height of the slope was H = const = 20.0 m.

We have the following results:

(i) Taking into account correlations between variables the probability of failures for equal geometrical values of slope and mean values $\bar{S}_M > 0$ is reduced (cases 2,4)

(ii) In contrary to the assumption of normal distributions, which are unlimited at both sides, conditions are so, that it seems real to use other, at both sides limited density functions (equal distribution, empiric distribution). With these assumption the probabilities of failures may be $P_{Br} = 0$ (case 3,4).

Limited distributions seem to represent a better approach than unlimited distributions.

(iii) Results show further, that even if there are safety distances $S_M > 0$

a failure of a slope is probably.

(iv) There are nearly no differences between the results when using the comparable variants of density functions in the range of larger slope angles ($\beta \geq 50^o$) and higher probabilities of failures (variants 1,3 and 2,4 resp.).

TABLE 2

Numerical results - probabilities of failures

β	A_1	A_2	$\bar{S}_M$	P^1_{Br}	P^2_{Br}	P^3_{Br}	P^4_{Br}
25^o	0,0242	2,587	1,396	≈ 0	≈ 0	0	0
30^o	0,0208	2,230	1,065	≈ 0	≈ 0	0	0
35^o	0,0204	1,810	0,825	0,0025	≈ 0	0	0
40^o	0,0179	1,681	0,643	0,0089	≈ 0	0	0
50^o	0,0170	1,267	0,392	0,0509	0,0047	0,0409	0
55^o	0,0164	1,115	0,287	0,0997	0,0228	0,1080	0,0333
60^o	0,0158	0,983	0,193	0,1820	0,0885	0,2010	0,1000
65^o	0,0153	0,866	0,107	0,2990	0,2236	0,3250	0,2000

CONCLUSION

A condition for using failure probability instead of safety factor for practical purposes seems to be a thourough investigation to get a knowledoe of the influences of different possible assumptions.
In this paper especcially analysed are the influences of different density functions for the random variables and correlative relations between the variables.

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